Bob Bush
27 Russell St.
W. Lafayette
Indiana, U.S.A.

1928

.3- Art. 29 to 32 inc.

28 - inc art 34. (work Peeb in class)
/28 -
/28 - Practice
/28 - Test.
/28 35 to 41. inc.

/28 - 42-43
/28 44-47
/28 48 to 52 inc.
/28 53 + 54
/28 55 & 56
28. " "
28
28 58-61 inc. 1/8/29: 94-95
62-65 inc. 1-9-29 96-97
8 68-69
8 70
8 71

/28 84-85
/28 85-87
/28 89-91
-28 92-93

STEEL STRUCTURES

STRESSES IN SIMPLE STRUCTURES

1929 – 1930

.3, 1929	Art.	127-128
.4, 1929		124-125
	"	129

STEEL STRUCTURES

STRESSES IN SIMPLE STRUCTURES

BY

LEONARD CHURCH URQUHART, C.E.

Professor in charge of Structural Engineering,
Cornell University

AND

CHARLES EDWARD O'ROURKE, C.E.

Assistant Professor of Structural Engineering
Cornell University

FIRST EDITION
SECOND IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK: 370 SEVENTH AVENUE
LONDON: 6 & 8 BOUVERIE ST., E. C. 4
1926

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE

In preparing this work, the authors have sought clearly to set forth the fundamentals of stress calculation in simple structures, without complicating the discussion by frequent reference to the economics of design. They feel that the student, in seeking to apply the principles of mechanics to the various types of trusses and loadings, should not be confused by such references. Those considerations will be discussed in another volume on design.

This volume is confined entirely to stresses in simple structures, for which both graphical and analytical analyses are given, except in those cases where the advantage of one method over the other is so great that the details of the more cumbersome method would serve no useful purpose. The graphical methods have, as far as possible, been separated from the analytical, so that each may be taken up separately if desired. The relation, however, between graphical and analytical methods is frequently mentioned. Both methods are illustrated by actual numerical problems in which the arithmetical work is made as simple as possible so that attention may be concentrated upon the theory involved.

The application of concentrated loads to bridge structures is first considered without the use of moment tables in order to avoid the danger of it becoming merely a mechanical operation with an incomplete understanding of the theory. The moment tables are somewhat different from those usually found, but are extremely simple of application. In order to be available for convenient use without the text, they are reprinted and included as a loose insert.

L. C. URQUHART.
C. E. O'ROURKE.

CORNELL UNIVERSITY
ITHACA, N. Y.
May, 1926

CONTENTS

STRESSES
IN
SIMPLE STRUCTURES

CHAPTER I

DEFINITIONS AND PRINCIPLES

1. A *force* is that which tends to change the motion, size, or shape of a body of matter. A force exerted by one body upon another is known as an external force with respect to the second body. A force acting in the interior of a body is known as an internal force. Forces may be further classified as concentrated or distributed forces. A concentrated force is one whose place of application is so small that it may be considered to be a point, while the place of application of a distributed force is an area. In many cases a distributed force may be considered as though it were concentrated at the center of the area of application.

In the structures to be considered, the external forces include all the loads, the weight of the structure itself, and the reactions, which act upon it and tend to distort it.

The tendency to distort, exerted by the external forces, is resisted by the internal forces, or, as they are usually designated, the internal *stresses*. These stresses are measured in the same units as the external forces, usually in pounds or tons. They may tend to pull portions of the structure apart, or push them together. The former tendency is known as *tension* and the latter as *compression*. Throughout this work tension will be considered as positive and compression as negative.

2. The distortion that is caused in any body or in any member of a structure by the application of external forces or by the development of the internal stresses is known as deformation or *strain*. It is measured in units of length, and is usually expressed as the change of length per unit of length. This change in length per unit of length is known as the *unit strain*.

3. For all elastic materials there is, within certain limits, a definite relation between stress and strain. That is, a certain unit stress in a material will cause a corresponding change in length per unit of length, and a unit stress of double that amount

will produce a change in length per unit of length of double that amount, etc. In all materials there is a limit of unit stress beyond which the strain increases more rapidly than the stress. This limit is known as the *elastic limit* of the material. For stresses below this limit the ratio of unit stress to unit strain is a constant or nearly so, and this ratio is the *modulus of elasticity* of the material.

Materials stressed below the elastic limit return to their original shape when the stress is removed. On the other hand, materials stressed beyond the elastic limit receive a certain amount of permanent distortion, and never return to their original form. Any such permanent change in the size and shape of a portion of a structure will, of course, set up additional stresses in other portions. It follows, therefore, that working stresses used in design should be well within the elastic limit, so that no such permanent distortion can take place.

4. Forces are *concurrent* when their lines of action meet in a point. They are *co-planar* when they lie in the same plane. A force is defined when its amount, its direction, and its point of application are known. In this volume, forces acting upward or to the right will be considered as positive, and those acting downward or to the left as negative.

The *moment of a force* about a point is its tendency to produce rotation about that point. It is measured by the product of the amount of the force into the perpendicular distance from the point to the line of action of the force. Tendency to produce rotation in a clockwise direction will be considered as positive, and tendency to produce rotation in a counter-clockwise direction as negative.

A *couple* is a pair of equal and opposite forces having different, but parallel lines of action.

The *resultant* of a system of forces is that single force which, when substituted for the original system of forces, produces the same effect upon the body as the original system of forces. A force equal and opposite to the resultant will balance or hold in equilibrium the original system. It is sometimes referred to as the *anti-resultant* or *equilibrant*. There is no resultant for a couple except another couple.

5. Conditions of Equilibrium. A system of forces acting upon a body is in *equilibrium* when the state of motion, size, or shape is not being changed.

In Fig. 1, P_1, P_2, and P_3 represent a system of concurrent forces applied at O. $X - X$ and $Y - Y$ are any two axes at right angles to each other. Each force may be resolved into components along the two axes. These components form a system equivalent to the original system. The algebraic sum of the horizontal components is indicated as ΣX, and that of the vertical components as ΣY. The resultant of ΣX and ΣY, namely, R, is also

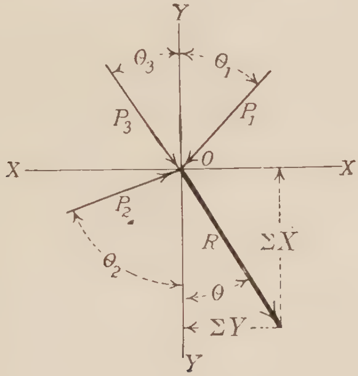


FIG. 1.

equal to the resultant of the original system of forces. R is evidently equal in amount to $\sqrt{\Sigma X^2 + \Sigma Y^2}$, and its direction is determined, since $\Sigma Y \div \Sigma X = \tan \theta$, while its point of application is at O.

For equilibrium, R must be zero, or $\sqrt{\Sigma X^2 + \Sigma Y^2}$ must be zero, which requires that

$$\Sigma X = 0$$

and

$$\Sigma Y = 0$$

These equations express the first two conditions of *static equilibrium*, (1) *the sum of the horizontal components equals zero;* and (2) *the sum of the vertical components equals zero.* These two conditions must exist in a system of concurrent forces which are in equilibrium.

In Fig. 2, P_1, P_2, and P_3 represent a system of non-concurrent forces. Each force may be resolved into components along the two axes. The amount of the resultant is $\sqrt{\Sigma X^2 + \Sigma Y^2}$ and its direction is determined, since $\Sigma Y \div \Sigma X = \tan \theta$. Its line of action is obtained by determining the distance from O that this

force must act in order to produce the same moment about O as the original system of forces, that is,

$$Ra = P_1a_1 + P_2a_2 + P_3a_3$$

In order to have equilibrium, a force equal and opposite to R and having the same line of action must be applied, so that about the point O

$$\Sigma M = 0$$

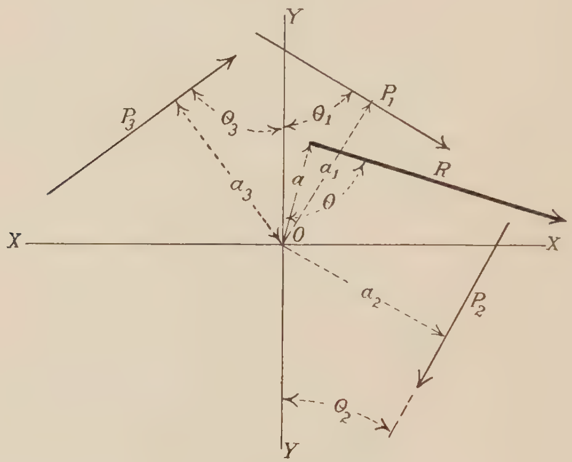


FIG. 2.

This equation expresses the third condition of *static equilibrium, i.e., the sum of the moments equals zero.* This condition, together with the first two, must exist in a system of non-concurrent forces, which are in equilibrium.

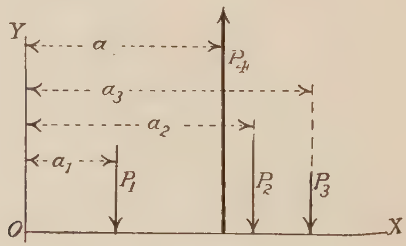


FIG. 3.

6. Parallel Forces in Equilibrium. In Fig. 3, P_1, P_2, and P_3 represent a system of parallel forces. In order to determine the amount and line of action of a force P_4 which will hold the system

in equilibrium, the second and third conditions of static equilibrium must be applied,

$$\Sigma Y = +P_4 - P_1 - P_2 - P_3 = 0$$

$$\Sigma M = +P_1a_1 + P_2a_2 + P_3a_3 - P_4a_4 = 0$$

In Fig. 4, the forces P_1 and P_2 are fully known, while only the lines of action of P_3 and P_4 are known. In this case the applica-

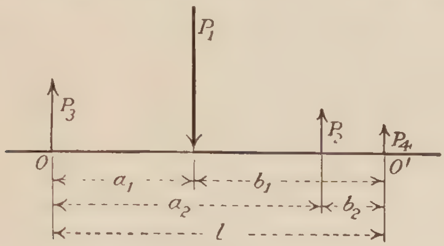


FIG. 4.

tion of the third condition of static equilibrium is sufficient to obtain the amounts and directions of P_3 and P_4. Taking moments of P_1, P_2, and P_4 about O

$$P_4 = \frac{P_1a_1 - P_2a_2}{l}$$

and moments about O' of P_1, P_2 and P_3 give

$$P_3 = \frac{P_2b_2 - P_1b_1}{l}$$

7. Center of Gravity. The *center of gravity* of a body is the point through which the resultant of all the gravity forces acting upon the body will pass. Since all gravity forces act in parallel lines, the problem of determining the center of gravity is one of determining the resultant of a system of parallel forces, as illustrated in Fig. 3. The term "center of gravity" is often applied to bodies which have no weight such as areas or lines. Such a determination is frequently necessary in structural analyses.

The center of gravity of the shaded area of Fig. 5 is obtained as follows: It is divided into the three rectangles, as shown, whose centers are their respective centers of gravity. Taking moments of their respective areas about any plane OY, the

distance $\overline{X}$, of the center of gravity of the whole area from OY is determined.

$$\overline{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$$

Similarly the distance

$$\overline{Y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A}$$

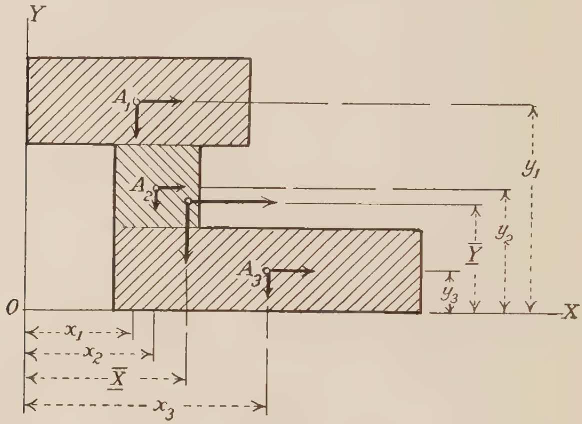


FIG. 5.

8. Moment of Inertia. The moment of inertia of a force with respect to any axis is the product of its magnitude into the square of its perpendicular distance from that axis. The total moment of inertia for any system of parallel forces is the sum of all such products for each force of the system. This sum will have its smallest value for an axis coinciding with the line of action of the resultant of the system of forces. For any other axis, the difference in the moment of inertia of the whole system is the product of the resultant into the square of the distance between the two axes.

Applying the above to the area of Fig. 5, the moment of inertia of the whole area about any horizontal axis will have its smallest value when the axis is taken through the center of gravity. The areas A_1, A_2, and A_3 are each made up of an infinite number of smaller areas, so that the sums of the products of these smaller areas into the distances of their centers from the centers of A_1, A_2, and A_3, respectively, have finite values. Hence the moment of inertia, I, of the whole area, about the axis through its center of gravity is $I_1 + A_1(\underline{\bar{x}} - x_1)^2 + I_2 + A_2(\underline{\bar{x}} - x_2)^2 + I_3 +$

$A_3(\bar{x} - x_3)^2$, where I_1, I_2, and I_3 represent the moments of inertia of the smaller areas about the respective centers of A_1, A_2, and A_3.

9. Radius of Gyration. The radius of gyration of a system of forces is the distance from the axis at which a force equal to the resultant of the system must act in order to give the same moment of inertia about that axis as the forces themselves.

Since the moment of inertia of an area has its smallest value about an axis through the center of gravity of the area, it follows that the least radius of gyration of an area is measured from an axis through the center of gravity of the area.

10. Beams. A beam is a single piece of material, or several pieces of the same or different materials, fastened together to act as a single piece, supported at one or more points, and sustaining loads or forces applied along its length. The simplest form of a beam is a single homogeneous piece, of uniform cross-section, laid horizontally on two supports, and subjected only to

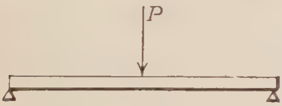


FIG. 6.

FIG. 6(*a*).

vertical loads or forces. Figure 6 is an illustration of a *simple beam.* Under the load P, the beam tends to assume the shape of Fig. 6(a). On the under or convex side, the longitudinal fibers of the material are in tension, while on the upper or concave side they are in compression. Figure 7(a) shows a *cantilever beam.* In this type of beam the compression of the fibers takes place in the lower portion and the tension takes place in the upper portion of the beam. Figure 7(b) shows an *overhanging beam* and Fig. 7(c) a *continuous beam.* In each of these latter types, except in cases of unusual loading, midway between supports the fibers are stressed as in a simple beam, compression occurring in the upper, and tension in the lower fibers, while over the supports the reverse is true and the fibers are stressed as in the cantilever beam.

In homogeneous beams of symmetrical cross-section the "common theory of flexure" assumes that, within the elastic limit, cross-sections remain plane surfaces during flexure, and that the relative amounts of stretching or contracting of the elementary fibers is proportional to their distances from a plane

midway between the extreme fibers of the section. This plane is known as the *neutral surface* and its intersection with the vertical plane, in which the longitudinal axis of the beam is contained, is known as the *neutral axis*.

FIG. 7(*a*).

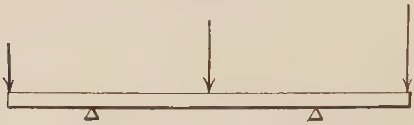


FIG. 7(*b*).

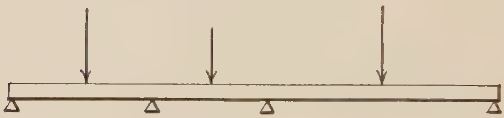


FIG. 7(*c*).

In homogeneous beams of unsymmetrical cross-section the neutral axis lies in the center of gravity of the cross-section, and there is often considerable difference in the distances from the axis to the extreme fibers on the opposite sides. Since the stress in every other fiber is less than that in the outermost fiber

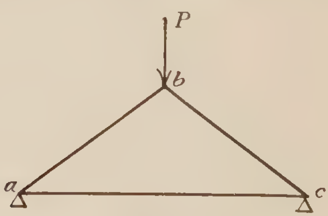


FIG. 8.

under the above assumption, it follows that the most economical homogeneous beam sections are symmetrical about their neutral axes and have the major portion of their sections concentrated as closely as possible to the outermost fiber.

11. Trusses. A truss is a jointed structure, designed to act as a beam. Each separate member of a truss is usually subjected

to stress only in the direction of its length. Figure 8 represents the simplest form of a truss. With the load P applied as shown, the inclined members ab and bc are in compression and the horizontal member ac is in tension.

Since the triangle is the only polygonal figure whose shape cannot be changed without altering the length of its sides, a truss must be composed of an assembly of triangles in order to form a rigid framework. It is not necessary that these triangles be

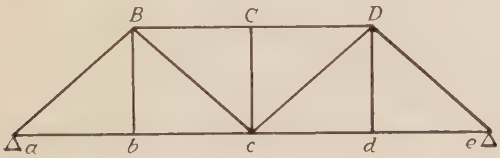


FIG. 9.

similar or symmetrical with one another. Figure 10 represents the truss idea as well as Fig. 9.

In order strictly to maintain the truss principle, all external forces or loads must be applied to the truss at the joints. If loads are applied elsewhere flexural stresses result. It is true that in horizontal or inclined members, the weights of the members themselves produce flexural stresses, but in simple trusses, these stresses are so small that they are usually neglected in design.

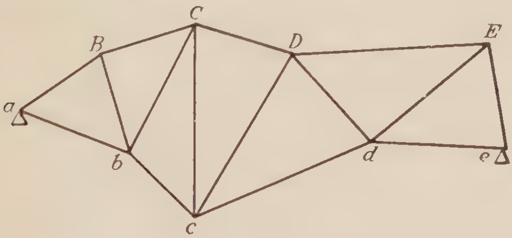


FIG. 10.

A *simple truss* is one which is supported only at its ends, and under vertical loads produces only vertical pressures or reactions upon its supports.

In general, all members in compression are known as *struts*, while those in tension are known as *ties*. The *upper chord* is the upper series of members, *i.e.*, in Fig. 8 the upper chord is abc, in Fig. 9, $aBCDe$, and in Fig. 10, $aBCDEe$. Similarly, the

lower chord is the lower series of members *ac* in Fig. 8 and *abcde* in Figs. 9 and 10. In a simple truss, under vertical loads, the upper chord is in compression and the lower chord in tension. Members connecting the upper and lower chords are designated as *web members.* They may be either struts or ties depending upon their relative positions with reference to one another. In a truss, such as Fig. 9, where the main portions of the upper chord are horizontal, or nearly so, the members *aB* and *Ee* are often referred to as the *end posts;* other inclined web members are called *diagonals,* while the vertical web members are designated as *posts* or *suspenders,* depending upon whether they are in compression or tension, respectively.

The points where the members meet are the joints. At these joints there are pins, rivets and plates, or other connections which are subjected to shearing stresses, or to combined stresses of shear, compression, and flexure. The spaces between the chord joints are called *panels,* and the joints themselves are known as *panel points* or *apexes.*

Application of the Conditions of Equilibrium

12. The external forces acting on a structure are due to the weight of the structure itself, to the loads sustained, and to the pressures or reactions occurring at the points of support. The loads sustained are usually known and the weight of the structure is assumed. The unknown reactions are then determined by applying the principles of Art. 5.

Since all portions of a structure are in equilibrium, the conditions of equilibrium may be applied to any portion of the structure in order to determine the internal stresses. In some cases the application of the first two conditions, $\Sigma X = 0$, and $\Sigma Y = 0$ is sufficient to determine all of the stresses in the members of a structure. This method is called the *resolution of forces.* In other cases the application of the third condition, $\Sigma M = 0$, is necessary or furnishes a simpler solution. This method is known as the *method of moments.*

13. Determination of Reactions. The reactions of a structure subjected to vertical loads only and resting freely on two supports present a condition similar to that of Fig. 4. With inclined loads or reactions, one or both of which are fixed, the determination of the value of the reactions requires the solution of more than two

moment equations. The additional work necessary may take the form of a third moment equation or one equation of resolution may be used.

The truss of Fig. 11 supports the inclined load P_1 and the vertical load P_2, applied in the positions indicated in the figure. At the left end the truss rests on rollers and is free to move horizontally; hence the left reaction is vertical. The right end of the truss is fixed and the right reaction has a horizontal component equal and opposite to the horizontal component of P_1. With the center of moments at a

$$P_1 s_1 + \frac{P_2 l}{2} - R_V l = 0, \text{ from which } R_V = \frac{P_1 s_1}{l} + \frac{P_2}{2}$$

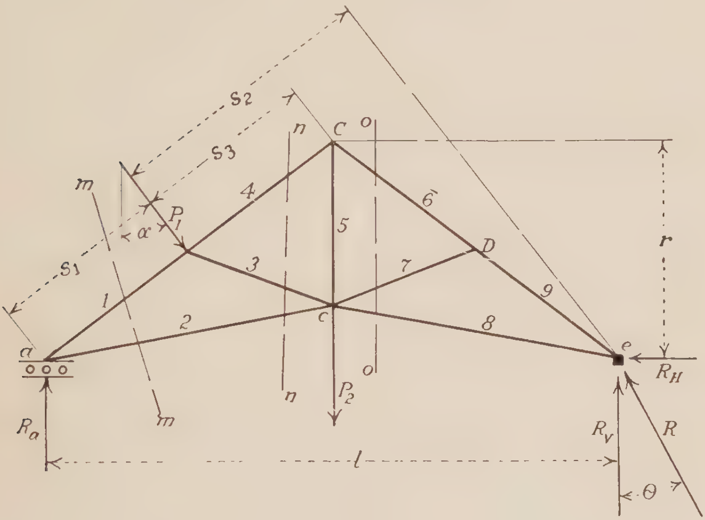


FIG. 11.

With the center of moments at e

$$R_a l - P_1 s_2 - \frac{P_2 l}{2} = 0, \text{ from which } R_a = \frac{P_1 s_2}{l} + \frac{P_2}{2}.$$

The determination of R_H may be made either by taking moments about any point, such as C, not in the same horizontal plane as a and e, and using the values of R_a and R_V obtained above, giving

$$\frac{R_a l}{2} - P_1 s_3 - \frac{R_V l}{2} + R_H r = 0$$

or, since P_1 is the only external load which has a horizontal component, $\Sigma X = 0$ gives

$$P_1 \sin \alpha - R_H = 0$$

In the first instance the center of moments could have been taken at any point whose location with reference to the lines of action of the other forces was known, but the point C was chosen since it was in the line of action of one of the forces P_2, and hence eliminated that force from the computations. Since in most cases, $\sin \alpha$ is easily determined, the second method is the simpler solution. R_V and R_H having been determined

$$R = \sqrt{R_V^2 + R_H^2}$$

and

$$\theta = \tan^{-1} \frac{R_H}{R_V}$$

14. Determination of Stresses. The reactions having been determined, any portion of the structure may be isolated from the remainder, and the conditions of equilibrium applied to determine the stresses in the various members. Two methods may be used: (*a*) the *method of successive joints*, or (*b*) the *method of sections.*

a. The Method of Successive Joints. In the truss of Fig. 11, whose reactions were determined in Art. 13, S_1, S_2, S_3, etc. represent the internal stresses in the respective members of 1, 2, 3, etc. and θ_1, θ_2, θ_3, etc. the angles between the respective members and the vertical. Isolating the joint a from the remainder of the structure (Fig. 11(a)) there are three concurrent forces acting at the point and the two equations of equilibrium may be written as follows:

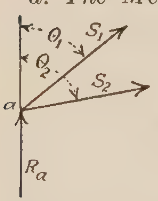


FIG. 11(a).

$$S_1 \sin \theta_1 + S_2 \sin \theta_2 = 0$$
$$S_1 \cos \theta_1 + S_2 \cos \theta_2 + R_a = 0$$

from which S_1 and S_2 may be determined. If the direction of the unknown stress is assumed as acting *away from* the section, the sign of the resulting values, as determined by the solution of the necessary equations, will indicate the character of the stresses, *i.e.*, a positive result indicates tension and a negative result, compression.

Next, taking joint B (Fig. 11(b)), S_1, determined above, and P_1 are known and the two equations are:

$$S_1 \sin \theta_1 + S_3 \sin \theta_3 + P_1 \sin \alpha + S_4 \sin \theta_4 = 0$$

and

$$S_1 \cos \theta_1 - S_3 \cos \theta_3 + S_4 \cos \theta_4 - P_1 \cos \alpha = 0$$

from which S_3 and S_4 may be computed.

Similarly at joint C (Fig. 11(c))

$$S_4 \sin \theta_4 + S_6 \sin \theta_6 = 0$$

and

$$S_4 \cos \theta_4 - S_5 - S_6 \cos \theta_6 = 0$$

The equations for joints C, D, and e are similar.

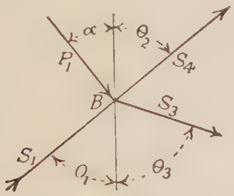


Fig. 11(b).

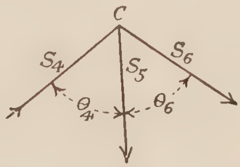


Fig. 11(c).

The computations at joint B can, in this example, be considerably simplified, by considering aC as the $X - X$ axis. Since P_1 is perpendicular to aC, $\Sigma X = 0$ gives

$$S_1 + S_4 + S_3 \sin (\theta_1 + \theta_3 - 90°) = 0$$

and $\Sigma Y = 0$

$$-P_1 - S_3 \cos (\theta_1 + \theta_3 - 90°) = 0$$

Symmetry of trusses, both as to form and loading, often reduces and simplifies the computations, and a full conception of the geometric and trigonometric relations involved will often lessen the labor.

b. The Method of Sections. In this method, instead of passing circular sections around successive joints, a section is passed through the structure cutting the members whose stresses are desired. The equations of equilibrium may be applied to either portion in order to determine the stresses in the members. Since each section usually contains more than one joint, there is often more than one external force acting on the portion of the structure considered, and the internal unknown stresses are not

necessarily concurrent. Since, however, the structure is in equilibrium, each portion must also be in equilibrium, so that in any section cut, the internal stresses hold in equilibrium the external forces upon either side of that section.

In the truss of Fig. 11, the stresses in the members 1 and 2 are found by passing a section *m-m,* cutting those members. Considering the portion of the structure on the left of the section, the equations are identical with those written for Fig. 11(a), on page 12. Passing a section *n-n* through members 4, 3, and

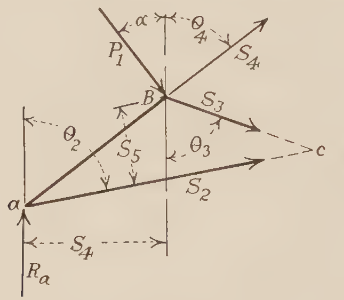


FIG. 11(d).

2, and considering the portion of the structure on the left (Fig. 11(d)), equilibrium equations may be written as follows:

$$\Sigma X = 0 \quad P_1 \sin \alpha + S_4 \sin \theta_4 + S_3 \sin \theta_3 + S_2 \sin \theta_2 = 0$$
$$\Sigma Y = 0 \quad R_a - P_1 \cos \alpha + S_4 \cos \theta_4 - S_3 \cos \theta_3 + S_2 \cos \theta_2 = 0$$
$$\Sigma M = 0 \text{ about } B \quad R_a s_4 - S_2 s_5 = 0$$

In the equation for $\Sigma M = 0$, the center of moments is taken at the intersection of two of the members whose stresses are unknown, in order to eliminate them from the equation. By substituting in each of the first two equations the value of S_2, obtained from the third equation, they may be solved simultaneously for the desired values of S_3 and S_4. In the third equation the center of moments could just as well have been taken at c, the intersection of members 2 and 3, and the value of S_4 determined in terms of the external forces.

The same results can also be obtained by considering the portion of the truss on the right of the section. In the example here given, the equations are slightly more involved than those for the portion on the left, and the latter are to be preferred. On the other hand, the stresses in members 6, 7, and 8 can be more easily determined by considering the section to the right of *o-o,*

as the equilibrium equations contain the functions of only one external force R, instead of those of the three external forces R_a, P_1, and P_2, if the section on the left is taken.

In most trusses a combination of the method of successive joints and that of sections will offer a simpler solution than either method alone.

15. Example of Previous Analyses. The truss of Fig. 12 has a span of 32 ft., a rise of 12 ft., and sustains loads as shown. The right end is fixed, but the left end is free to move horizontally. The lower chord is horizontal and member 3 is normal to the upper chord.

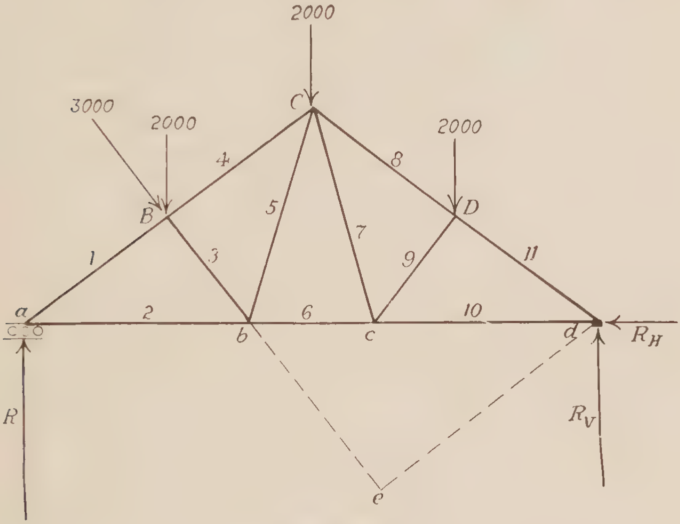


Fig. 12.

As in the previous articles, θ_1, θ_2, θ_3 etc. will be the designation of the angles which the respective members make with the vertical.

$$\sin\theta_1 = \sin\theta_4 = \sin\theta_8 = \sin\theta_{11} = 0.800$$
$$\cos\theta_1 = \cos\theta_4 = \cos\theta_8 = \cos\theta_{11} = 0.600$$
$$\sin\theta_3 = \sin\theta_9 = 0.600$$
$$\cos\theta_3 = \cos\theta_9 = 0.800$$
$$\sin\theta_5 = \sin\theta_7 = 0.280$$
$$\cos\theta_5 = \cos\theta_7 = 0.960$$

From similar triangles, de, the lever arm of the inclined load about d, is 15.6 ft.

Considering the whole truss $\Sigma M = 0$ about a gives

$3000 \times 10 + 2000(8 + 16 + 24) - R_V \times 32 = 0$, from which

$$R_V = 3940$$

$$\Sigma M = 0 \text{ about } d \text{ gives}$$

$-3000 \times 15.6 - 2000(8 + 16 + 24) + R \times 32 = 0$, from which

$$R = 4460$$

$$\Sigma X = 0 \text{ gives}$$

$$3000 \sin \theta_3 - R_H = 0, \text{ from which } R_H = 1800$$

Isolating the joint a, $\Sigma Y = 0$ gives

$$4460 + S_1 \cos \theta_1 = 0, \text{ from which } S_1 = -7740$$

and

$$\Sigma X = 0 \text{ gives}$$

$$S_2 - 7440 \sin \theta_1 = 0, \text{ from which } S_2 = +5950$$

Isolating the joint B and considering aBC as the $X - X$ axis $\Sigma Y = 0$ gives

$$-3000 - 2000 \cos \theta_3 - S_3 = 0, \text{ from which } S_3 = -4600$$

Cutting a section through members 4, 5, 6, and considering the portion of the truss on the left of the section, $\Sigma M = 0$ about C gives

$4460 \times 16 - 3000 \times 10 - 2000 \times 8 - S_6 \times 12 = 0$ from which

$$S_6 = +2120$$

$$\Sigma Y = 0 \quad \text{gives}$$

$$S_4 \cos \theta_4 + S_5 \cos \theta_5 + 4460 - 2000 - 3000 \cos \theta_3 = 0$$

and

$$\Sigma X = 0 \quad \text{gives}$$

$$S_4 \sin \theta_4 + S_5 \cos \theta_5 + 2120 + 3000 \sin \theta_3 = 0$$

Solving these two equations simultaneously,

$$S_4 = -6240 \text{ and } S_5 = +3830$$

Cutting a section through members 6, 7, and 8 and considering the portion of the truss on the right of the section

$$\Sigma Y = 0 \text{ gives}$$

$$S_7 \cos \theta_7 + S_8 \cos \theta_8 + 3940 - 2000 = 0$$

and

$$\Sigma X = 0 \text{ gives}$$

$-S_7 \sin \theta_7 - S_8 \sin \theta_8 - 2120 - 1800 = 0$ from which

$$S_7 = +1330 \text{ and } S_8 = -5370$$

Isolating the joint D, $S_9 = -1600$

At joint d

$$\Sigma Y = 0 \text{ gives}$$

$$+3940 + S_{11} \cos \theta_{11} = 0 \text{ and } S_{11} = -6570$$

$$\Sigma X = 0 \text{ gives}$$

$$6570 \sin \theta_{11} - 1800 - S_{10} = 0 \text{ and } S_{10} = +3460$$

16. Analysis of Beams. In order to determine the maximum fiber stresses existing in beams subjected to external forces, the fundamental conditions of equilibrium must be applied to the whole beam to determine the reactions, and to portions of the beam to determine the stresses. The actual distribution of the internal stresses in a beam will not be considered here, but will be taken up in another volume having to do with design.

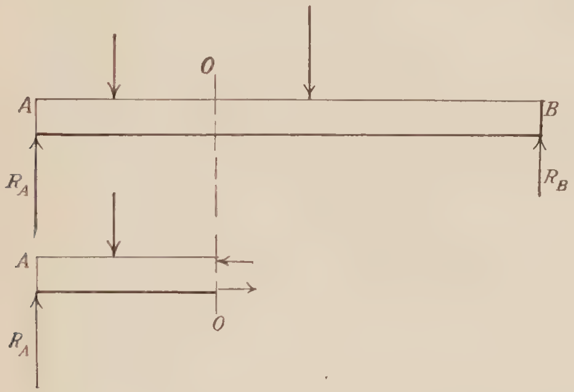


FIG. 13.

Figure 13 represents a simple beam loaded in any manner. The reactions R_A and R_B are determined by applying the principles of Fig. 4, *i.e.*, R_A is found by taking $\Sigma M = 0$ about B and R_B by taking $\Sigma M = 0$ about A. At any section of the beam, the sum of the moments of the external forces acting on either side of the section about that section is known as the *bending moment.* This bending moment is balanced by the moment of the stress couple at the section, shown in the figure, which is known as the *resisting moment* of the beam. The bending moment is called positive when it causes convexity downward, as illustrated in Fig. 6(a), which produces compression in the upper, and tension in the lower, fibers of the beam. Bending moments causing tension in the upper, and compression in the lower, fibers are called

negative. Such moments exist in a beam such as that of Fig. 7(*a*) and in portions of the beams of Figs. 7(*b*) and 7(*c*). In a simple beam all downward forces cause positive bending moment.

At any section of the beam, such as *O-O*, the sum of the vertical components of the external forces on either side of the section is known as the *shear* on the section. Since the $\Sigma Y = 0$

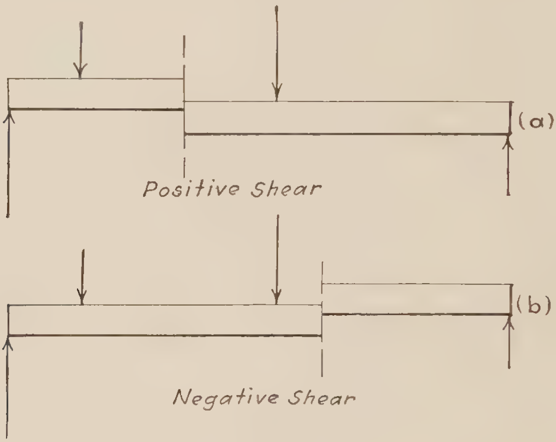


FIG. 14.

for any portion or for the whole of the beam, the sign of this shear is plus for one section and minus for the other. It is usual to consider the section on the left and to give the shear the same sign as the resultant of the external forces acting on that portion. The shear, therefore is positive when the left-hand portion tends to move upward (Fig. 14(*a*)) and negative when the reverse is true (Fig. 14(*b*)).

CHAPTER II

FUNDAMENTAL PRINCIPLES OF GRAPHIC STATICS

17. Graphical Representation and Graphic Statics. Graphical representation has long been used to make visible the results of an investigation—to present a compact picture of already known facts, from which picture certain analytical relations and conclusions may be deduced. When used in this manner, it may be defined as the geometrical expression of analytical knowledge.

To the designing engineer, however, the greatest value of graphical representation lies in the fact that by the application of proved geometrical constructions to the linear representations of numerical quantities, graphical solutions of structural problems may be obtained, in some instances with much less effort than is required in the corresponding mathematical solutions.

The relative advantage of the graphical solution as compared with the analytical varies as the irregularity of the given relations. Unequal forces acting at varying spacings and in irregular directions complicate the analytical solution of a primarily simple problem, but are matters of indifference in the graphical solution.

The theory involved in such graphical solutions when applied to statical engineering problems, *i.e.* problems dealing with structures every part of which (and therefore the structure as a whole) is at rest when the structure is subjected to the action of external forces, is called *graphic statics.*

Before proceeding with a description of the methods used in analyzing graphically any given structure, certain general notations, fundamental principles and geometric constructions, will be explained. The discussion will be limited to forces acting in one plane only, the condition existing in practically all of the modern engineering structures to which the graphical solution is desirably applicable.

18. Representation of a Force. Since a force is completely determined when it is known in amount, in direction, and in point of application, any force may be represented by the length, direction, and position of a straight line. The length

of the line is fixed by the relation between the amount of the given force and a prearranged unit of force. If the unit of force be taken as 1 ton to the inch, a line 3 in. long represents a force of 3 tons.

The graphical representation of a force may be designated in several different ways. In the following discussion, two general means of identification will be used. (1) The line representing a force will be marked with the letter P, followed by a subscript in some instances (see Chapter I and Fig. 15). (2) Each extremity of the line will be indicated by a letter, and the force referred

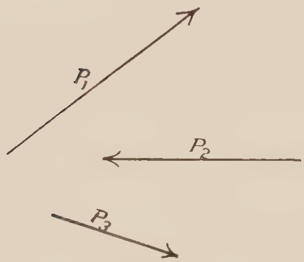


FIG. 15.

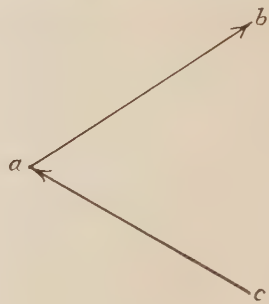


FIG. 16.

to by means of these letters (see Fig. 16). The direction of the force will be given, where necessary, by an arrow on the line representing the force. In (2) the order of the letters given in referring to a force will indicate its direction, *i.e.*, in Fig. 16, referring to the upper force as *ab* indicates that it acts from *a* toward *b*, while reference to the lower force as *ca* indicates that it acts from *c* toward *a*. A diagram, such as Fig. 15 or Fig. 16, which shows the analytical relation of the given forces in any problem—their positions, directions, and amounts—is called a *force diagram.*

19. Composition and Resolution of Forces. In Fig. 17, P_1 and P_2 represent two forces acting at the point *a*. The amount of the resultant of the two forces is given by the length of the diagonal of the parallelogram *acbd* constructed upon the two given forces as sides. The geometric proof of this conclusion is evident from the figure.

A single force applied at point *a*, acting in the direction of point *b*, of an amount represented by the length of the line *ab*, will replace P_1 and P_2 insofar as accomplishing work is con-

cerned. A force of the same amount but acting in the opposite direction will hold the forces P_1 and P_2 in equilibrium, *i.e.* at rest. If the force represented by the line ab acts from a toward b it is the resultant, and if it acts from b toward a it is the anti-resultant of the given forces. If another force P_3 were acting at point a, it could be combined in a similar manner with the resultant of P_1 and P_2 to determine the resultant of P_1, P_2, and P_3. The

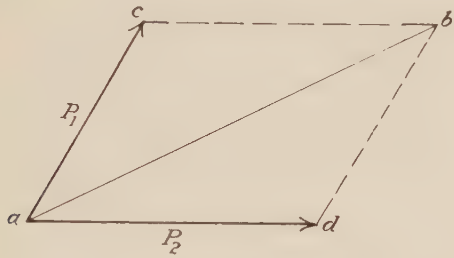


Fig. 17.

process of replacing forces P_1 and P_2 (and more if present) with the single force ab is called the *composition of forces*—two (or more) forces are combined into one. On the other hand, any single force may be resolved into two components acting in any given directions. The solution of this problem, called the *resolution of forces*, is the reverse of that employed in the composition of forces.

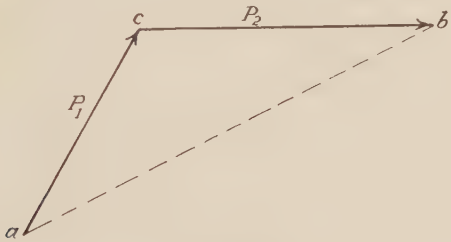


Fig. 17(a).

20. The Force Triangle. A study of Fig. 17 shows that in order to determine the amount and direction of the resultant of P_1 and P_2 it is not necessary to construct the complete parallelogram $acbd$. From c of the force P_1 (Fig. 17(a)), a line cb is drawn parallel and equal to P_2. A force equal to that represented by the length of the line ab, applied in the direction from a to b, will produce the same effect as the two original forces, and a

similar force applied in the direction from b to a will hold the original forces in equilibrium. The triangle acb is called the *force triangle* and represents graphically the relation between the forces P_1, P_2, and ab.

Thus it is seen that if in addition to forces P_1 and P_2, a third force ba is applied at point a in the direction from b to a, these three forces will be in equilibrium. The point a will then remain at rest. It may now be concluded that *if three forces meeting in a point are in equilibrium, they will form a closed force triangle,* that is, not only one in which the three lines parallel and equal to the three forces, respectively, form a closed, three-sided figure, but one in which the arrows representing the direction of the forces all point in the same direction around the triangle.

By applying this principle, any one of the three forces may be found in amount and direction if the other two are fully known,

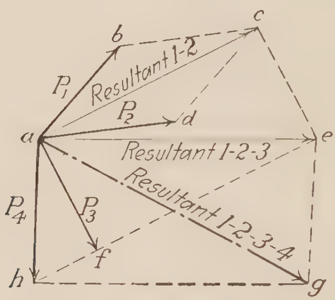


FIG. 18.

or any two of the forces may be found in amount if their directions are given and the third force is fully known. The force triangle is the basic principle on which the graphical analysis of structures depends.

21. The Force Polygon. If instead of two forces, three or more were acting at point a, the resultant of these forces could be found by the successive application of the principle of the force triangle; two forces are first composed into one and this one combined with the third force; this process is continued until the last force is combined with the resultant of all the other forces to determine the resultant of the entire group.

Figure 18 represents four forces, P_1, P_2, P_3, and P_4, acting in a common point a. Assuming that they are not in equilibrium, let it be required to find the force necessary to produce a condi-

tion of equilibrium. Forces P_1 and P_2 are combined as described in the preceding article by completing the parallelogram *abcd*. The resultant of these two forces is the force *ac* acting in the direction from *a* to *c*. It is now assumed that P_1 and P_2 have been replaced with the one force *ac* and there are but three forces acting at *a*, that is, P_3, P_4, and *ac*. The force *ac* is combined with P_3 by constructing the parallelogram *acef*, the diagonal *ae* being the force necessary to replace *ac* and P_3. The forces *ae* and P_4 are then combined to find *ag*, the resultant of the four original forces.

A study of Fig. 18 shows that it is not necessary to complete all of the parallelograms in order to find *ag*. The polygon *abceg*, four of whose sides are respectively parallel and equal to forces P_1, P_2, P_3, and P_4, gives the required information. This

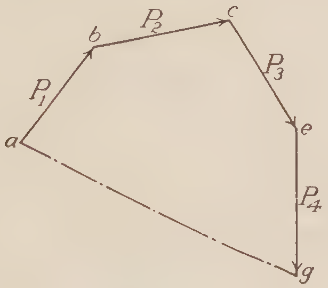


Fig. 18(*a*).

polygon is repeated for clearness in Fig. 18(*a*). The line drawn from the beginning *a* to the end *g* of the last known line *eg* of the polygon, contrary to the order of the forces, gives the intensity and direction of the resultant of the four given forces. A force of equal intensity applied in the opposite direction, from *g* to *a* (in the direction of the forces), would hold the four given forces in equilibrium. Point *a* would remain at rest under the action of five forces, P_1, P_2, P_3, P_4, and *ga*.

The order in which the forces are combined does not affect the final resultant. In Fig. 18, P_1 and P_4 could be first combined, then P_3 and finally P_2 could be considered in that order. The final result would be the same as that obtained in Fig. 18, but the polygon would be of different shape, as shown in Fig. 19. By taking the forces in regular order, as in Fig. 18, a polygon without intersecting lines is obtained. Obviously this is more desir-

able than one in which the sides cross each other as in Fig. 19. Intersecting lines cannot always be avoided, but by combining the forces in regular order a minimum number of intersections occurs.

The polygon *abceg*, formed by the successive laying off of lines parallel and equal to the given forces is called the *force polygon*—the force triangle expanded. By its use, any one of a number of forces acting in a point in a static structure may be determined in amount and direction if the others are completely known, or any two may be found in amount if they are known in direction and the other forces in amount and direction. The force polygon serves a similar purpose for a series of forces not meeting in a point; however, it does not in this case determine

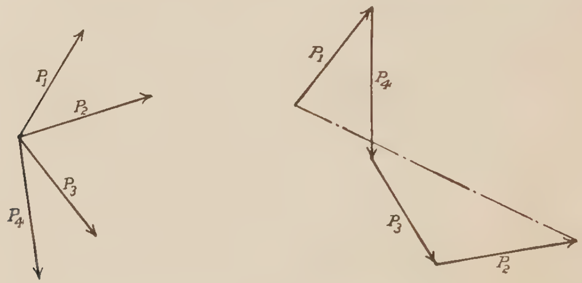


Fig. 19.

the point of application of the unknown force. This is considered separately in Art. 23.

22. Application of the Force Polygon. In order to illustrate the application of the principle of the force polygon to the determination of the internal forces or stresses in the members of a static structure, let it be required to find the stresses in the component parts of the crane truss shown in Fig. 20. This truss consists of a vertical post *bd*, supporting a boom *dc*, from which the external load P_1 is suspended. The boom is anchored to the post by the tie rod *bc*, and the post is in turn anchored at its upper end by the back-stay *ab* fastened to the ground.

Cut a section in such a manner as to completely isolate the joint *c* from the rest of the truss. The members *bc* and *dc* (designated by the letters T_2 and T_1) cut by this section must be replaced by means of the stresses S_2 and S_1 in these members, respectively, as shown in the upper right hand portion of Fig. 20.

Since this structure is in equilibrium, the stresses S_1 and S_2 and the external load P_1, a system of three forces meeting at a point which is at rest, must form a closed force triangle. Hence to find S_1 and S_2 construct the triangle *mnk* by laying off the line *mn* equal and parallel to P_1, and drawing through its two extremities *m* and *n*, lines parallel to T_2 and T_1, respectively. The intersection of these two lines fixes the amounts of the stresses S_2, represented by *mk*, and S_1, represented by *nk*.

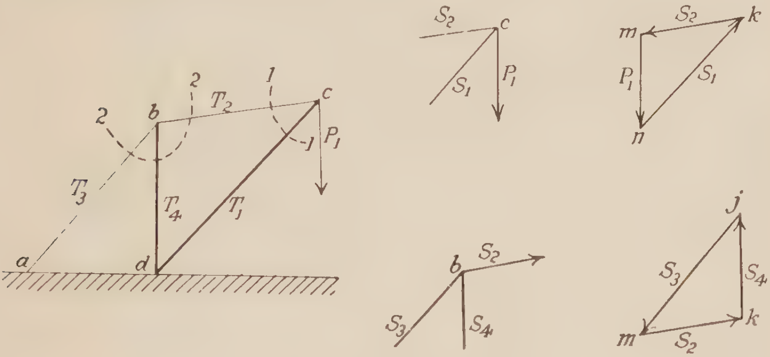


FIG. 20.

Since the point *c* is at rest under the action of the forces S_1, S_2, and P_1, the direction of these forces must be such that they all point the same way around the force triangle. The direction of P_1 is known to be downward, therefore S_1 acts from *n* to *k*, and S_2 from *k* to *m*. Transferring these directions to the truss diagram, the stress in *bc* acts away from *c*, and the stress in *dc* acts toward *c*. The member *bc* is therefore in tension since it is pulling away from the joint, and the member *dc* is in compression since it is pressing against the joint.

Passing now to joint *b*, a section is cut, completely isolating joint *b* from the rest of the truss. Since the stress in *bc* is known both in amount and direction, a force triangle may be drawn, which gives the amounts and directions[1] of the stresses S_3 and S_4 in the members T_3 and T_4, respectively. In this force triangle for joint *b*, the stress S_2 in member *bc* acts away from joint *b*—the opposite direction from that used when joint *c* was consid-

[1] In any of the members of the truss the line of action of the stress is of necessity along the member itself. By "direction" in this discussion is therefore meant the direction in which the stress acts along the member, *i.e.*, toward, or away from, the joint considered.

ered.[2] This force triangle shows not only the amounts of the stresses S_3 and S_4 in the members *ab* and *bd* respectively, but also indicates that *ab* is in tension (the stress S_3 acts away from joint *b*) and *bd* is in compression (the stress S_4 acts toward joint *b*).

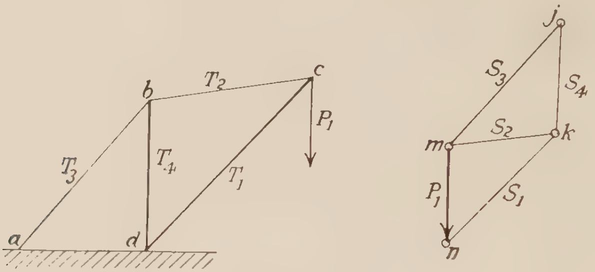


Fig. 20(*a*).

Instead of drawing the two force triangles required in the complete solution of this crane truss as separate figures, the work may be somewhat simplified by uniting them in one figure. This has been done in Fig. 20(*a*) by drawing the second force triangle *mkj* on the side *mk* of the first triangle. The truss diagram has been repeated in this figure for convenience of reference. For the

[2] This is made clear by a discussion of a simple column under direct load (Fig. I). In order to keep the column in equilibrium, the bearing surface under the column must exert an upward pressure equal in amount

Fig. I.

to the downward load. The tendency is therefore to shorten the column—to compress it. The column fibers must resist this compression, otherwise the column will yield. Therefore the internal stress in the column acts in the opposite direction from the external forces; upward at the upper joint and downward at the lower joint. It is this internal stress that is required in the problem under consideration.

reasons stated above, in the triangle *mnk*, S_2 acts from *k* to *m*, while in the triangle *mkj*, S_2 acts from *m* to *k*. This arrangement of the graphic solution is desirable from the standpoint of both accuracy and clearness and will be used in all succeeding problems.

The stresses in the members of the truss shown in Fig. 21 are

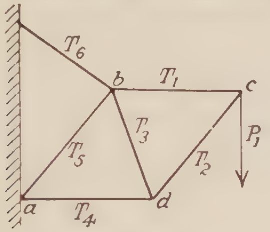


FIG. 21.

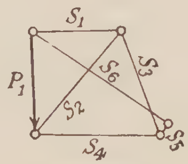


FIG. 21(*a*).

determined in a manner similar to that described for the crane truss in the preceding problem. The complete solution is given in Fig. 21(*a*). Members T_1, T_3, and T_6 are in tension, and members T_2, T_4, and T_5 are in compression. The order in which the joints must be considered is as follows: first *c*, second *d*, and third *b*. Joint *b* could not be solved before joint *d* because there

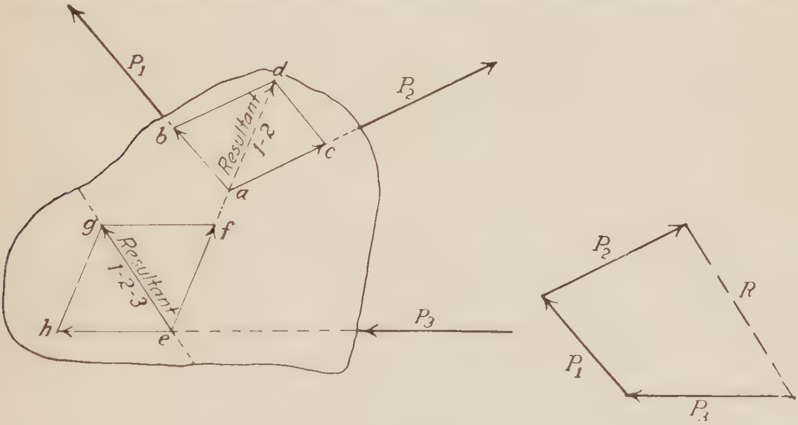


FIG. 22.

would be too many unknown quantities (more than two) in the construction.

23. Non-concurrent Forces. The preceding discussion has been limited to a series of forces acting in a common point. Figure 22 represents a system of three *non-concurrent* forces. It is assumed that these forces are not so balanced as to create a condition of equilibrium in the body on which they act. The force polygon, constructed by drawing lines equal and parallel

to the given forces, determines both the amount and direction of the resultant R of the three forces, but does not fix its point of application. This force polygon is shown in the right portion of Fig. 22.

One method of determining this point of application[3] is by means of the rather cumbersome construction indicated in the beginning of Art. 21. In Fig. 22 this construction is carried on as follows: The forces P_1 and P_2 are produced to their intersection at a. The parallelogram $abdc$ is then drawn, as explained in Art. 21, to determine the resultant ad of P_1 and P_2. The force ad is produced to intersect P_3 at point e. The diagonal eg of the parallelogram $efgh$ constructed on the sides ef and eh parallel and equal to ad and P_3, respectively, determines the amount, direction, and point of application of the resultant of ef and P_3, and hence the resultant of P_1, P_2, and P_3. A force equal in amount to eg but acting in the opposite direction, that is, from g to e, would produce a condition of equilibrium if applied at any point along the prolongation of the line eg. The above construction has been reduced in scale for clearness.

A neater, quicker, and more accurate method of arriving at the same conclusion is described in the following article.

24. The Equilibrium Polygon. The method used in the preceding article for determining the resultant of a group of forces not meeting in a common point is not applicable if the various points of intersection involved in the construction do not fall within the limits of the drawing. Such a condition is typified by the case of a series of parallel or nearly parallel forces. The method is undesirable from the standpoints of expediency and accuracy if a large number of forces are to be considered, or if the intersections must be located by lines which converge at slight angles.

A method of solution which may be applied to all problems involving co-planar, non-concurrent forces is indicated in Fig. 23. The four forces P_1, P_2, P_3, and P_4 acting upon the given body are not in equilibrium. It is desired to find their result-

[3] Any force may be considered as acting at any point in its line of action. By "point of application" is meant, therefore, any point in the line of direction of the force in question. For this reason when only three forces which are in equilibrium act upon a given body, their lines of action will intersect in a point even though the forces act at different points on the body. Such a system comes under the general classification of forces meeting at a point and the force polygon alone gives a complete analysis of the relations between the forces.

ant in amount, direction, and point of application. By means of the force polygon *abcde*, the amount and direction of this resultant are determined, the closing line *ae* of the polygon giving the necessary information. As yet the line of action of this force is not fixed.

From some convenient point O in the force polygon, draw a line to each of the vertices of the polygon. Since the lines Oa and Ob form a closed triangle with the force P_1, they represent two forces which will hold P_1 in equilibrium—two forces which may replace P_1 in the force diagram. In Fig. 23, at any point m on the line of action of P_1, lines mn and mv drawn parallel to S_1 and S_5, respectively, represent the lines of action of these two forces. Similarly S_1 and S_2 represent two forces which may replace P_2. The line of action of S_1 has already been fixed by the line mn. From the point of intersection, n, of the line mn with the

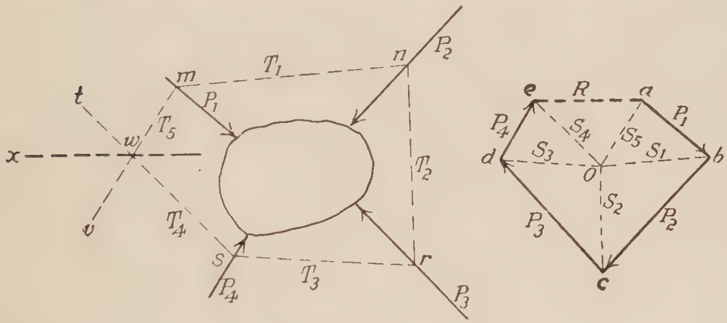


FIG. 23.

force P_2, draw nr parallel to S_2. From r draw rs parallel to S_3, and from s draw st parallel to S_4. Lines mv and st, which are parallel to S_5 and S_4, respectively, represent the lines of action of S_5 and S_4. These latter forces together with the resultant ae form a closed force triangle. S_5, S_4, and ae therefore represent a series of three forces in equilibrium. To fulfill this condition where three forces only are involved, the three forces must meet in a point. The line of action of the resultant ae must therefore pass through the point of intersection w of the lines mv and st. The resultant of the four given forces is thus fully determined. A force of equal magnitude but acting in the opposite direction, *i.e.* from e to a (in the direction of the forces in the force polygon) will hold P_1, P_2, P_3, and P_4 in equilibrium and keep the body on which they act at rest.

The polygon *mnrsw* represents a jointed frame which, by means of the stresses of tension and compression in its various members,

could hold the given forces in equilibrium. It is called, therefore, an *equilibrium polygon.* The amount and character of stress in each of the members of the jointed frame are given by the lines $S_1 \ldots S_5$ in the force polygon. The point O is called the "pole" and the lines $S_1 \ldots S_5$ are called the "rays" of the force polygon. Obviously, since an infinite number of poles may be selected, and an infinite number of starting points may be used, an infinite number of equilibrium polygons may be drawn for a given group of forces; the final result will, however, be the same for all, *i.e.*, the line of action of the resultant of any given force group, determined by one equilibrium

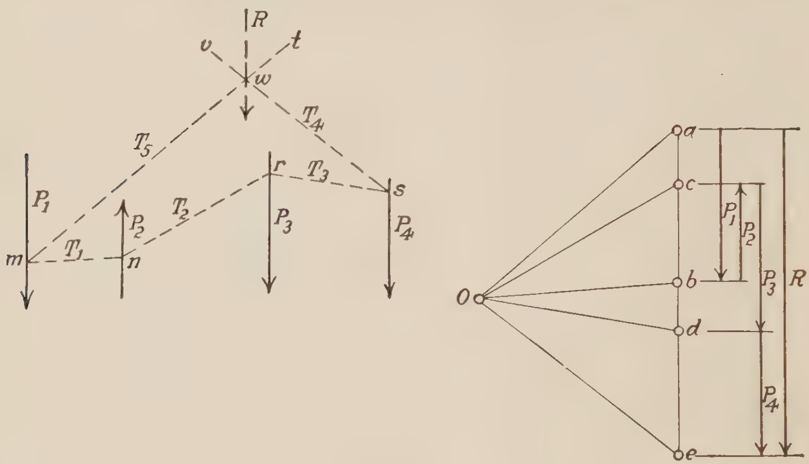


FIG. 24.

polygon, will coincide with that found by any of the other possible polygons.

Besides being a means of determining the line of action of the resultant of a series of non-concurrent forces, the equilibrium polygon possesses several properties which render it extremely valuable in the analysis of static structures. These properties are considered in detail in Art. 28.

25. Resultant of Parallel Forces. In the case of a system of parallel forces, the force polygon becomes a straight line. Figure 24 represents a system of four such forces, not in equilibrium. It is required to determine their resultant. Draw the force polygon *abcde.* This polygon is a straight line, and *ae*, its closing line, gives the magnitude and direction of the required resultant. For convenience in the following construction, the forces repre-

sented by each of the lines of the force polygon in Fig. 24 are indicated at the right of the polygon. To hold the given forces in equilibrium a force equal in amount to ea, acting upward from e to a, is necessary. The line of action of this force remains to be found. This is determined from the equilibrium polygon drawn according to the method described in Art. 24.

The pole O is selected and the rays drawn to the vertices of the force polygon. The fact that these vertices are in the same straight line does not alter the principle involved. From any point on the line of action of P_1, lines mn and mt are drawn parallel to Ob and Oa (the rays that form with P_1 a closed force triangle), respectively. From n, the point of intersection of mn

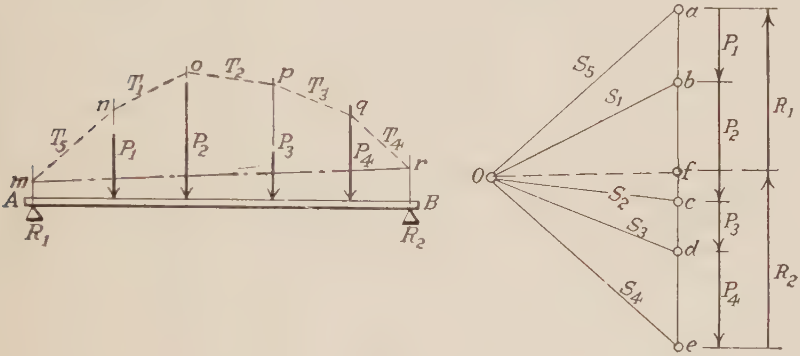


FIG. 25.

and the line of action of P_2, a line nr is drawn parallel to Oc; from r, a line rs is drawn parallel to Od; and from s, a line sv is drawn parallel to Oe. The point of intersection w, of the lines mt and sv, is a point in the line of action required. An upward force equal and parallel to ea, applied so that its line of action passes through point w, will hold the forces P_1 . . . P_4 in equilibrium. A downward force of the same amount will replace the given forces.

26. Reactions of Beams. Figure 25 represents a horizontal beam supported at its extremities and loaded with the four vertical forces P_1 . . . P_4. In order that equilibrium be maintained, the two supports must furnish reactions R_1 and R_2, the sum of which is equal to the sum of all the downward forces acting on the beam. The amount of each of the reactions is required. The problem involves a series of six parallel forces,

all of which are known in direction and point of application, and all but two in amount. The force polygon and equilibrium polygon together furnish the necessary information.

The portion *abcde* of the force polygon is drawn by laying off the given loads in order. When applied to a series of parallel forces, this portion of the force polygon is referred to as the "load line." The sum of the two reactions is represented by the line *ea*. A pole *O* is selected and the rays are drawn to the intersections of the forces in the load line. A part of the equilibrium polygon is now constructed by drawing lines on the force diagram parallel to these rays in the proper order (see Art. 25). This polygon is then closed by drawing the line *mr*. In the force polygon, the line *Of* is drawn parallel to *mr*. Point *f* divides the load line

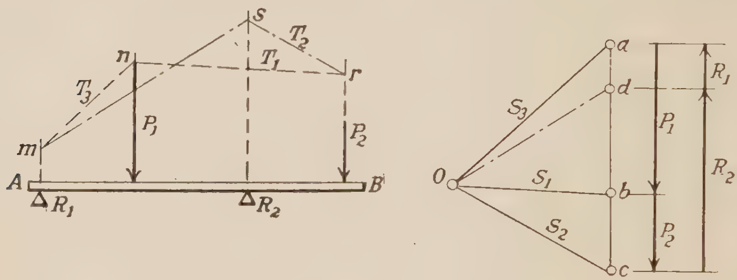


FIG. 26.

into the two reactions, R_2 represented by *ef*, and R_1 represented by *fa*. The force polygon *abcdef* closes, and the equilibrium polygon *mnopqr* closes, hence a condition of equilibrium exists in the beam.

The point of application of the resultant of the forces $P_1 \ldots P_4$ could be found by producing the lines *mn* and *rq* to their intersection (compare with Art. 25). The amount of this resultant is given by the line *ae*.

As a further example of the determination of beam reactions, let it be required to determine R_1 and R_2 in the overhanging beam *AB* shown in Fig. 26. The ray *Od*, parallel to the closing line *ms* of the equilibrium polygon *mnrs*, divides the load line *ac* into the required parts; R_2 is represented by *cd* and R_1 by *da*.

27. Conditions Necessary for Equilibrium. From the foregoing paragraphs it is evident that the only condition necessary for the equilibrium of a system of forces meeting in a point is that the force polygon must close. For a system of non-concur-

rent forces, both the force and the equilibrium polygons must close. For the concurrent forces an equilibrium polygon is unnecessary since the force polygon gives all of the required relations between the forces.

28. Properties of the Equilibrium Polygon. Figure 27 represents a body at rest under the action of the forces P_1 . . . P_5. The given forces in turn are held in equilibrium by the jointed frame or equilibrium polygon *mnqrs.* This polygon was drawn for the pole O, shown in Fig. 27(a), as explained in Art. 24. The character of the stress in each of the members of the jointed frame is given by the force polygon. In this case each member is in compression.

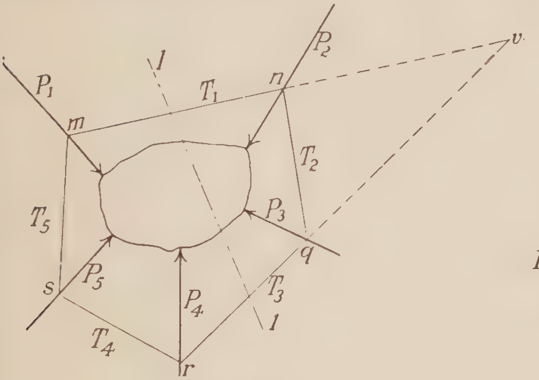


FIG. 27.

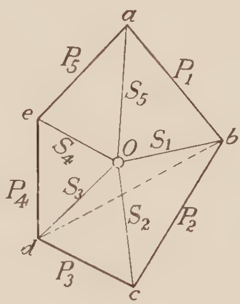


FIG. 27(a).

Pass a section 1-1 through any two of the members of the frame. The rays S_1 and S_3, which in the present section represent the stresses in the members T_1 and T_3 of the equilibrium polygon, form a closed force polygon with forces P_2 and P_3, and also with P_1, P_5, and P_4. The lines T_1 and T_3 obviously meet in a point when produced. The equilibrium polygons for the forces P_3 and P_2, and for P_1, P_4, and P_5 are therefore closed by the lines T_1 and T_3. The stresses in T_1 and T_3 thus complete the conditions necessary for equilibrium for each of the two groups of external forces indicated above. It may now be concluded that *the internal stresses in the members of a static structure cut by a given section hold in equilibrium the external forces on either side of that section.*

In the force polygon, the line db completes a triangle with the rays S_1 and S_3. It represents, therefore, the resultant of the

stresses in the members T_1 and T_3. In order that a group of three forces be in equilibrium it is necessary that they meet in a point. The resultant of the stresses in T_1 and T_3 must act through the point of intersection of T_1 and T_3. Since T_1 and T_3 hold the forces P_1, P_4, and P_5 in equilibrium, it follows that the resultant of the former is equal and opposite to that of the latter. The resultant of the forces P_1, P_4, and P_5 is therefore given in amount and direction by the line db; it acts from b toward d and its line of action passes through the point of intersection of T_1 and T_3. Therefore, it may be stated that *the resultant of the external forces on the left of any section passes*

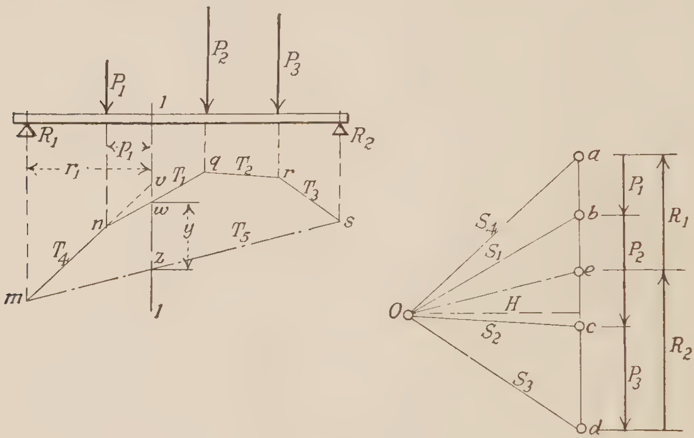


FIG. 28.

through the point of intersection of the sides of the equilibrium polygon cut by the section; its amount and direction are given in the force polygon by the diagonal which separates the forces on the left from those on the right. The resultant of the external forces on the right of the section is equal and opposite to that of the forces on the left of the section.

A third property of the equilibrium polygon applies to a system of parallel forces. Figure 28 represents a horizontal beam supporting the downward vertical loads P_1, P_2, and P_3. The reactions R_1 and R_2 hold the beam in equilibrium. The line Oe drawn parallel to the closing line ms of the equilibrium polygon $mnqrs$ divides the load line ad into two parts de and ea, which represent the reactions R_2 and R_1, respectively (see Art. 26). Cut a vertical section 1-1 through any two sides, T_1 and

T_5, of the equilibrium polygon. Produce the side T_4 to intersect the section in the point v. Let the distance from R_1 to the section be called r_1, the distance from P_1 to the section, p_1 and the vertical ordinate of the section intercepted between T_1 and T_5, y. Also let the horizontal distance from the pole O to the load line—the "pole distance"—be called H. The bending moment at section 1-1 is

$$M = R_1r_1 - P_1p_1$$

The triangle mvz is similar to the triangle Oae and the triangle nvw is similar to the triangle Oab. Hence

$$\frac{R_1}{H} = \frac{vz}{r_1}$$

and

$$\frac{P_1}{H} = \frac{vw}{p_1}$$

Thus

$$R_1r_1 = H(vz)$$

and

$$P_1p_1 = H(vw)$$

Therefore

$$R_1r_1 - P_1p_1 = H(vz - vw)$$

or

$$M = Hy$$

Thus it is seen that *in a static structure acted upon by a series of parallel forces, the bending moment in any section parallel to the forces is equal to the product of the ordinate y in the equilibrium polygon and the pole distance H in the force polygon.* The equilibrium polygon therefore represents the moment diagram for the given structure. The ordinate y is measured to the same linear scale as the force diagram, and the distance H is measured by the same unit as the forces in the force polygon. By adopting convenient scales, the numerical value of the bending moment may readily be determined from the graphical solution.

29. Bending Moment and Shear Diagrams for Beams. At any section in a beam supporting vertical loads, the vertical shear is equal to the algebraic sum of the external forces on the left of the section. The force polygon will give this summation for any section if the loads are arranged in the proper order.

In Art. 28 it was shown that the ordinate of the equilibrium polygon at any section along the beam is a measure of the bending moment at that section. The equilibrium polygon therefore serves as a moment diagram, from which the bending moment at any section may be directly measured, if the proper scale is adopted.

The application of the force and equilibrium polygons to the determination of moments and shears at any point in a horizontal beam supporting vertical loads will be shown in the following articles.

30. Simple Beams with Concentrated Loads. Let it be required to draw the moment and shear diagrams for the simple

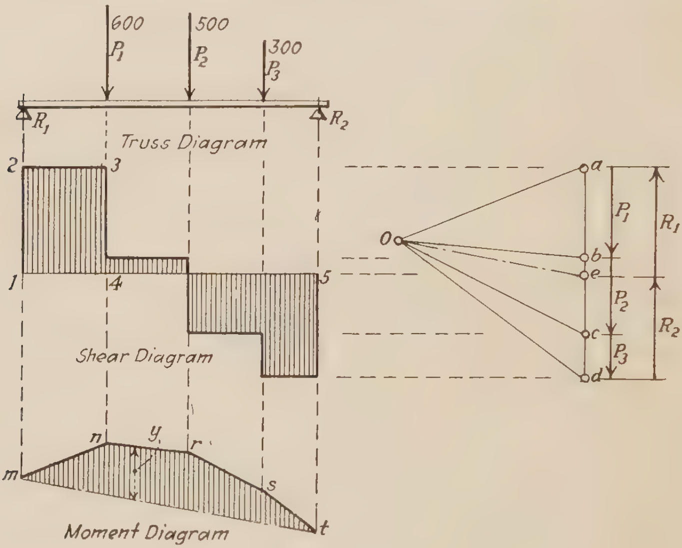


FIG. 29.

beam, loaded and supported as shown in Fig. 29. Lay off the load line *abcd*. From any convenient pole *O*, draw the rays *Oa* . . . *Od*. The equilibrium polygon *mnrst* may then be constructed by drawing lines parallel to these rays between the lines of action of the forces R_1, P_1, P_2, P_3, and R_2 respectively. A line through *O* parallel to the closing line *mt* of the equilibrium polygon, divides the load line into the two parts *de* and *ea*, representing R_2 and R_1 respectively. Between the left support and the first load the vertical shear is equal to R_1, and is given in the force polygon by the distance *ea*. Draw horizontal lines

through a and e to intersect the lines of action of R_1 and P_1. The shear at any section between these two points is then represented by the ordinates of the shaded area 1-2-3-4. Between P_1 and P_2 the shear equals $R_1 - P_1$, or in the force polygon, $ea - ab$, or be; between P_2 and P_3 it equals $ea - ac$, or ec, and between P_3 and R_2 it equals $ea - ad$, or de. By extending the points b, c, and d horizontally until they intersect the lines of action of P_1, P_2, P_3, respectively, the shear at any section is represented graphically as shown in the shaded figure 1-5; this figure is called the shear diagram. When measured above the line 1-5 the shears are considered as positive, and below this line as negative. A positive shear thus means that the left segment of the beam tends to move upward with respect to the right segment—the upward external forces on the left of the section being greater than the downward forces on that portion of the beam.

As stated above, the ordinates of the equilibrium polygon *mnrst* are proportional to the bending moments at the various sections. This polygon may be called the moment diagram. The maximum ordinate of the moment diagram is the vertical through r. The point r is the intersection of the sides of the equilibrium polygon which are parallel to the rays Ob and Oc respectively. The rays Ob and Oc lie on opposite sides of and adjacent to the ray Oe which is drawn parallel to the closing line of the equilibrium polygon. At this point the shear changes from positive on the left (measured from e to b) to negative on the right (measured from e to c). The maximum moment is thus shown to be at the point where the shear passes through zero—a graphical proof of one of the fundamental principles of the mechanics of static structures.

In the original construction, the scale of forces used was 500 lb. to the inch, and the pole distance was made 1000 lb., *i.e.*, the actual distance from O to the load line was 2 in. The scale of distances used in the beam diagram was 4 ft. to the inch. Hence, the moment at any section is equal to the value of y at the section, measured to the scale of 4 ft. to the inch, multiplied by 1000 lb. If the value of y when measured to the proper scale is expressed in feet, the moment will be given in foot-pounds when the multiplication indicated above is completed. Any ordinate in the shear diagram measured to a scale of 500 lb. to the inch gives the value of the shear at the corresponding section in pounds.

31. Simple Beams with Uniform Loads. Let the moment and shear diagrams be required for the simple beam of Fig. 30 which supports a uniform or distributed load over its entire length. Each reaction equals one-half of the total load, represented by the length of the line ab. The shear at the ends of the beam, therefore, equals $\frac{1}{2}wl$, in which w equals the load per linear foot and l the span of the beam in feet. At any point x ft. from the left support the shear equals $\frac{1}{2}wl - wx = w\left(\frac{l}{2} - x\right)$. At the center of the beam, $x = \frac{l}{2}$ and the shear is zero. Between

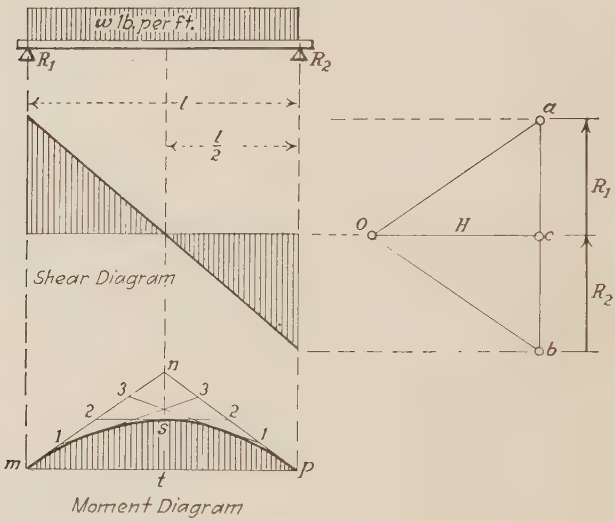


Fig. 30.

the support and the center, the shear decreases uniformly from $\frac{1}{2}wl$ to zero. The shear diagram is therefore as shown in Fig. 30.

In order to construct the moment diagram first assume that the uniform load is replaced with one single concentrated load W at the center of the beam, of an amount equal to the total uniform load wl. The moment diagram for this condition, constructed for the pole O, is shown by the triangle mnp. The pole is selected for convenience at any point on a horizontal line through the middle of the load line ab. The triangle mnp is therefore an isosceles triangle constructed upon a horizontal base.

Analytically, the maximum bending moment in the beam due to the assumed concentrated load equals

$$M = R_1 \cdot \frac{l}{2} = \frac{W}{2} \cdot \frac{l}{2} = \frac{Wl}{4}$$

The maximum bending moment due to the uniform load equals

$$M = R_1 \cdot \frac{l}{2} - \frac{W}{2} \cdot \frac{l}{4} = \frac{Wl}{4} - \frac{Wl}{8} = \frac{Wl}{8}$$

The value of W in this equation is the same as in the one above, in accordance with the assumption made in a previous paragraph. From these two equations it is seen that the maximum moment in a beam with a single concentrated load at the center is twice as great as that for the same load when uniformly distributed throughout the length of the beam. The bending moment due to the uniform load at any section x distant from the left support equals

$$M_x = \frac{wl}{2} \cdot x - \frac{wx^2}{2} = \frac{w}{2}(lx - x^2)$$

which is the equation of a parabola. With the information obtained from these three equations, the bending moment diagram for the uniform load may be constructed.

In Fig. 30 divide the line *mn* into any number of equal parts and the line *np* into the same number of equal parts. Connect points 1, 2, and 3 on *mn* with points 3, 2, and 1, respectively, on *np*. The lines thus drawn and the lines *mn* and *np* are tangents to the parabola *msp*. The accuracy of the parabolic curve increases with the number of tangent lines used in the construction. The ordinate *st* of the parabola equals one-half of *nt*. Therefore the ordinates from the line *mp* to the curve *msp* fulfill the requirements of the bending moment variation outlined above for the uniform load, and the diagram *mspm* is the bending moment diagram for the given beam and loading. The amount of the moment at any section is equal to the ordinate of the moment diagram, measured to the scale of the beam diagram, multiplied by the value of H, measured to the scale of the forces in the load line.

32. Simple Beams with Uniform and Concentrated Loads. Figure 31 represents a simple beam supported at its ends and loaded with a concentrated load and a uniform load over a portion

of the beam. The reactions R_1 and R_2 are found in the usual manner, assuming the uniform load concentrated at its center of gravity. For this assumed condition the equilibrium polygon is *mnsr*. The portion *mnop* of this polygon represents the actual moment diagram for the left portion of the beam from the left

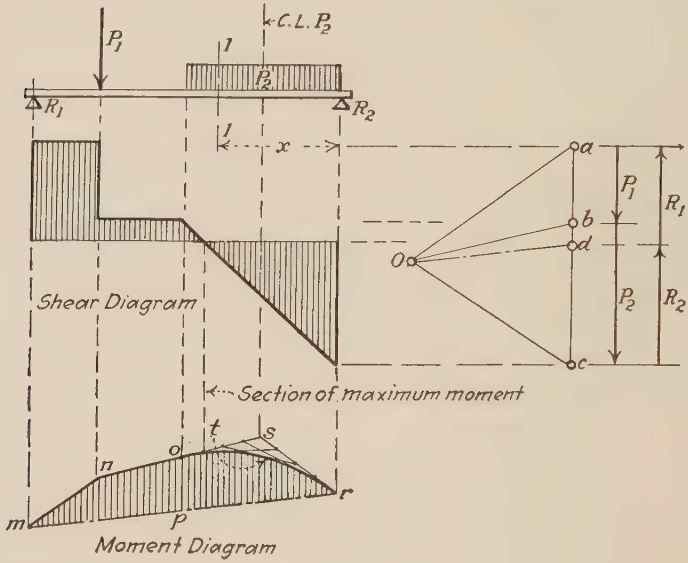


FIG. 31.

reaction to the beginning of the uniform load. From this point to the right reaction the diagram will assume the form of a parabola. This is evident from a consideration of a segment of the beam to the right of a section such as 1-1, which passes through the uniform load. The equation for the moment at any section x ft. from the right support, considering the right segment, is

$$M_{1-1} = R_2 \cdot x - \frac{wx^2}{2}$$

This is the equation of a parabola, as in the preceding problem. The moment at the right support is zero, and at the left end of the uniform load the moment is represented by the ordinate *op*. Lines *os* and *sr* are tangent to the parabola at points *o* and *r*, respectively. The parabola is constructed, therefore, by dividing these tangent lines into the same number of equal parts and connecting the points of division in proper order as shown in the

diagram. The final moment diagram for the loading shown is then given by the figure *mnotr*.

The shear diagram is constructed as explained in the previous examples. It should be noted that the maximum ordinate of the parabola—the point at which the moment is a maximum—occurs directly under the point at which the shear passes through zero.

The simplest method of constructing the moment diagram for a beam which supports a uniform load and a concentrated load applied at some point in the uniformly loaded area, consists in constructing two separate diagrams, one for the uniform load and one for the concentrated load, and adding these two diagrams graphically. This graphical addition is performed by adding the ordinate of one diagram at any section, to the corresponding ordinate of the second diagram. By taking a sufficient number of points along the beam, an accurate composite moment diagram may be obtained in this manner.

33. Overhanging Beams. Beams supporting a system of loads, some of which are placed beyond the reactions, require special consideration in order to secure a moment diagram which may easily be interpreted. The beams in the preceding articles were subjected to positive moments only—moments which cause tension in the lower fiber and compression in the upper fiber. The fact is shown in the preceding diagrams, since the moments at all points along the beam are measured in the same direction from the closing line. An overhanging beam may be subjected to positive moment over some portions of its length and negative moment over the remaining portions. In order to show clearly the regions of positive and negative moments, a slight modification of the moment diagram as drawn for a simple beam supported at its extremities is necessary.

Let a beam be taken with one overhanging end, supported and loaded as shown in Fig. 32. The reactions R_1 and R_2 are determined graphically by means of the equilibrium polygon *mnqrs*, constructed for the pole O of the load line *abcd*, the loads P_1, P_2, and P_3 being laid off in this order. Line *Oe*, drawn parallel to the closing line *ms* of the equilibrium polygon, divides the load line into the two reactions, R_2 represented by *de* and R_1 by *ea*.

The polygon *mnqrs* represents the moment diagram for the given beam, but is not in the most convenient form for easy interpretation. In order to obtain a moment diagram whose ordinates may be measured from a horizontal line $m'r'$, draw

another force polygon or load line by laying off the loads and reactions in the order in which they occur from left to right on the beam. Thus in the lower load line, R_1 is laid off from e' upward to a', the length of $e'a'$ being the same as ea in the upper load line. Then P_1 is laid off from a' downward to b'; R_2 from b' upward to c'; P_2 from c' downward to d', and finally P_3 from d' downward to e' to close the polygon. This construction corresponds to that used in the simple beams of the preceding

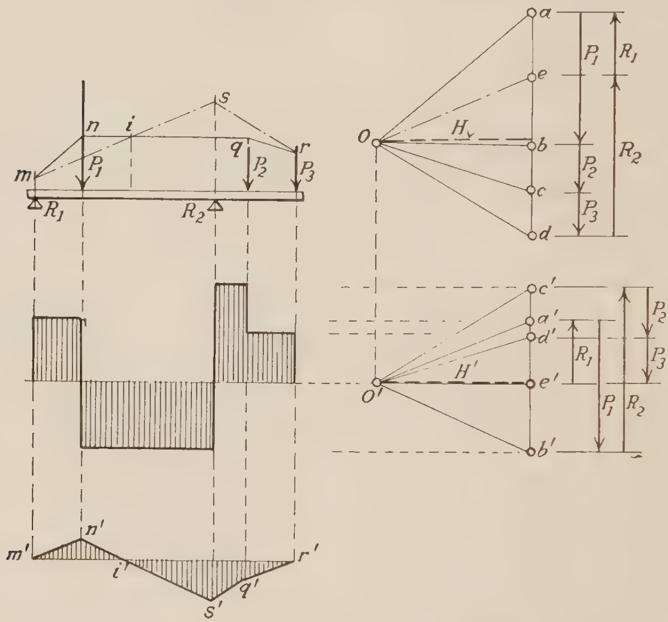


Fig. 32.

articles, assuming that the reactions are R_1 (upward) and P_3 (downward), and that the loads are P_1 and P_2 (downward) and R_2 (upward).

The equilibrium polygon $m'n's'q'r'$, drawn for the pole O' in the lower force polygon, also represents the moment diagram for the given beam. If the pole distance H' to the lower load line equals the pole distance H to the upper load line, each of the ordinates in the lower equilibrium polygon will be equal to the corresponding ordinate in the upper equilibrium polygon. The ordinates in the lower polygon are measured upward from the line $m'r'$ from m' to i', and downward from i' to r'. The

ordinates from m' to i' represent positive moments and those from i' to r' represent negative moments. The point i', where the line $n's'$ crosses the line $m'r'$, is thus the point of inflection—the point at which the moment changes from positive to negative. The point i' is in the same vertical line as the corresponding point i in the upper moment diagram. The lower moment diagram shows the portions of positive and negative moments more clearly than does the upper, and for this reason the additional construction is justified for beams with complicated loadings.

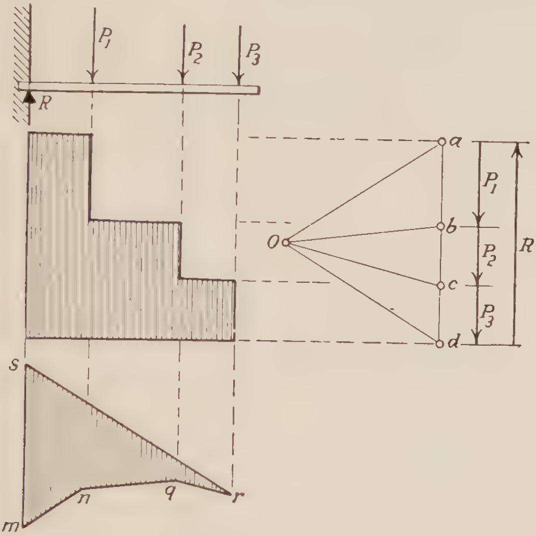


Fig. 33.

The shear diagram is constructed from the lower force polygon by projecting horizontal lines from the points of division on the load line to the proper verticals through the loads and reactions in the beam diagram. The shear passes through zero at two points, one at which the positive moment is a maximum, and the other at which the negative moment is a maximum. The latter occurs at the right support R_2.

For the case of a uniform load, the moment diagram may be constructed by first replacing the uniform loads over the inside and projecting portions with equivalent concentrated loads at the center of each portion, and drawing the moment diagram for this condition as outlined above; the corresponding diagram

for the uniform load is then found by drawing parabolas as indicated for simple beams under uniform loads.

34. Cantilever Beams. Let it be required to construct the moment and shear diagrams for the cantilever beam shown in Fig. 33. Since there is but one reaction, it is equal to the sum of the three loads P_1, P_2, and P_3. The equilibrium polygon *mnqrs*, constructed in the usual manner for the pole O, is the required moment diagram. This diagram could be so arranged that the ordinates would be measured from a horizontal line if it were so desired, by selecting the pole on a horizontal line through point d in the force polygon—the point which represents the end of the outer load on the beam.

The shear diagram is found by projecting horizontal lines from points a, b, c, and d in the force polygon to the lines of action of the loads and the reaction in the beam diagram.

CHAPTER III

ROOF TRUSSES

35. Introductory. The principal function of a roof truss is to support the roof covering and any external loads sustained by the latter, and to transmit these loads through its reactions to the walls of the buildings or to the supporting columns. Several of the more common types of roof trusses are shown in Fig. 34. The framing details are so arranged as to cause the loads to be

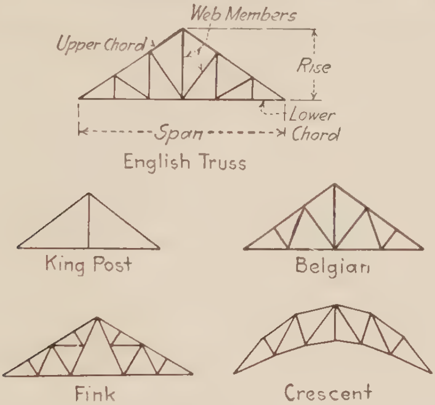


FIG. 34.

brought to the truss only at the points of intersection of the center lines of the principal members—the *panel points*—and thus to cause stresses of direct tension or compression in these members, unless prevailing peculiarities of the construction require other distribution of the load.

The distance between the centers of the supports is called the *span*, and the distance from the highest point (the peak) to the line on which the span is measured is called the *rise*. The uppermost line of members extending from one support to the other, through the peak, is called the *upper chord*, and the lowermost line of members the *lower chord*. The other diagonal and vertical elements which furnish the necessary bracing for the chords at the joints are called the *web members*.

By the application of the fundamental principles outlined in Chapter II the stresses in the members of a roof truss may be found graphically with very little labor and with sufficient precision for the purposes of design. The constructions involved in such graphical solutions will be indicated and explained in detail in the following articles. For convenience, the trusses will be lettered in the spaces between the members and loads, and each member and load designated by means of the letters in the adjoining spaces. Thus in Fig. 35 the vertical web member will be referred to as *EF*, the lower chord as *ER* or *FR*,

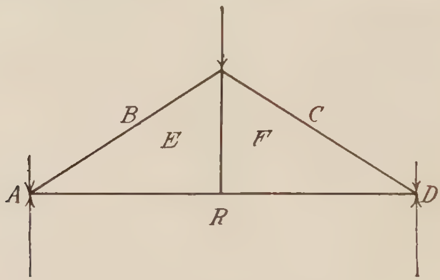


FIG. 35.

the left reaction as *AR*, the load at the left support as *AB*, and so on.

36. Loads on Roof Trusses. The total load for which a roof truss must be designed consists of the weight of the truss, the weight of the roof covering, the probable weight of the snow, and the pressure of the wind. In addition, the truss may be called upon to support some special load, such as a suspended ceiling or floor or heavy machinery.

The weight of the truss must be assumed before the truss is designed. To aid the designer in making this assumption several formulas have been devised, based upon the exact weights of previously designed trusses. The weight of the truss will vary with the span and rise of the truss, the distance between trusses, the material used in the truss, the type of roof covering, the geographical location of the structure as affecting the snow and wind pressures involved, as well as other obvious design factors. No practical formula can be obtained which includes all of these variables. The span and spacing of trusses and the material used in the construction cause the greatest variation in weight, and since the dead weight of the truss itself forms but a small

part of the total design loads, the error involved in neglecting the other factors in a weight formula is of no great consequence.

For wooden roof trusses, a simple and reasonably accurate formula representative of average conditions is given by N. Clifford Ricker as follows:

$$w = \frac{l}{25} + \frac{l^2}{6000} \tag{1}$$

in which w = weight of truss per square foot of horizontal covered surface and l = span of truss in feet. This equation has been found in the authors' experience to be equally applicable to steel trusses of ordinary spans.

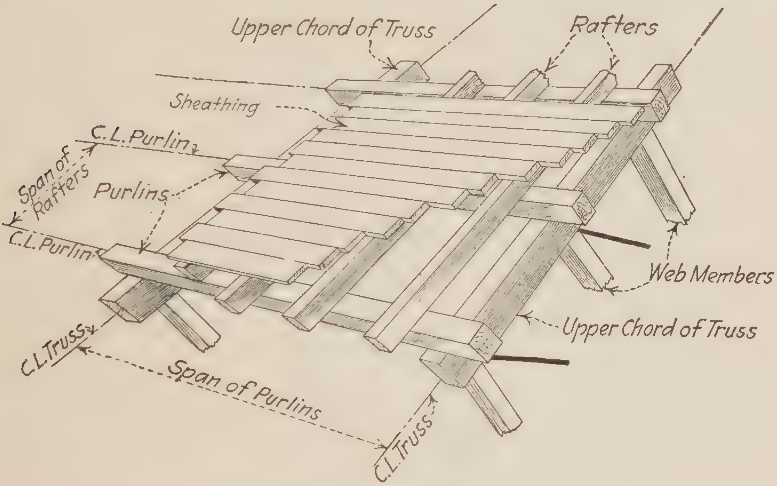


FIG. 36.

For wooden roof trusses, Merriman and Jacoby recommend the following:

$$w = 0.5(1 + 0.1l) \tag{2}$$

in which w and l represent the same factors as in the preceding equation.

For steel roof trusses, M. A. Howe gives

$$w = \tfrac{3}{4}(1 + 0.1l) \tag{3}$$

The weight of the truss is usually divided equally between the panel points of the upper chord; for large trusses, however, a part of this load should be applied at the lower chord panel points.

The type of roof covering depends upon the character of the building, whether monumental, public, residence, or mill. The most common materials used for sloping roofs are slate, tile, tin, and wooden shingles resting on 1-in. wooden sheathing. The sheathing is supported on rafters of fairly close spacing placed parallel to the upper chord, and these in turn are supported by purlins resting on the upper chords at the panel points (see Fig. 36). The purlins are either wooden beams or steel shapes. For mill and shop buildings the roof covering usually consists of corrugated metal sheathing, with insulating material on the under side, supported directly on the purlins. The exact weight of roof covering and its supporting framing can be determined only by computation for any specific case. For preliminary computations and estimates, the following values of the weights in lb. per square foot of roof surface will prove satisfactory.

Tin shingles	1
Wooden shingles	2
Slates	10
Tiles	10–20
1-in. wooden sheathing	3
Wooden rafters	1½–3
Wooden purlins	2–4
Steel purlins	3–5
Corrugated steel	1 2

The snow loads to be used in the design vary with the geographical location and altitude of the structure and with the slope of the roof. Where snow is likely to occur, a minimum value of 25 lb. per square foot of horizontal covered surface should be used for all slopes up to 20 deg.; this load may be reduced 1 lb. for each degree of slope above 20 deg.

The wind pressures also vary with the geographical location and the slope of the roof. The wind is assumed to blow horizontally with a velocity sufficient to cause a pressure of 30 to 40 lb. per square foot on a vertical surface, which represents a velocity of approximately 86 to 100 miles per hour. Since the friction of air on comparatively smooth surfaces is very slight, it may be assumed without appreciable error that only the normal effect of this pressure need be considered. The amount of normal pressure for varying slopes is given by Duchemin in the following formula.

$$P_N = P_H\left(\frac{2 \sin A}{1 + \sin^2 A}\right)$$

in which P_H = horizontal pressure per square foot on a vertical surface

P_N = normal pressure per square foot of sloping surface

A = angle of inclination of the sloping surface

For a horizontal pressure of 30 lb. per square foot, the values of P_N for different slopes as computed from this formula are tabulated below:

Slope A (degrees)	Normal pressure P_N (pounds per square foot)	Slope A (degrees)	Normal pressure P_N (pounds per square foot)
5	5.19	35	25.90
10	10.11	40	27.29
15	14.55	45	28.28
20	18.37	50	28.97
25	21.51	55	29.41
30	24.00	60	29.69

For intermediate angles of inclination the normal pressures may be obtained by interpolation. For horizontal pressures other than 30 lb. per square foot, the values given above change in proportion.

The wind pressure to be used in the design of a sloping roof is normal to the surface of the roof. The other loads on the roof are vertical. These two types of loadings need separate consideration in determining the stresses in the trusses, as will be shown later.

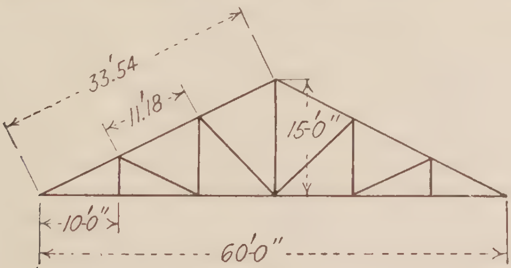


Fig. 37.

37. *Illustrative Problem.* Let it be required to determine the dead, snow, and wind panel loads for the wooden roof truss shown in Fig. 37. The span of the truss is 60 ft.-0 in., the rise 15 ft.-0 in., and the distance between adjacent trusses 12 ft.-0 in. The roof covering consists of slates laid on 1-in. wooden

sheathing supported on 2 by 6-in. rafters, 24 in. on centers; the rafters in turn are supported on 8 by 10-in. purlins resting on the upper chords at the panel points.

From equation (2) the probable weight of the truss per square foot of horizontal covered surface is

$$w = 0.5(1 + 0.1 \times 60) = 3.5 \text{ lb.}$$

and the total weight of one truss $3.5 \times 12 \times 60 = 2520$ lb. The proportion of the total weight assumed as concentrated at each intermediate upper chord panel point is $\frac{1}{6} \times 2520$ or 420 lb. One-half of this amount, or 210 lb., is assumed as acting at each end panel point.

Each intermediate panel point supports 11.18×12, or 134.0 square feet of roof surface, and 10.0×12, or 120.0 square feet of horizontal projection of roof surface. The weight of 1-in. sheathing supported by each panel point is $134.0 \times 3 = 402$ lb., and the weight of slate $134.0 \times 10 = 1340$ lb. Since each purlin, and therefore each panel point, supports $\frac{12}{2}$ or 6 rafters, the panel load due to the rafter weight, assuming the weight of timber as 36 lb. per cubic foot, is $6 \times \frac{2 \times 6}{12} \times 11.18 \times 3 = 204$ lb. The weight of one purlin is $\frac{8 \times 10}{12} \times 12 \times 3$, or 240 lb.

The angle of inclination of the roof surface is 26°–34′, $\left(\tan^{-1} \frac{15}{30}\right)$, and from the table on page 49 the normal wind pressure is 22.3 lb. per square foot. The wind panel load is therefore 134.0×22.3 or 2990 lb. The snow panel load equals $(25 - 6) \times 120.0$ or 2280 lb.

The total panel loads computed above, taken to the nearest 10 lb. are summarized below.

	Pounds
Slate	1340
Sheathing	400
Rafters	200
Purlins	240
Truss, per panel	420
Total dead panel load	2600
Snow panel load	2280
Wind panel load	2990

The end panel load in each case is one-half of the amounts shown above, and the wind panel load at the peak is one-half of the value computed for the intermediate panel points.

For a truss of the form shown in Fig. 38 the roof area and the horizontal area supported by each panel point must be determined and the weights of roof covering and snow computed from these areas, each panel load consisting of one-half of the load on the adjacent panels. The wind panel load must be obtained by combining, by means of the parallelogram of forces, the wind loads from the panels adjacent to the panel point under consideration. The normal pressures on the two upper chord panels are different since their angles of inclination are not equal. The computations required in the determination of the wind panel loads for this truss are as follows: The spacing of trusses is 10 ft.-0 in.

Angle of inclination, panel 1–2 $= \tan^{-1} \frac{15}{15} = 45° - 0'$.

Angle of inclination, panel 2–3 $= \tan^{-1} \frac{5}{15} = 18° - 25'$.

Normal pressure, panel 1–2 = 28.3 lb. per square foot of roof surface.
Normal pressure, panel 2–3 = 17.2 lb. per square foot of roof surface.
Roof area, panel 1–2 = 10 × 21.2 = 212 sq. ft.
Roof area, panel 2–3 = 10 × 15.8 = 158.0 sq. ft.
Total wind pressure on panel 1–2 = 212 × 28.3 = 6000 lb., one-half of which is applied at each end of the panel as shown in Fig. 38.
Total wind pressure on panel 2–3 = 158 × 17.2 = 2720 lb., one-half of which is applied at each end of the panel.

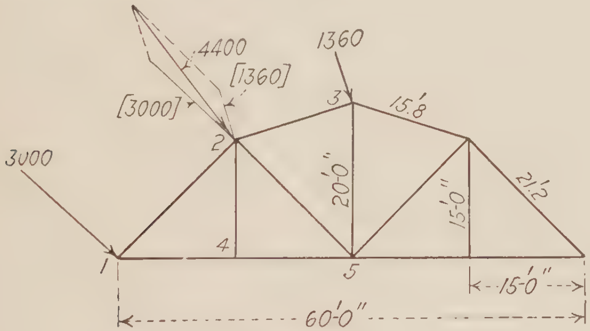


FIG. 38.

The wind panel load at point 1 is therefore 3000 lb., and at point 3, 1360 lb. At point 2, the resultant wind load is equal to 4400 lb., the diagonal of the parallelogram constructed with the 3000- and 1360-lb. loads as its sides. This construction is indicated in Fig. 38.

38. Stresses Due to Dead and Snow Loads. Having determined the loads that are brought to each panel point of the

truss, the stresses caused by these loads in each member of the truss are found graphically by the application of the principle of the force polygon to each joint in succession. Since both the snow and the dead loads are vertical, the stresses found for one may be multiplied by the ratio between the two panel loads to determine the stresses caused by the other, provided this ratio is constant at all panel points. For example, if the dead load at each panel point in a given truss were 1000 lb. and the snow panel load 1500 lb., the snow-load stresses would be equal to the dead-load stresses multiplied by the ratio $^{1500}/_{1000}$. In the event that it is not necessary to determine the dead-load stresses and the snow-load stresses separately, the combined stresses may be found for any type of truss by adding the dead and the snow loads at each panel point and constructing one group of force polygons for the resulting loads.

In order to illustrate the method of determining dead-and-snow-load stresses in a roof truss, let it be required to find these stresses in the truss shown in Fig. 37, with panel loads as computed in Art. 37. The truss is repeated in Fig. 39 and lettered in accordance with the system explained in Art. 35. One set of force polygons only will be used to find the sum of the dead-and-snow-load stresses, hence the panel load at each intermediate panel point used in the following construction is equal to the sum of the dead and snow panel loads given in Art. 37, *i.e.*, 2600 + 2280, or 4880 lb. The load at each end panel point is $\frac{1}{2} \times$ 4880, or 2440 lb.

For equilibrium, the external forces on the truss, consisting of the downward panel loads and the upward reactions, must form a closed force polygon. On account of the symmetry of the loads, both in amount and position, each reaction equals one-half of the total downward load on the truss.[1] Since the external forces are parallel, the sides of the force polygon lie in a straight line $a \ . \ . \ . \ a'ra$ (Fig. 39), constructed by laying off the panel loads in regular order from left to right, and the reactions equal to $a'r$ and ra at the right and left ends, respectively, point r being midway between a and a'.

Cut a circular section around the joint at the left support. The external forces AB and AR are held in equilibrium by the internal stresses in the members BE and ER. These four forces

[1] See Art. 45 for determination of dead-load reactions for unsymmetrical trusses.

therefore form a closed force polygon. The vertices of the force polygon will be lettered with the lower case letters corresponding to the upper case letters of the truss diagram. Through the point b draw a line be parallel to BE and through r a line re parallel to RE. The lengths of the lines be and re, measured to the scale of the load line, give the magnitudes of the stresses in

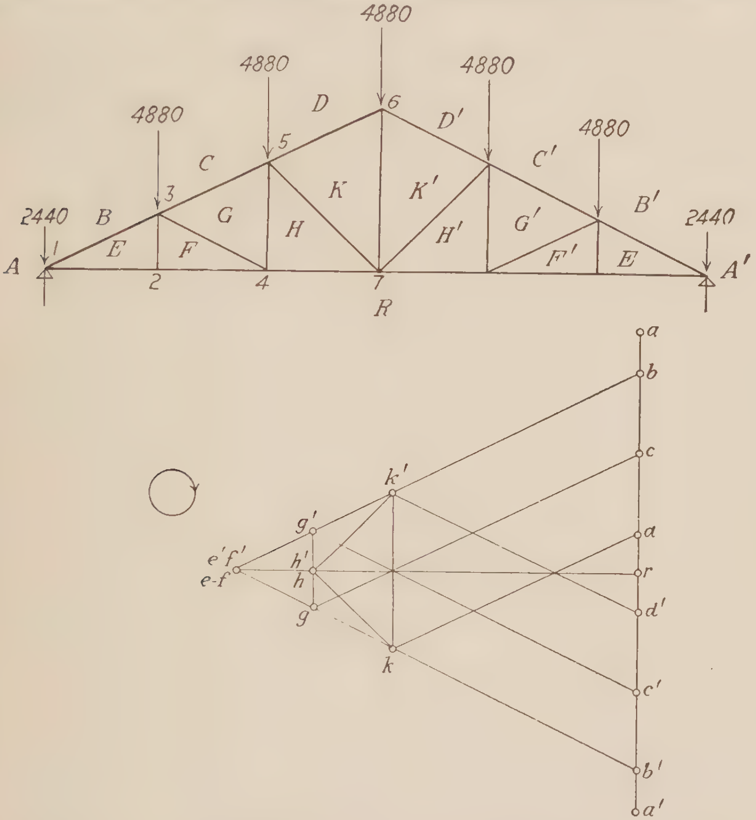


FIG. 39.

BE and RE, respectively. To find the character of the stresses it is only necessary to trace the forces around the polygon in regular order. For equilibrium the forces must act in the same direction around the polygon. The reaction ra acts upward and the load ab downward hence the stress be must act from b to e and the stress er from e to r to close the polygon. Transferring these two directions to the truss diagram, the stress in BE acts downward toward the joint under consideration and the stress in

ER acts away from this joint. *BE* is therefore in compression and *ER* in tension.

Next isolate joint 2 from the rest of the truss by cutting a circular section around this joint. For equilibrium, the stresses in *RE*, *EF*, and *FR* must form a closed force polygon. The stress in *ER* is already known in amount and direction and hence the amounts of the remaining two stresses can be found. A line through *e* parallel to *EF*, and a line through *r* parallel to *RF* determine, by their intersection, the location of the point *f*, and therefore, the amounts of the stresses in *EF* and *FR*. In this case the stress in *EF* is zero and that in *FR* equal to that in *ER*. The stress in *ER* has already been found to be tension, therefore the force in this member acts away from joint 2, toward the left, or from *r* to *e*. The stress in *FR* must act from *f* to *r* to close the polygon. This direction, when transferred to the truss diagram indicates that member *FR* is in tension. The amount and character of each of these stresses are obvious if the summations of the horizontal and vertical components of the forces involved are taken. Since there is no vertical force other than the stress in *FE* in the section cut, this stress must be zero. Similarly the stress in *FR* must equal that in *ER*.

The stress diagram is completed for the given truss by constructing a force polygon for each of the remaining joints in regular order. Joint 3 must be considered before joint 4 in order to determine the stress in *FG*; otherwise there would be three unknowns at joint 4 and the solution of this joint would be impossible. The complete stress diagram is shown in Fig. 39. When measured to the scale of the load line *aa′*, each of the lines in this diagram represents the magnitude of the combined dead- and- snow-load stress in the corresponding member of the truss.

It is possible to determine the character of the stress in each member of the truss without reference to any other member or load and thus save considerable time in the stress determination. In Fig. 39, the loads and reactions were laid off on the load line in the order *AB*, *BC* . . . *A′R*, *RA*—clockwise around the truss. This is indicated by the circular arrow on the stress diagram. In passing around joint 3 in this same direction the members and external load occur in the order *BC*, *CG*, *GF*, *FE*, *EB*. The force polygon for this joint is *bcgfeb*, and must be followed in this same order since the load *bc* acts downward and is

laid off from b to c. Hence the stress in GF acts from g to f; when transferred to the truss diagram this direction is toward the joint and GF is in compression. At joint 4, to determine the character of the stress in GH, it is only necessary to note that in passing clockwise around the joint (following the circular arrow) the letters in the truss diagram occur in the order G to H; hence in the stress diagram the stress in this member must act from g

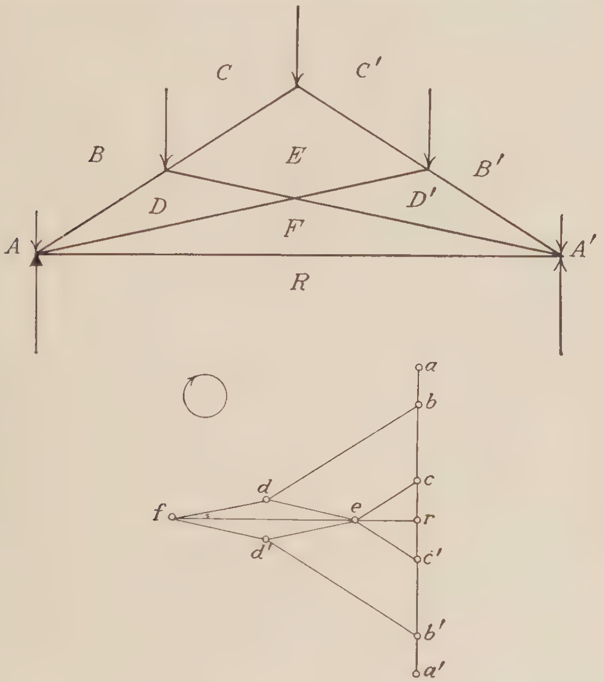


Fig. 40.

to h, upward in this case. This direction, transferred to the truss diagram indicates that GH is in tension. In like manner the character of the stress in any member may be found without reference to any other. This method of determining the character of the stresses as obtained from the stress diagram is known as the *principle of the circular arrow.*

It is not always possible to begin the stress diagram at one of the end joints. In the truss shown in Fig. 40 the stress diagram is begun by cutting a circular section at the peak joint. In this section only two unknown quantities exist and the determination of the stresses in CE and $C'E$ is possible. If the construction

were attempted by first cutting a section around the end joint, three unknown stresses would make the analysis of the joint impossible. By starting at the peak and working toward the end this condition is obviated. The complete stress diagram for this truss is shown in Fig. 40. All of the members are in compression except the tie FR, which is in tension.

39. Stresses Due to Wind Loads. As stated in Art. 36, the wind pressure on sloping roofs acts at right angles to the roof surface. The load line is, therefore, not vertical and the reactions are not equal. Before the stress diagram can be drawn it is necessary that each reaction be determined. Two independent problems must therefore be considered in the complete analysis of a truss under wind loads. First, the reactions must be found for the given external loads by considering the truss as a whole, and second, the internal stress in each member must be determined by considering each joint in succession to be isolated from the rest of the truss.

If both ends of the truss are anchored to the walls in such a manner that each wall will furnish resistance to a horizontal thrust, both reactions will be inclined. Since there are four unknown quantities involved in the solution of the reactions (the amount and direction of each), and since, analytically there are but three equations available for the solution (the summation of horizontal components, vertical components, and moments), some assumption must be made regarding the distribution of the total horizontal thrust between the two reactions. In all of the following problems dealing with trusses of this type, the reactions will both be assumed parallel to the resultant wind load on the truss. Thus, if all of the wind panel loads are parallel as is the case in the truss in Fig. 41, each reaction will be parallel to these loads; if the wind panel loads are not all parallel, as in Fig. 42, the reactions will be parallel to the closing line of the force polygon formed by laying off the wind panel loads in regular order from the end joint to the peak, *i.e.*, in Fig. 42 point r will be located on the right line joining points a and d. Another arbitrary assumption that is sometimes made is that the horizontal components of both reactions are equal. The former assumption is as reasonably accurate as the latter, and permits of a somewhat easier graphical solution.

In steel trusses, especially for the longer spans, provision is usually made for expansion and contraction by allowing one end

to rest on rollers in such a manner that the truss is free to move horizontally at this end.[2] In such cases the reaction at the free end must be vertical, since no provision is made for resisting a horizontal pressure at this end. The entire horizontal component of all the wind panel loads must be resisted by the other support, which is fixed rigidly to the wall in order to furnish the required horizontal resistance. The reaction at the fixed end will, therefore, be inclined at some angle, unknown until the complete analysis of the external forces is made.

In steel trusses of shorter spans the provision for expansion and contraction is made by merely allowing one end of the truss to rest freely on a steel plate securely fastened to the wall. Neglecting friction, the reaction at the free end in this type of construction will be vertical as in the preceding type. For heavy trusses with this condition of end support the friction at the end free to slide would be of sufficient magnitude to cause a horizontal resistance that should not be disregarded.

In the following articles the wind-load reactions for the various conditions outlined above will be determined, and later, the stress diagrams for the trusses involved will be constructed.

40. Wind-load Reactions, Both Ends Fixed. Ordinarily the analytical solution for determining truss reactions will be found somewhat easier than the graphical solution, and also more exact. The values of the reactions are determined by taking moments first about one reaction point and then about the other. The graphical solution, however, may prove more desirable if the lever arms of the given loads about the centers of moments are not easily obtained.

The graphical solution for determining wind-load reactions for trusses with both ends fixed, assuming each reaction parallel to the resultant wind load in accordance with the statement made in Art. 39, is illustrated in the following problem, the truss there used being that of Fig. 37 and the wind panel loads those computed in Art. 37. The truss diagram is repeated in Fig. 41.

[2] This provision is often more theoretical than practical. Once the truss is erected very little attention is given to the maintenance of the roller bearings in a frictionless condition. Where a ceiling is suspended from the trusses there is often no provision made for the effect of a movement of the free end of the truss upon the ceiling. If it were not for the fact that the walls supporting the trusses are considerably more elastic than they are theoretically considered, a good many more cracked ceilings would result.

Both ends of the truss are fixed. It is required to find the reactions caused by the given wind loads.

Considering the truss as a free body acted upon by the loads and reactions as shown, the load line ae is constructed by laying off the loads AB . . . DE in order. According to the assumption made in the preceding paragraph, the line ae represents the sum of the two reactions ER and RA. The point r must be located in order to determine the amount of each of these

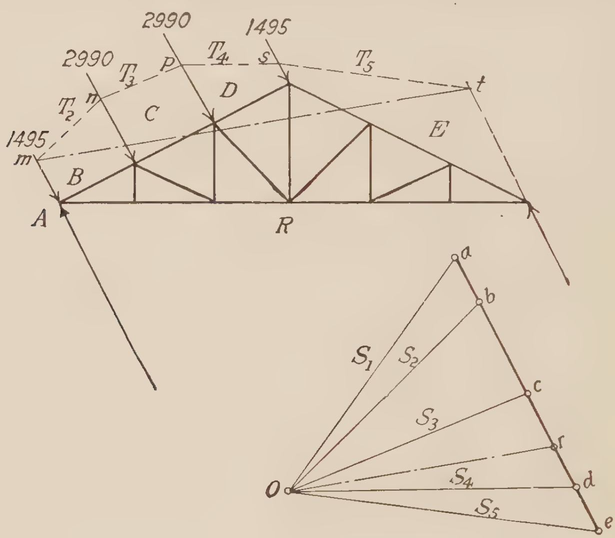


FIG. 41.

reactions. A convenient pole O is selected and the rays Oa . . . Oe are drawn to the load line. The equilibrium polygon $mnpst$ is constructed for the pole O by drawing, on the truss diagram, lines parallel to the rays S_1, S_2 . . . S_5 in order. Each of these lines terminates on the lines of action of the wind panel loads corresponding to the loads on either side of the parallel ray in the force polygon—for example, line T_2, which is parallel to the ray S_2, is drawn from the line of action of AB to that of BC. This polygon may be commenced anywhere on the line of action of the force AB. Since the lines of action of AB and AR coincide, the line of the equilibrium polygon which is parallel to the ray S_1 becomes a point. This explains the apparent omission of the parallel to S_1 from the construction. The line Or, parallel to the closing line mt of the equilibrium polygon, divides the load line into the required reactions, er representing the right reaction and

ra the left reaction. The proof of this construction and the fundamental principles involved are the same as those given in Chapter II for simple horizontal beams with vertical loads. The scale of the original drawing was 100 lb. to the inch. Line *er* measured 2.80 in. and line *ra* 6.17 in. The right reaction is therefore 2800 lb. and the left reaction 6170 lb.

As a further illustration of the application of the principles of graphic statics to the determination of wind-load reactions in fixed-end trusses, let it be required to determine the reactions for the truss of Fig. 38 with the wind panel loads as computed in Art. 37. The truss diagram is repeated in Fig. 42.

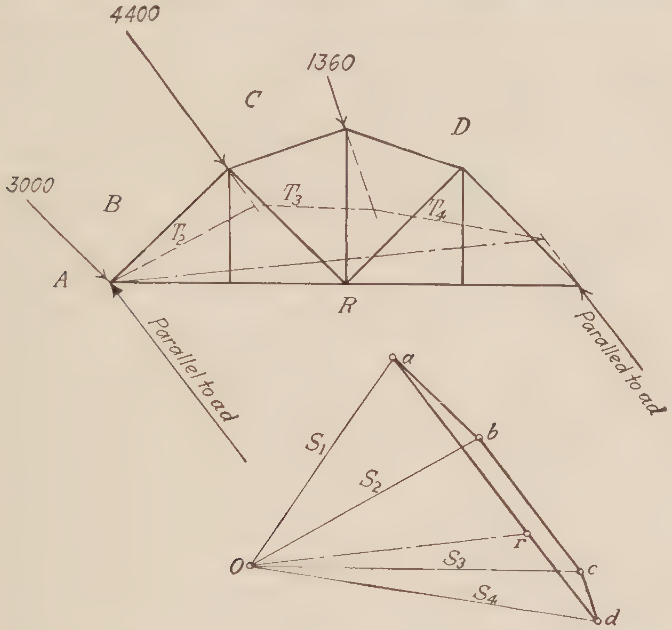


Fig. 42.

The load line *abcd* is first drawn by laying off the lines *ab*, *bc*, and *cd* equal and parallel to the loads *AB*, *BC*, and *CD*, respectively. The line *da* represents the sum of the two reactions *AR* and *DR* in accordance with the assumption that each reaction is parallel to the resultant wind load. The point *r*, which divides the line *da* into the two required reactions, must be located. From any convenient pole *O*, the rays S_1 . . . S_4 are drawn to the vertices of the force polygon. Since the lines of action of *AR* and *AB* coincide at the left reaction point, for convenience in

construction the equilibrium polygon is begun at this point. A line parallel to ray S_1 then becomes a point at the left-end joint, and the first apparent line of the equilibrium polygon is T_2, parallel to S_2, the ray to the intersection of the forces *ab* and *bc* in the force polygon. Line *Or*, parallel to the closing line of the equilibrium polygon, divides the line *ad* into the two required reactions, *dr* representing the right, and *ra* the left reaction.

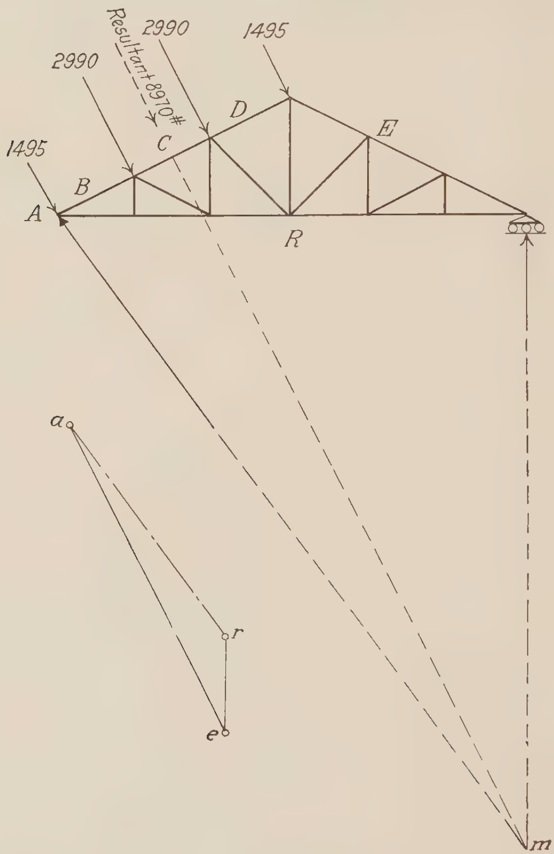


Fig. 43.

41. Wind-load Reactions, One End Free. When one end of the truss is free to move horizontally without restraint, the reaction at this end is vertical. The entire horizontal component of the wind pressure must therefore be resisted by the fixed end. Since the wind may blow on either side of the truss, eventually the stresses must be determined for two conditions,

first, when the wind is blowing on the fixed side, and second, when the wind is blowing on the free side. The reactions must therefore be found for these two conditions.

Let it be required to determine the wind-load reactions for the truss of Fig. 37, repeated in Fig. 43, with the wind panel loads as computed in Art. 37. One end of the truss is assumed to be on rollers so that no horizontal resistance is present at this

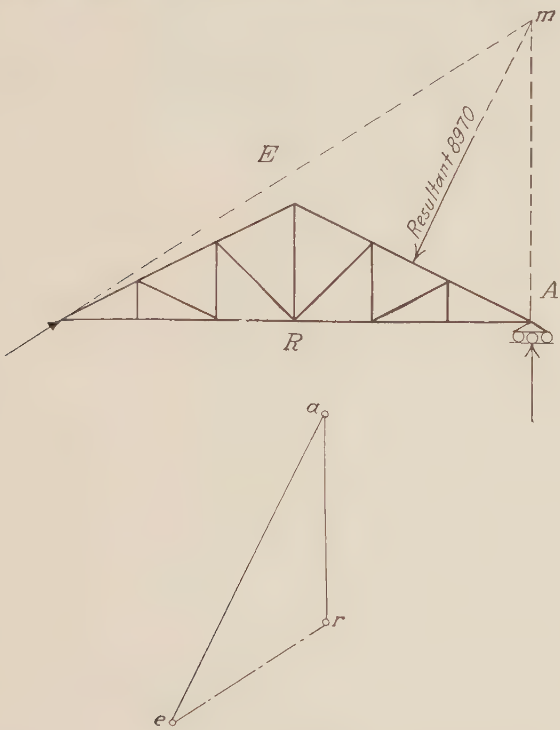


Fig. 44.

end. Assume first that the right end is free and that the wind is blowing on the left, or fixed, side. Replace the four wind panel loads AB . . . DE with one single load equal in amount to the sum of the four loads, and applied at the center of gravity of these loads. The condition of equilibrium of the truss as a whole is not changed by this substitution. Three forces are now holding the truss in equilibrium—the resultant wind load and the two reactions. The right reaction is vertical by hypothesis. As shown in Chapter II, when three forces act on a body, one

condition necessary for equilibrium is that the three forces must meet in a point. Therefore the line of action of the left reaction is found by first producing the resultant wind load to intersect the known line of action of the right reaction at m, and then drawing a line from m through the left reaction. Since points m and A are necessarily points in the line of action of the left reaction, the line mA represents this line of action. In order to determine the magnitude of each of the reactions, draw the load line ae equal and parallel to the resultant wind load. From a draw ar parallel to the direction of the left reaction as found above, and through e draw the vertical er to intersect ar at r. The left reaction is represented in amount and direction by the

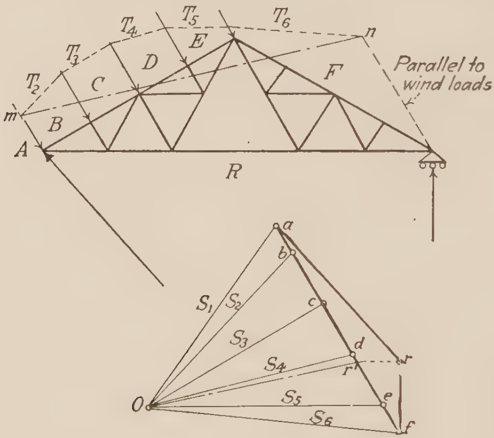


FIG. 45.

line ra, and the right reaction by the line er. This construction is justified because of the fact, already proved, that when three forces act upon a body in a common point, the only condition necessary for equilibrium is that their force polygon must close.

Figure 44 shows the same construction applied to the determination of the reactions for the same truss when the wind is blowing on the free side. The method of procedure is identical with that outlined above.

It is not always desirable or practical to use the simple method just described. The upper chord might be of such a shape that the complete determination of the resultant load is of too complex a nature, or the point of intersection, m, of the reactions might fall outside of the limits of the drawing. In such cases, the

force and equilibrium polygons furnish the necessary construction for the determination of the reactions. Two general methods may be used as shown in the following paragraphs. These methods may be applied to the determination of wind-load reactions for any type of truss and any condition of loading, one end of the truss being free and friction at this end neglected.

Let it be required to determine the reactions for the truss of Fig. 45, the wind panel loads being as shown. The wind is assumed as acting on the left side; the right end is free. The loads *AB* . . . *EF* are laid off in order on the load line *af*. From any convenient pole *O* the rays are drawn to the points of intersection of the loads. Assuming temporarily that both ends are fixed, an equilibrium polygon *m* . . . *n* is drawn, the starting

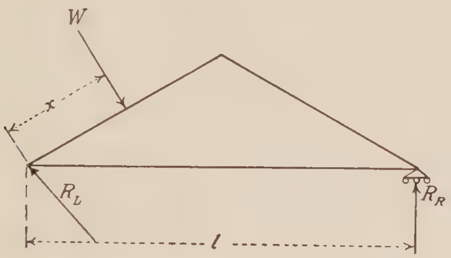


FIG. 46.

point *m* being anywhere on the line of action of the load *AB*. Under this assumed condition of support, both reactions will be parallel to the resultant wind load. These reactions are determined by the intersection of line *Or'*, parallel to the closing line *mn* of the equilibrium polygon, with the load line *af*.

Thus far the method of solution has been the same as indicated in Art. 40, for trusses with both ends fixed. For the reason there given, the line T_1, parallel to the ray S_1, becomes a point and thus, apparently, is omitted from the construction. Having determined the reactions *fr'* and *r'a*, assuming both ends fixed, let the right end be placed on rollers in accordance with the actual condition as stated in the original problem. The right reaction will then be vertical and the left reaction inclined at some angle, unknown as yet. To determine fully these two reactions, draw a horizontal line through *r'* to intersect a vertical through *f* at *r*. The right reaction is then represented by *fr* and the left reaction by *ra*, the closing line of the force polygon

afr. The proof of this last statement is given in the following paragraph.

For equilibrium, the sum of the moments of the external forces acting upon the truss of Fig. 46 about the left reaction and

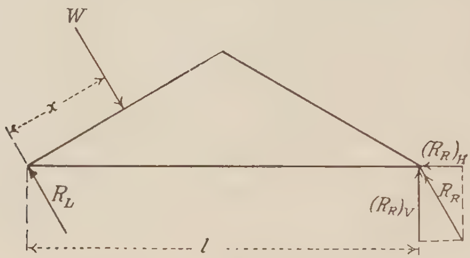


Fig. 47.

(or about any other point) must equal zero. Hence $R_R l = Wx$,

$$R_R = \frac{Wx}{l} \qquad (a)$$

In the above equation, W represents the resultant wind load and x the perpendicular distance from its line of action to the left

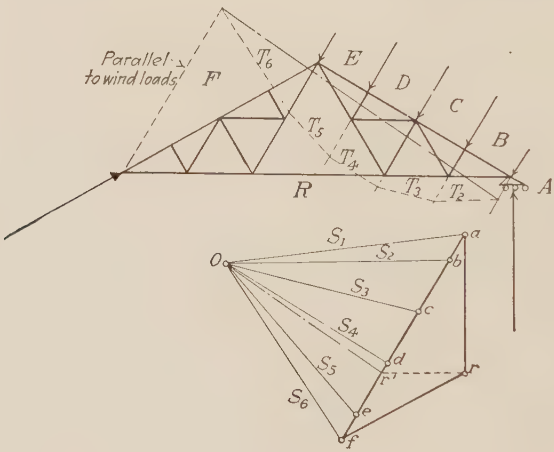


Fig. 48.

support. Considering next the same truss with both ends fixed, the external forces acting on the truss are as shown in Fig. 47. For convenience, the right reaction has been broken up into its horizontal and vertical components $(R_R)_H$ and $(R_R)_V$, respectively.

For equilibrium, the sum of the moments of the external forces about the left reaction must equal zero as in the case of Fig. 46. Since the horizontal component of the right reaction passes through the center of moments, the required equation becomes

$$(R_R)_V\, l = Wx$$

from which

$$(R_R)_V = \frac{Wx}{l} \qquad (b)$$

By comparing equations (a) and (b) it is seen that the vertical reaction at the free end of a truss with one end free is equal to the

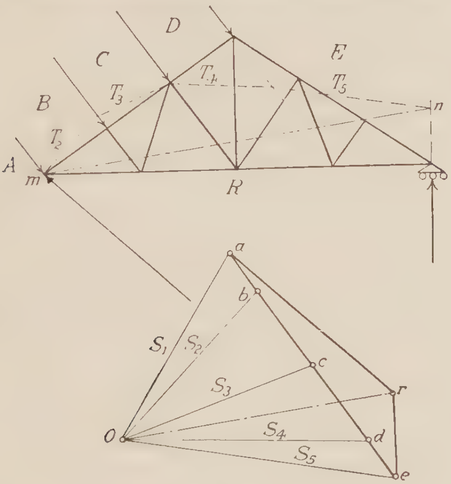


FIG. 49.

vertical component of the corresponding reaction if both ends are fixed thus proving the construction as shown in Fig. 45.

The application of this same method to the determination of the reactions when the wind is blowing on the free side is indicated in Fig. 48. The method of procedure is identical with that described above.

Instead of first assuming both ends fixed, as was done in the preceding method, the wind-load reactions for a truss with one end free may be found by beginning with the assumption that the reaction at the free end is vertical. The necessary construction for this method is described below.

Let it be required to find the wind-load reactions for the truss of Fig. 49. The wind is blowing on the left side; the right end is

free. Lay off the load line *ae*, select a convenient pole *O* and draw the rays *Oa* . . . *Oe*. The right reaction *ER* is vertical, hence point *r* will be located on a vertical line through point *e*. The line of action of the left reaction *RA* is as yet unknown. It is known, however, that the left reaction point is a point in the line of action of this force. This fact furnishes the key to the solution of the present problem. Construct the equilibrium polygon *m* . . . *n* for the pole *O*, beginning with point *m* at the left reaction point. Thus the line parallel to the ray *Oa*, which is drawn from the line of action of the force *AB* to the line of action of the reaction *RA*, becomes a point at *m*. The intersection of the

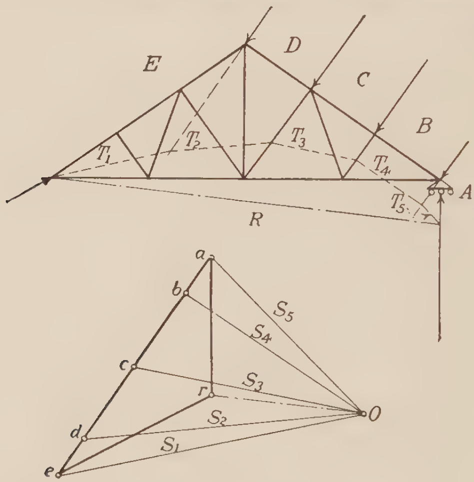


FIG. 50.

line *Or*, parallel to the closing line *mn* of the equilibrium polygon, with the vertical through *e*, determines the location of point *r*, and hence the amounts of the two reactions *er* and *ra*.

It is essential that the equilibrium polygon be commenced at the left reaction point; otherwise the points at which the line parallel to the ray *Oa* should terminate in the equilibrium polygon could not be located. It should also be noted that in this method the line T_5 of the equilibrium polygon is drawn from the line of action of the load *ED* to the known line of action of the right reaction *i.e.* to a vertical through the right support. Compare carefully this solution with that of the preceding problem.

Figure 50 shows the application of the last described method to the same truss when the wind is blowing on the free side.

The fundamental principles involved are unchanged. The equilibrium polygon is begun at the left reaction point as before, for the reason that this is the only known point in the line of action of the left reaction. The line from this point to the line of action of force *ED*, parallel to the corresponding ray *Oe* of the force polygon, is not a point as was the case in the solution with the wind on the fixed side. The reason for this difference is obvious from a study of Fig. 50.

42. Wind-load Stress Diagrams. After the wind-load reactions, for any given conditions of loading and support, have been determined as outlined above, the stresses produced in the members of the truss by the wind loads may be determined by the application of the principle of the force polygon to each joint in succession in a manner similar to that explained in Art. 38 for dead loads. The only difference between the dead-load and wind-load stress determinations is that in the former case the load line is vertical, while in the latter case the load line is inclined. When one end of the truss is free, an additional difference in the solution exists—the wind-load reactions do not coincide with the load line. This condition also exists in the case of trusses with fixed ends, if the upper chord is broken. These differences, however, are matters of no consequence in the stress determination as will be seen in the following examples.

Let it be required to determine the wind-load stresses in the members of the truss of Fig. 37, both ends fixed, with the loads as found in Art. 37. The reactions for the given conditions were found in Fig. 41. The truss is repeated in Fig. 51. Construct the external load force polygon *abcdera* by laying off the loads *ab*, *bc*, *cd*, *de*, and the reactions *er* and *ra* in order. Next isolate joint 1 from the rest of the truss by cutting a circular section around this joint. The internal stresses in *BF* and *FR* must hold the external forces *AB* and *RA* in equilibrium. They meet in a point, hence the only condition necessary for equilibrium is that they must form a closed force polygon. Through *b* draw a line *bf* parallel to *BF*; through *r* draw a line *rf* parallel to *FR*. Point *f*, located at the intersection of these two lines, is the last required vertex of the closed force polygon *abfra*. The line *fb* represents the amount of the stress in *FB* and the line *rf* represents the stress in *RF*.

As explained in Art. 38 the direction of each stress must be the same around the polygon *abfra*. Hence, since force *AB* acts

downward, that is from a to b, the other forces must continue in this direction. Transferring these directions to the truss diagram, the stress in BF acts toward the joint 1. BF is therefore in compression. The stress in FR acts away from joint 1. Fr is in tension. Taking the joints in the order indicated by the numbers at the joints in Fig. 51, the complete stress diagram is drawn as there shown.

It is seen that the stress is zero in each of the web members on the leeward side of the truss except in the one leading up to the

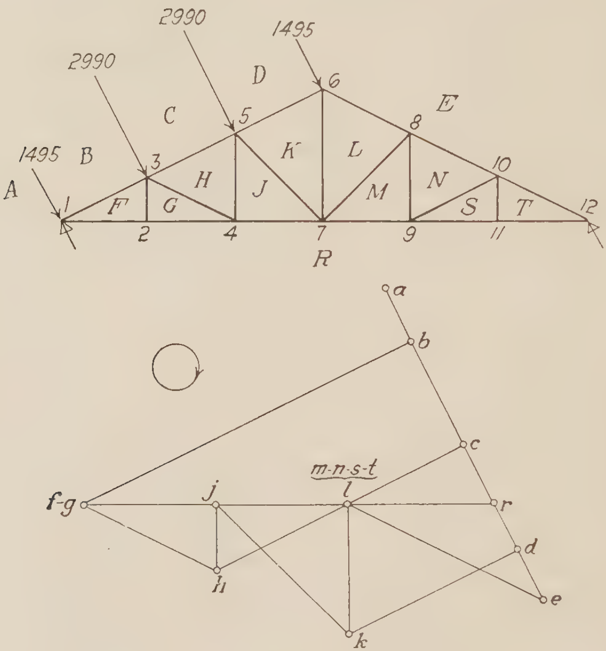


Fig. 51.

peak. The stresses in all upper chord members on this side are equal; similarly, the stresses in the lower chord members on this side are equal. The fact that on the leeward side of a truss the stresses in the web members are zero, while the chord stresses are constant throughout their respective lengths, may be checked analytically as follows. Summation of vertical components about joint 11 shows that there is no stress in the vertical, while summation of horizontal components shows equal stresses in the two chord segments. Similarly at joint 10, a summation of the

forces normal to the upper chord, shows no stresses in either of the web members meeting the chord at that point, while a summation of the forces parallel to the chord shows equal stresses in the two segments. Similar results may be obtained by taking summations at joints 9 and 8.

By means of the principle of the circular arrow the character of the stress in each of the various members is found to be as follows:

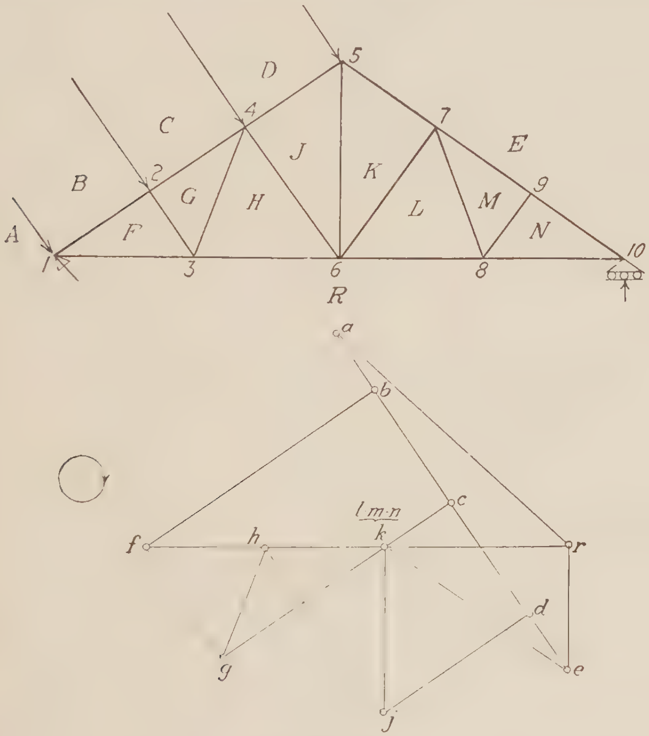


FIG. 52.

in each upper chord member, compression; in each lower chord member, tension; in web members *HJ* and *KL*, tension; and in *GH* and *JK*, compression.

Next let it be required to determine the wind-load stresses in the members of the truss of Fig. 49, with one end free and the wind blowing on the fixed side. The truss is repeated in Fig. 52. Draw the closed external load force polygon *abcdera*, the point *r* having been found in Fig. 49. Isolate joint 1 from the rest of the truss as in the preceding problem. Construct the closed

force polygon *abfra* for the external loads *AB* and *RA* and the internal stresses *BF* and *FR* acting at this joint, by drawing through *b* a line *bf* parallel to *BF*, and through *r* a line *rf* parallel to *RF*. The point of intersection of these two lines completely determines the required polygon. The stress in *FR* is represented by the line *fr* and the stress in *BF* by the line *bf*. By means of the principle of the circular arrow, *FR* is seen to be in tension—

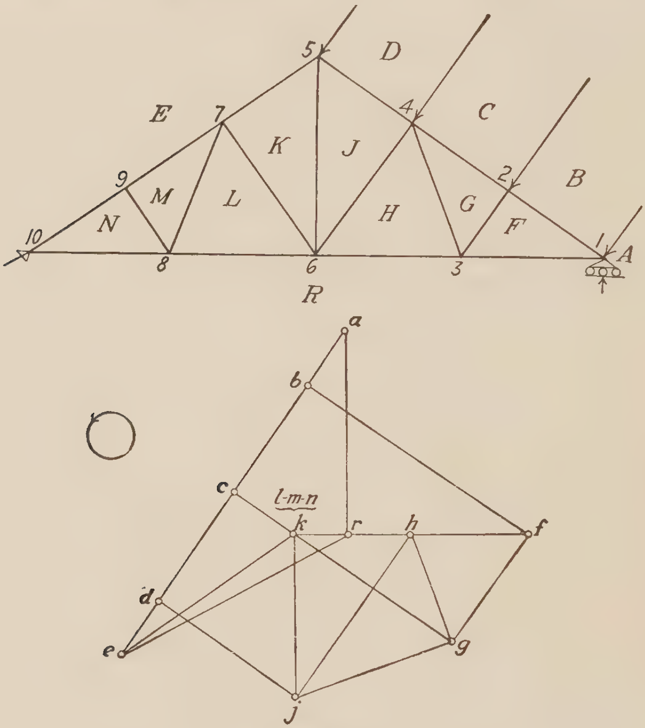


Fig. 53.

the stress acts from *f* to *r*, toward the right, or away from joint 1. Similarly *BF* is in compression.

By applying the same method to each of the remaining joints in proper order, the stress in each of the members of the truss is found as shown in Fig. 52. Here, again, the stress in each web member (except the one leading to the peak) on the leeward side of the truss is zero; the stresses in all the upper chord members on this side are equal; similarly the stresses in the lower chord members on this side are equal.

As a third illustration of the method of determining wind-load stresses, let it be required to construct the stress diagram for the same truss as used in the preceding problem, with the same loads but with the wind blowing on the free side instead of on the fixed side. The reactions for this condition were found in Fig. 50. In Fig. 53 construct the closed external load force

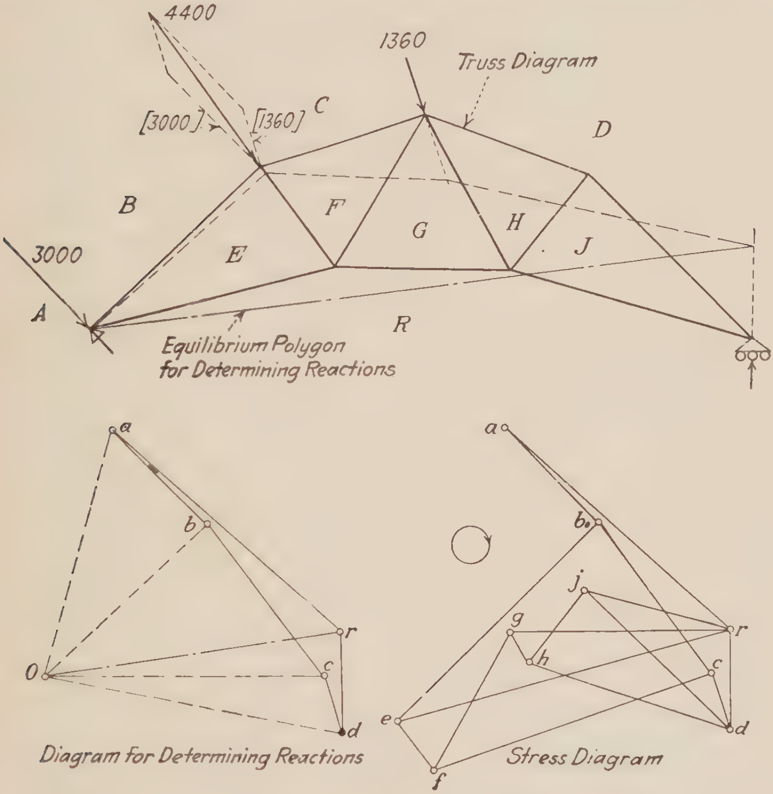


FIG. 54.

polygon *abcdera;* cut a circular section around joint 1 and draw the closed force polygon *abfra* for the loads and stresses involved in this section. The stress in *RF* is represented by the line *rf* and the stress in *BF* is represented by the line *bf*.

The complete stress diagram is drawn as shown in Fig. 53. The upper chord members are all in compression; the lower chord
upper chord members are all in compression; the lower chord

members HR and FR are in tension, whereas LR and NR are in compression; the web members GF and JH are in compression, GH and KJ in tension, and KL, LM, and MN, zero.

A broken upper chord and inclined lower chord do not materially complicate the internal stress determination. Let the right end of the truss of Fig. 54 be assumed as free and let the wind act on the left side. The amount of each of the wind panel loads is computed in a manner similar to that used in Art. 37 in determining the wind apex loads for the truss of Fig. 38. The reactions for the assumed conditions are found in Fig. 54 by means of the principles outlined in Art. 41. The internal stresses in the members are then determined by isolating each joint in succession, commencing at the left reaction point, and constructing a force polygon for each joint. The complete construction is shown in Fig. 54. The method followed in this construction is identical with that used in the preceding problem. The final check on the construction lies in the fact that the last line drawn in the stress diagram, *i.e.* line jr in the present case, must be parallel to the corresponding member JR in the truss diagram.

In all problems similar to the above, two distinct steps are essential to a complete solution; first, the unknown reactions must be determined by taking the entire truss as a free body, and second, the internal stresses must be determined by isolating each joint in succession from the rest of the truss. In the first step, two conditions are necessary for equilibrium—the force polygon must close, and the equilibrium polygon must close. In the second step, since all of the forces involved at any one joint meet in a point, only one condition is necessary for equilibrium—the force polygon must close.

43. Ambiguous Trusses. Some types of trusses are apparently graphically indeterminate, that is, a graphical solution seems impossible, because more unknown conditions exist than can be solved by the intersections of known lines. With some of these trusses, a solution is possible if the indeterminate condition is temporarily replaced with a determinate condition from which sufficient data may be obtained to enable a complete solution of the original truss to be made.

The Fink truss is a typical structure in which such a condition exists. In its simplest form this truss consists of straight upper chords and either a horizontal or an arched lower chord, web braced as shown in Figs. 55 and 58. The two short, and one

long, compression braces on each side are perpendicular to the upper chord that they support. The upper chord panels are not necessarily equal, nor is the truss necessarily symmetrical about its vertical center line.

In the graphical solution of the truss shown in Fig. 55, when the second upper chord joint is reached the stresses in the members *CH* and *HJ* are known and those in the members *DL*, *LK*, and *KJ* are unknown. As stated in Art. 20, not more than two unknown quantities can be determined at any one joint in the graphic analysis of the forces acting at that joint. Here, then, an indeterminate condition exists with the webbing shown, unless other conditions may be brought into play, such, for example, as making use of the knowledge of symmetry between any of the members involved, etc. Since these extra conditions do not exist in all forms of Fink trusses, a general solution is preferred—a solution that may be applied to any irregular truss of the Fink type. In the method given below, the webbing is temporarily changed and the stress diagram for this changed system carried beyond the point where the change produces any effect upon the stresses in the members of the original truss. The original system of webbing may then be restored and the stress diagram for the actual truss completed.

Let it be required to determine the stresses in the members of the Fink truss of Fig. 55 with the dead panel loads as shown. Lay off the loads *AB*, *BC*, *etc.*, on the load line *a* . . . *a′* (Fig. 57). Because of the symmetry of the truss and loads, point *r* is midway between *a* and *a′*. Next isolate each of the joints 1, 2, and 3 from the rest of the truss in that order, and construct the stress diagram *abgrjhc* for this portion of the truss. This portion of the stress diagram is shown by the heavy lines in Fig. 57. Further construction is impossible since three unknown stresses exist at either the next upper chord joint, or at the next lower chord joint.

Remove the members *KL* and *LM* and temporarily replace them with the one member *XM* as shown in Fig. 56. The stresses previously found are not affected by this substitution, and the remainder of the stresses in the revised truss may be determined, since, as the construction proceeds from joint to joint in the usual manner, at no joint will there exist more than two unknown stresses. The complete stress diagram for the left half of the revised truss of Fig. 56 is shown in Fig. 57, omitting point *k*.

Next compare the truss of Fig. 55 with that of Fig. 56. A circular section around the peak joint shows that, since no changes were made at this joint either in the amount of the external load or the position of the internal members, the stresses in members *EM* and *MN* (Fig. 55) are equal, respectively, to the stresses in *EM* and *MN* (Fig. 56). The stress in member *EM*

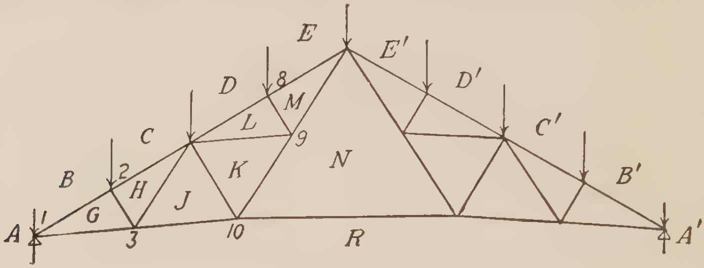


Fig. 55.

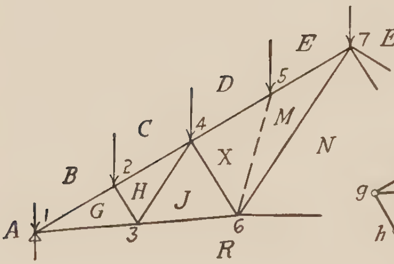


Fig. 56.

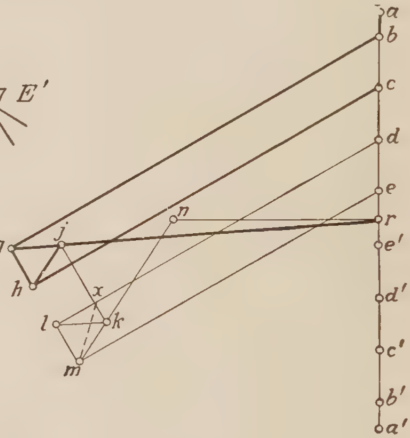


Fig. 57.

of the original truss is, therefore, represented by the line *em* in the stress diagram just drawn for the revised truss. The stresses in *LM* and *DL* (Fig. 55) are now found by drawing the force polygon *delm* for the forces acting at joint 8. Next, the stresses in *KL* and *KN* are determined by means of the force polygon *klmn*, drawn for joint 9, and then the only remaining unknown stress, that in the member *NR*, is obtained from the force polygon *nrjk* for joint 10. The complete stress diagram for the original truss is shown in Fig. 57, omitting point *x*. The character of the

stress in any member may be determined by the principle of the circular arrow, as previously explained. The joints in the two truss diagrams have been numbered to correspond to the order in which the construction must proceed. The stress diagram is

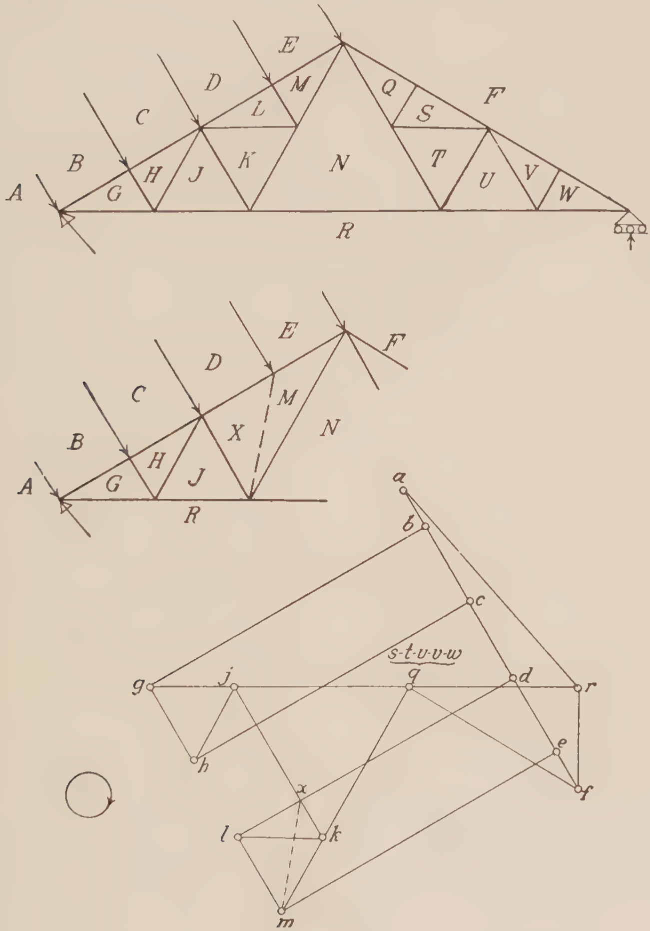


FIG. 58.

shown for one-half of the truss only. If the loads and members are symmetrical, the stress diagram for the entire truss will be symmetrical about a horizontal line through point r in Fig. 57.

If the loads BC and DE are equal, and the upper chord panels are all of the same length, HJ and LK will have equal stresses on account of their symmetrical positions in the truss, and gh and

lm will therefore lie in the same straight line. If the loads *BC* and *DE* are unequal, and the upper chord panel lengths equal, the stress diagram may be completed by noting that the point *k* must lie midway between the parallels *dl* and *em*. This is true because of the fact that *LM* is normal to the upper chord and hence is the altitude of an isosceles triangle, the equal sides of which are *LK* and *MN*. Triangle *lkm*, two of whose sides are parallel to *LK* and *KM*, respectively, must also be of the same form, and point *k* must lie as noted above. If *DL* and *EM* are of unequal lengths, then neither of these abbreviated solutions may be applied, and the general solution outlined above must be used.

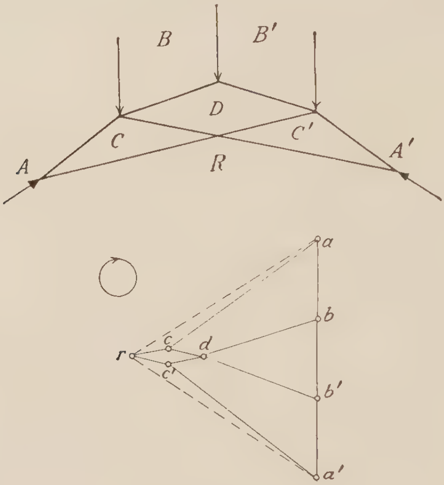


FIG. 59.

The determination of wind-load stresses in a Fink truss is carried on in a manner similar to that described above for dead-load stresses. Figure 58 shows the necessary construction for determining the stresses in the members of the truss shown, under the given panel loads. The right end is free and the wind is blowing on the left side. The reactions for this truss with the given loads and method of support, were found in Fig. 45. Point *x* was used in the construction only to make possible the solution of the indeterminate joint on the upper chord, and is not to be considered a part of the final stress diagram.

Other forms of trusses may have peculiarities which require special consideration. In the truss of Fig. 59 the reactions must be inclined in order to maintain a condition of equilibrium.

For this reason, three unknown quantities exist at each reaction point. The stress diagram must therefore be commenced at the peak joint in order to obtain a complete solution of the truss. The necessary construction is indicated in Fig. 59. The inclined reactions are found to be ra and $a'r$; point r is located by the intersection of lines cr and $c'r$, parallel to the truss members CR and $C'R$, respectively. If the solution had been commenced with the assumption that the reactions are vertical, the stress diagram would not close if carried to completion. If it is impossible to secure the necessary horizontal force against the truss at the supports, a horizontal tie could be placed between the two reaction points; this additional member would then provide for the horizontal force, and the reactions would be vertical as was the case in the truss of Fig. 40.

44. Trusses with Ceiling Loads. When a ceiling is suspended from the lower chord, the load brought by it to the truss must be considered in determining the total stresses in the truss members. Since the ceiling is a fixed part of the structure, its weight may be combined with the other vertical loads in the dead-load stress diagram, thus avoiding the necessity of drawing a separate ceiling-load diagram. In order to make the solution of this combined diagram possible, it is necessary that the loads and reactions be laid off in regular order around the truss. Thus, in Fig. 60, the force polygon $afgka$ is drawn by laying off the loads AB, BC, CD, DE, and EF, downward in that order; then the reaction FG upward; next the downward loads GH, HJ, and JK, followed by the upward reaction KA to complete the force polygon. This sequence is shown at the right of the load line in the figure.

The amounts of the lower chord apex loads are determined by considering that one-half of the ceiling weight for each panel is transmitted to the apex at either end of that panel. The reactions may be determined analytically or graphically. In this case since the loads and truss are symmetrical, each reaction is equal to one-half of the sum of the upper chord and lower chord apex loads. This value may be obtained graphically by drawing a line, the length of which is equal to the sum of the loads AB, BC, CD, DE, EF, GH, HJ, and JK; each reaction is equal to one-half of the amount represented by this line.

After the loads and reactions are laid off in the proper order, as explained in the second paragraph above, the construction of

the stress diagram is completed in a manner identical with that used for trusses with upper chord loads only—each joint is isolated in succession from the rest of the truss and a series of connected force polygons drawn, one for each joint. The complete stress diagram for the above truss and loading is shown in Fig. 60. The force polygon for the left end joint is *alka;* for the first upper chord joint *abmla, etc.*

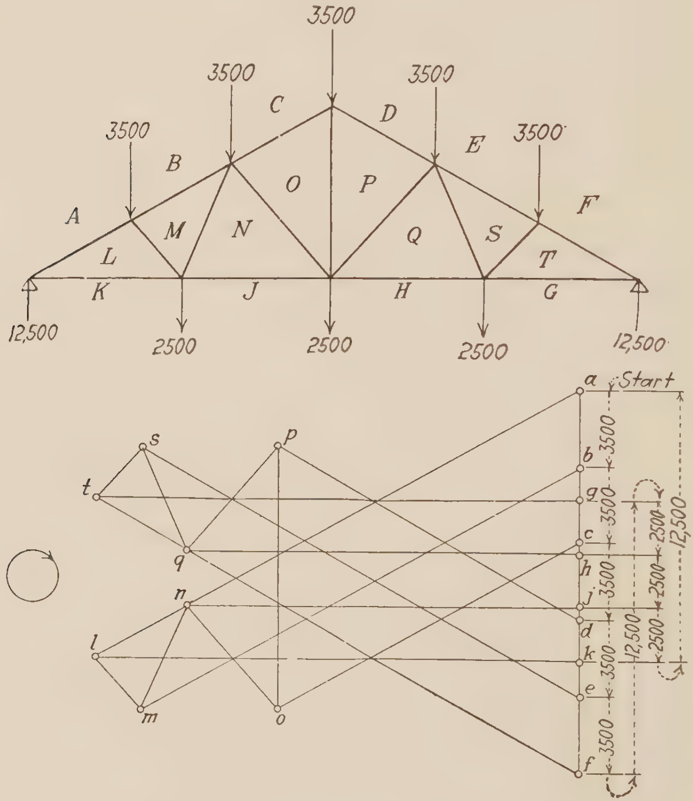


Fig. 60.

45. Unsymmetrical Loads and Trusses. All trusses are not symmetrical. Figure 61 illustrates a form of truss used in "saw-tooth" factory roof construction. The truss weight is divided among the panels *BF*, *CF*, and *DG* in proportion to the lengths of these panels; there is no snow load on panel *BF* because of the steep slope of this panel; the apex load at the peak is one-half of the sum of the loads on panels *BF* and *CF*; load *CD* is one-half

of those on panels CF, and DG, and loads AB and DE one-half of those on panels BF and DG, respectively.

The reactions for the loads shown in the figure are found by means of the force and equilibrium polygons in a manner similar to that explained in Chapter II for simple beams, point r being located as shown. The stress diagram is obtained by isolating

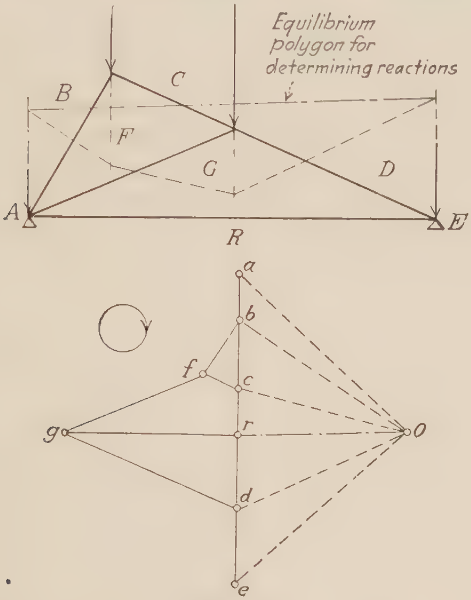


FIG. 61.

each joint in succession from the rest of the truss, commencing either at the peak or at the right reaction point, and drawing a series of force polygons for the loads and stresses involved. The construction could not have been begun at the left reaction point because three unknown stresses exist at this point. The complete stress diagram is shown in Fig. 61. All of the members of the truss are in compression, except the horizontal tie GR, which is in tension.

Very often a truss, either symmetrical or unsymmetrical in form, is called upon to support a single concentrated load at one of its lower chord apexes. Heavy shafting in a machine shop is a typical example of such a condition. The stresses due to this load alone are found in the same manner as for other vertical loads; the reactions are first found by means of the force and

equilibrium polygons drawn for this single load, and then the stresses in the various members are obtained by drawing the force polygons for successive joints. In Fig. 62, *ab* represents the given load *AB*. Line *Oc*, drawn parallel to the closing line *ms* of the equilibrium polygon *mns*, two of whose sides are parallel to the rays *Oa* and *Ob*, divides the load line *ab* into the two parts *bc* and *ca*, representing the right and left reactions, respectively. The complete stress diagram, obtained as in the preceding problems, is given in the figure.

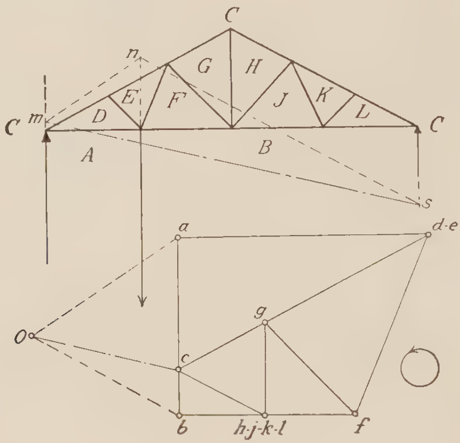


Fig. 62.

46. Maximum Stresses. The greatest stress that will occur in any member of a roof truss is the algebraic sum of the individual stresses caused by the simultaneous action of the dead loads, snow loads, wind loads, ceiling loads, and special loads. The resulting stress is called the maximum stress.

Since the wind may come from either side, the character of the stress caused by its pressure in any one member is variable—it may be tension with the wind on one side and compression with the wind on the other side. The character of the stress caused in any one member by a vertical load in any one position on a simple truss is constant—the stress may be tension or compression, but if tension for one vertical load in this position; it will be tension for any other vertical load in this same position, and *vice versa*. The stress due to wind pressure, which is added to the other stresses to obtain the maximum stress, must therefore be the greatest stress of the same

character as that caused by the vertical loads. A complete analysis of any roof truss necessarily requires that the stresses due to wind pressure be found for two conditions; when the wind comes from one side of the truss, and when it comes from the other side.

It is generally assumed that full snow-load stresses are not likely to occur when the full force of the wind is considered. The combination of dead and snow loads must then be compared with the combination of dead, partial snow, and wind loads.

47. Bridge Trusses. The procedure to be followed in the analysis of bridge trusses for dead loads is similar to that used in the foregoing articles. However, because of the comparative ease with which the analytical solution may be made, it is to be preferred for most types of bridge trusses (See Chapter V).

CHAPTER IV

STRESSES IN FRAMED BENTS

48. Introduction. In shop and mill buildings each roof truss is usually supported on two columns, one at either end of the truss. The trusses are generally braced transversely by diagonal members, called *knee braces,* which join the lower chords with the columns. The trusses are braced laterally, generally in units of two, by means of diagonal and longitudinal members in the plane of the lower chords and in the plane of the upper chords. Each of these braced units of two trusses is connected to the adjacent units with transverse struts at the panel points of the upper and

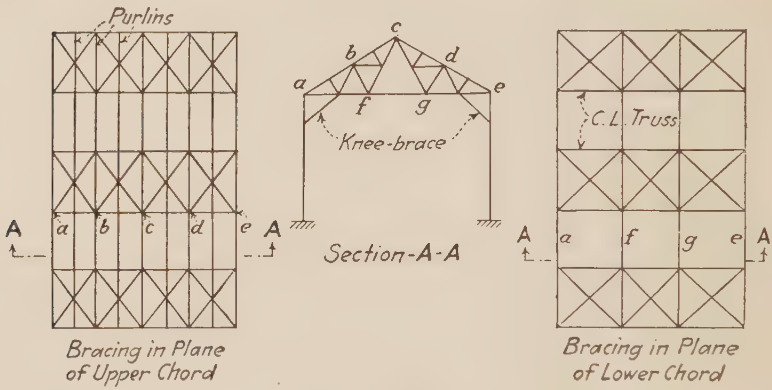


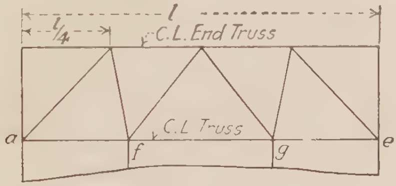
Fig. 63.

lower chords to increase further the rigidity of the entire structure. Only the bent with knee bracing will be considered in the following discussion since it involves all of the theory that applies to bents without such bracing, and in addition requires the further consideration of the stresses in the knee braces and the effect of these braces on the columns.

The truss framing and lateral bracing of a typical mill building are shown in Fig. 63. One truss and its supporting columns constitute a transverse *bent;* the framing of the entire building is made up of a series of parallel bents and the bracing

connecting these bents. The portion of the structure between two successive bents is referred to as a *bay*. In some cases, the bracing in the plane of the lower chord for the end bays is modified in order to produce equal lateral truss panel lengths on the ends of the building. The vertical members of the end framing may then be spaced equally across the building, supported at their upper ends by the lateral bracing. A common form of lateral bracing in the plane of the lower chords, for the end bays, is shown in Fig. 63(*a*). Very often the diagonal members of the lateral bracing system are omitted in all bays except those at the ends of the building.

In order to give better ventilation and light an auxiliary framework, or monitor, may be placed at the peak of the building.



Bracing in Plane of Lower Chord End Bay

Fig. 63(*a*).

The vertical sides of the monitor are fitted with either glazed sash or slats of metal or wood, called louvres, so placed as to permit air to enter the building without admitting rain and snow. Louvres may be fixed or movable. The monitor may extend the full length of the building or may be placed over any portion of the building. A typical bent with a monitor is illustrated in Fig. 64.

The determination of stresses in a framed bent is complicated by the fact that the horizontal component of the loads on the roof and sides of the building causes bending in the columns; the bending moment causes distortion of the columns and the distortion affects the stresses in the truss members. The analysis of a framed bent therefore requires a general solution which differs considerably from that used in simple trusses supported on masonry walls.

49. Loads on Mill Building Bents. The loads for which the stresses in a framed bent must be determined are the dead load, snow load, wind load and miscellaneous loads. The dead load includes the weights of the roof covering and its supporting pur-

lins, the lateral bracing, and the truss proper. While any of the types of roof covering mentioned in Art. 36 may be employed for the mill building bent, the type of roof covering most widely used consists of corrugated steel sheathing with some form of anticondensation lining on the under side, supported directly on the purlins. The purlins are placed only at the panel points if this does not make too great a span for the corrugated steel covering. If the upper chord panels are too long to permit of this arrangement, intermediate purlins are placed on each upper chord member. In such cases the upper chord members must be designed for bending stresses in addition to the direct stresses. If such an arrangement is undesirable, additional web braces may be used in the truss to obtain a shorter panel length. This arrangement is shown in Fig. 64, the additional braces being indicated by the broken lines.

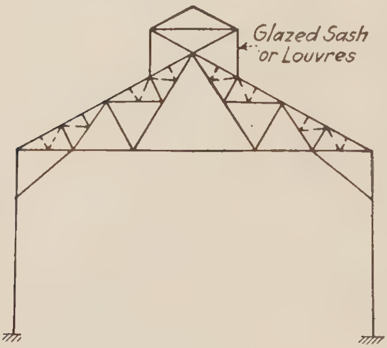


Fig. 64.

The exact weight of the roof covering and purlins may be computed after the type of covering has been selected and the purlins designed for the given loads. The weight of the bracing varies between fairly narrow limits. A value of from 0.5 to 1.0 lb. per square foot of horizontal covered area will usually provide for both the upper chord and lower chord bracing. The weight of the truss proper may be calculated by empirical formulae or it may be obtained from a comparison with existing structures of similar type and size. Milo S. Ketchum recommends the following formula for computing the probable weight of steel mill building trusses of the Fink type.

$$w = \frac{P}{45}\left(1 + \frac{l}{5\sqrt{A}}\right)$$

in which w = weight of truss in pounds per square foot of horizontal covered area

P = estimated load in pounds per square foot of horizontal projection of roof area

l = span of truss in feet

A = distance between trusses in feet.

A value of $P = 40$ lb. per square foot in the above equation is satisfactory for buildings in normal climates. As in the case of trusses in Chapter III, all of this load is usually assumed as concentrated at the upper panel points.

The snow and wind loads on the truss are figured as in Arts. 36 and 37 for simple trusses supported on masonry walls. The normal wind pressure on the sides and ends of the building is usually taken as 20 lb. per square foot of exposed vertical surface. This load is transmitted to the columns by means of horizontal beams, called girts, which are placed between the columns in the planes of the side walls. The girts are spaced sufficiently close vertically so as to furnish proper support for the covering of the side walls.

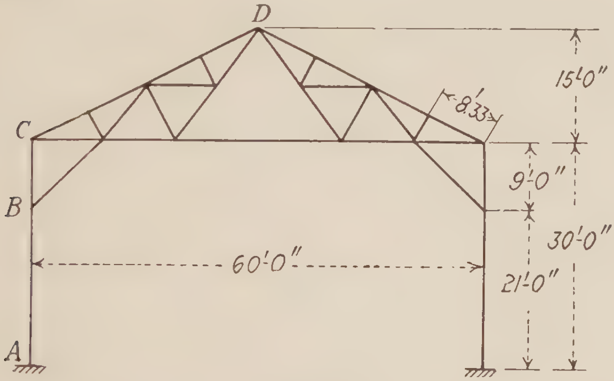


Fig. 65.

Miscellaneous loads include any loads that are supported by the truss other than those previously considered. These miscellaneous loads generally require special attention and each problem must be solved so as to satisfy the existing conditions.

Let it be required to determine the dead, snow, and wind panel loads acting on the bent of Fig. 65. The trusses are spaced 16 ft.-0 in. center to center. The weight of the roof covering will be assumed as 3.2 lb. per square foot, the weight of purlins, 3.3 lb. per square foot, and the weight of bracing 0.5 lb. per square foot of horizontal covered area.

The unit weight of the truss, computed from the formula on page 84 is

$$w = \frac{40}{45}\left(1 + \frac{60}{5\sqrt{16}}\right) = 3.6 \text{ lb. per square foot}$$

of horizontal covered area.

The total unit dead load is 10.6 lb. per square foot of horizontal covered area and the total load on one truss $10.6 \times 16 \times 60$, or 10,200 lb. Each intermediate panel load is $\frac{10,200}{8}$, or 1280 lb. and each end panel load 640 lb.

The snow panel loads are computed according to the same specifications as given in Art. 36. The angle of inclination is $\tan^{-1} (15/30)$ or 26 deg., 34 min., and the corresponding snow load is 19 lb. per square foot of horizontal covered area. The snow panel load for each intermediate point is $19 \times 16 \times 60/8 =$ 2280 lb. and for each end apex 1140 lb.

Assuming a horizontal wind pressure of 30 lb. per square foot on a vertical surface, the normal component of the wind acting on the truss, from the table on page 49, is 22.4 lb. per square foot of roof surface. Each intermediate wind panel load is $22.4 \times 8.33 \times 16$, or 2980 lb., and the wind load at each of the points C and D is 1490 lb. Assuming a horizontal pressure, due to the wind, on the sides of the building of 20 lb. per square foot, the load at B from this source is $\frac{9 + 21}{2} \times 16 \times 20$, or 4800 lb.; the load at A is $21/2 \times 16 \times 20$, or 3360 lb., and the load at C, $9/2 \times 16 \times 20$, or 1440 lb.

50. Stresses Due to Dead and Snow Loads. Under the action of dead and snow loads there are no horizontal forces acting on the bent. The stresses in the truss members are therefore the same as though the truss were supported on masonry walls, and the stress diagram is drawn as explained in Arts. 38 and 43. The stresses in the columns are those caused by the load from the roof and the weight of the side walls supported by the columns. These are direct compression unless the columns are assumed to be fixed at the top, in which event the deflection of the truss will cause additional stresses due to bending. Since, however, the joint at the top of the column is usually not made sufficiently rigid as to warrant the assumption of fixity at this point, a hinged condition may be considered in the usual case.

The stresses in the members of the bent, shown in Fig. 65, due to the dead apex loads as computed in Art. 49 are found in the stress diagram shown in Fig. 66. On account of the symmetry of the truss and loads acting on it, each reaction is equal to one-half of the total load on the truss. These stresses are tabulated on page 104 for the purpose of finding the maximum

stresses as explained in Art. 56. Since the ratio between the snow and dead panel loads is constant at all panel points, the snow-load stresses given in the table referred to above are

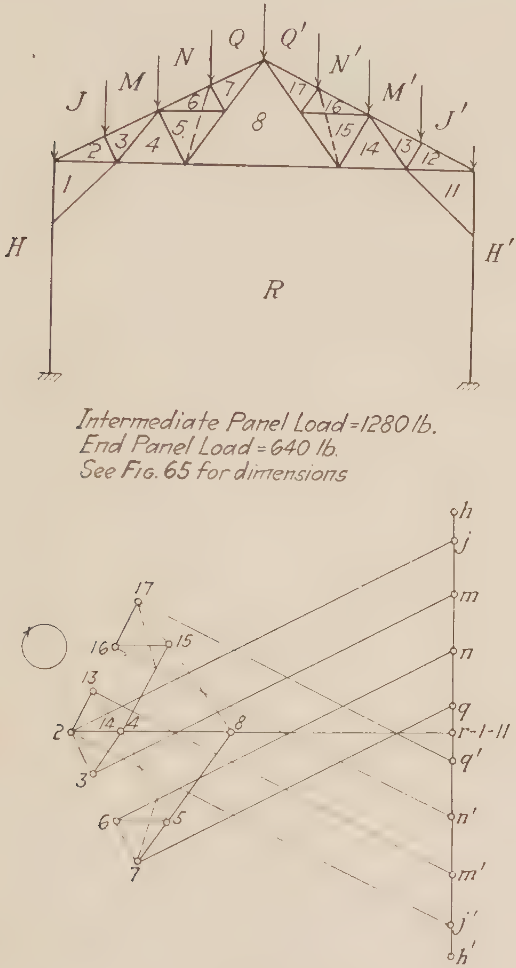


Fig. 66.

obtained by multiplying the dead-load stresses by the ratio of the snow to dead loads, or

$$\text{Snow-load stress} = \text{Dead-load stress} \times \frac{2280}{1280}$$

For each of the above classes of loading the stress in the knee brace is zero. This may be seen to be true by taking the summation of horizontal components at the foot of the knee brace.

51. Stresses Due to Wind Loads. The horizontal components of the inclined loads acting on a framed bent are transmitted to the foundations by the columns. These horizontal forces tend to cause distortion of the bent. This tendency is partially resisted by the bending stresses developed in the columns. The base of each column is so anchored to the foundation as to prevent lateral displacement, and a shearing stress is thereby developed in the column, thus making a total of three kinds of stress—direct stress, bending stress, and shearing stress.

The graphical solutions of roof trusses with wind loads as given in Chapter III, apply only to structures in which each member is subjected to but one kind of stress—direct tension or compression. The conditions outlined above will therefore materially modify the solution of roof trusses supported on columns, knee braced to the columns, when inclined loads act upon the bent thus formed.

The solution is further affected by the method in which the columns are supported on the foundations. Two cases will be considered in the following discussion: (1) each column is so anchored to its foundation as to prevent horizontal displacement of the base, but not in such a way as to prevent rotation at this point; when constructed in this manner the base of the column is considered as *hinged:* (2) each column is rigidly anchored to its foundation in such a manner as to prevent both translation and rotation at the base; such a column is considered to be *fixed* at this point. In both cases the top of the column will be assumed as hinged for the reason given in Art. 50.

In order to determine the internal stresses, all of the external forces must be known. It is therefore necessary to find the reactions at the column bases by considering the bent, as a whole, before the stress diagrams can be drawn.

52. Reactions Due to Wind Loads. Bases of Columns Hinged. Figure 67 represents a knee-braced bent with the columns hinged at the bottom. For clearness, the web members of the truss are omitted. Loads are assumed equal to P_1, perpendicular to the side walls, and P_2, normal to the roof surface. These loads cause reactions R_1 and R_2 at the left and right supports, respectively. It is required to find these reactions.

The given loads may be combined, by means of the principle of the parallelogram of forces, into one force P, acting through the point of intersection of the lines of action of the two forces in a direction parallel to the diagonal of the parallelogram formed on these two forces as sides. Three equations of static equilibrium are available for the solution, whereas four unknown quantities exist, namely, the vertical and horizontal components of both R_1 and R_2. The problem is therefore indeterminate unless some assumption is made which will establish a relation between two or more of the required forces.

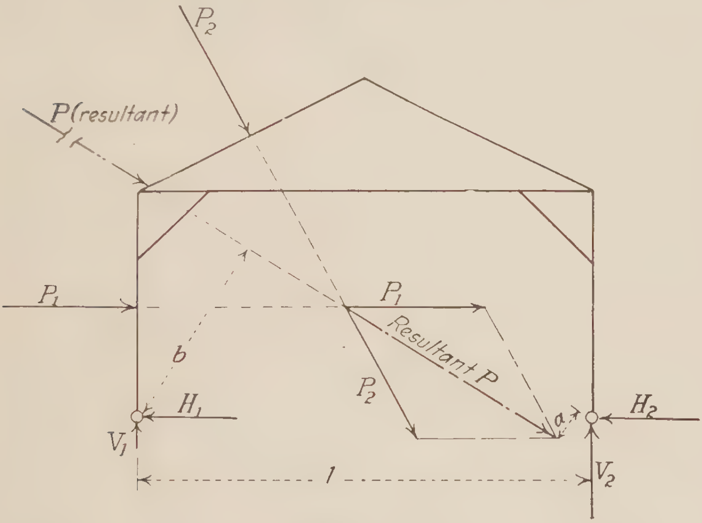


Fig. 67.

The loads are transmitted to the foundations by the columns. If the truss is sufficiently rigid and the columns are of equal size and equal rigidity, it is reasonable to assume that the total horizontal force will be transmitted equally by the columns. Thus it may be assumed that the horizontal component of R_1 will be equal to the horizontal component of R_2, and the fourth relation necessary to the solution of the present problem is obtained.

If H_1 and H_2 represent the horizontal components of R_1 and R_2, respectively, and H the horizontal component of the resultant wind load P,

$$H_1 = H_2 = \frac{H}{2}$$

The vertical component V_1 is obtained by taking moments about the right reaction, and

$$V_1 = \frac{Pa}{l} \tag{1}$$

Similarly by taking moments about the left reaction

$$V_2 = \frac{Pb}{l} \tag{2}$$

The values of R_1 and R_2 are then found in amount and direction by combining their vertical and horizontal components. As a check on the construction it should be noted that R_1 and R_2 should intersect on the line of action of the resultant load P, since for equilibrium, the three forces acting on the bent must meet in a point.

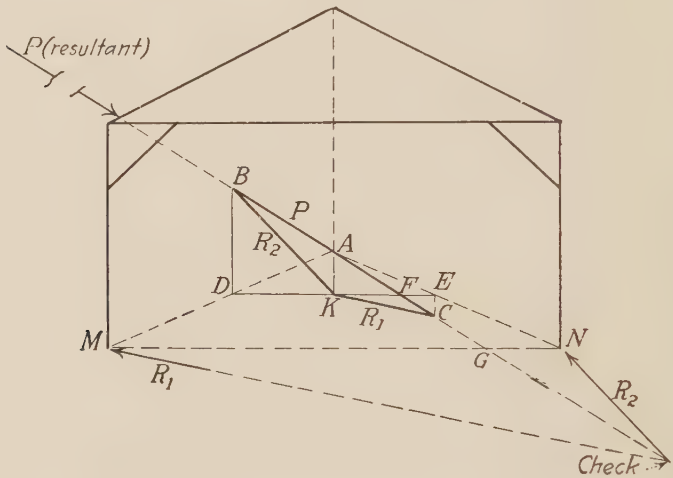


FIG. 68.

The reactions are obtained graphically with much less effort than is required in the analytical solution outlined above. In Fig. 68, P represents the resultant wind load as determined in Fig. 67. The line of action of this force is produced to intersect the vertical center line of the truss at A (Fig. 68). The load is laid off through A so that this point becomes the mid-point of the load, that is $AB = AC$, and $BC = P$. From the extremities B and C, vertical lines BD and CE are drawn to intersect the lines drawn from A to the left and right reaction points, respectively. BD and CE represent V_2 and V_1, the vertical compo-

nents of R_2 and R_1, respectively. This statement is proved as follows: Continue BC until it intersects the horizontal MN at G and consider BC resolved into its vertical and horizontal components at this point. By taking moments about each reaction point it follows, since the moments of the horizontal component of BC about the respective centers of moments are zero, that the vertical components of the reactions are inversely proportional to their respective distances from G, that is

$$\frac{V_1}{V_2} = \frac{GN}{GM} \tag{a}$$

From the similar triangles BDF and FEC,

$$\frac{V_1}{V_2} = \frac{FE}{FD} \tag{b}$$

Since DE is horizontal by construction,

$$\frac{FE}{FD} = \frac{GN}{GM} \tag{c}$$

Hence, from (b) and (c)

$$\frac{V_1}{V_2} = \frac{GN}{GM}$$

which satisfies the analytical relation given in (a).

The intersection K of DE with the vertical through A determines the horizontal component of each reaction, and the lines KB and KC completely establish the reactions R_2 and R_1, respectively, since KB represents the result of the composition of H_2 and V_2, and KC results from the combination of H_1 and V_1.

Another graphical solution of the same problem may be obtained as follows: In Fig. 69(b) lay off the line ab equal and parallel to the resultant force P acting on the bent of Fig. 69(a). From any convenient point m on the line of action of P, draw lines mn and mk to the left and right reaction points, respectively. Since n and k are points in the lines of action of R_1 and R_2, respectively, the figure mnk represents an equilibrium polygon for the external forces acting on the bent. The closing line nk is horizontal. Through point a in Fig. 69(b) draw a line aO parallel to mn, and through point b draw a line bO parallel to mk. From the point O, located by the intersection of these two lines, draw a horizontal line Oc to intersect a vertical dc through the midpoint of ab at c. The right reaction is represented by the line bc and the left reaction is represented by the line ca.

The proof of this construction is evident from a study of the figure; the triangle mnk of Fig. 69(a) represents an equilibrium polygon drawn for the pole O and the force polygon abc of Fig. 69(b), and by construction the horizontal component H_2 of R_2 is equal to the horizontal component H_1 of R_1, and $H_1 + H_2 = H$, the horizontal component of the resultant wind load. The assumed division of H, and the required conditions for static equilibrium, namely, that the force and equilibrium polygons must close, are satisfied, thus proving the construction.

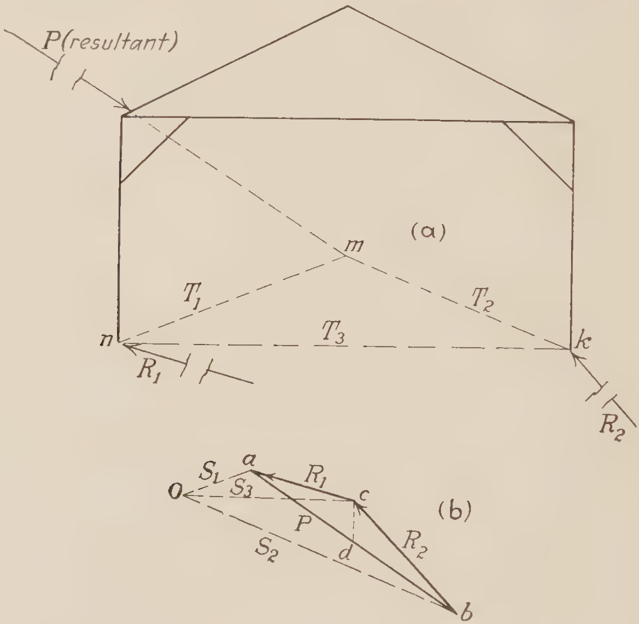


Fig. 69.

It should be noted that the above solution is the reverse of that used in Chapter III, since in the latter case the pole O in the force polygon was first assumed, and the equilibrium polygon drawn for that pole; in the solution just given the pole in the force polygon is located after the equilibrium polygon is drawn. This order of procedure is necessary since only one point in the line of action of each reaction is known, and the equilibrium polygon must pass through this known point in each line.

In order to illustrate the construction necessary to a complete determination of the reactions by the latter method, let it be

required to determine the reactions for the bent shown in Fig. 70 due to the given wind apex loads. The two forces acting at each of the points 1, 2, and 3, due to the wind pressure on the adjoining panels, are combined in each case by completing the parallelogram for the two forces as indicated. This procedure simplifies the construction, as only one force need be considered acting at each apex.

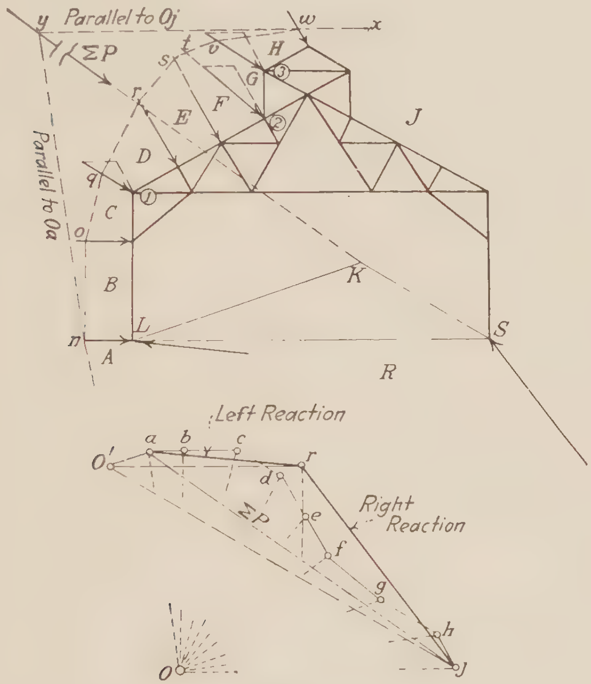


Fig. 70.

Construct a force polygon *ab* . . . *j* for the given loads by laying them off in regular order. Close the figure thus formed with the right line *ja*, which represents the amount and direction of the resultant wind load. To determine its line of application construct an equilibrium polygon *mn* . . . *wx*, for any pole *O*, and extend the sides *mn* and *xw* to their intersection at *y*. Point *y* is then a point in the line of action of the resultant force. (Compare with Art. 28.)

From any point *K* on the line of action of the resultant load, draw lines *KL* and *KS* to the left and right reaction points,

respectively. Locate the pole O' that corresponds to this new equilibrium polygon LKS by drawing through a a line aO' parallel to KL and through j a line jO' parallel to KS; the intersection of these two lines fixes O'. Through O' draw a horizontal line $O'r$ to intersect a vertical through the midpoint of ja at r. The right reaction is then given by the line jr, and the left reaction by the line ra.

53. Effect of Fixing Column Bases. Figure 71 represents the distorted position assumed by a knee-braced bent with hinged column bases when an inclined load P is applied to it. Since the construction is so made as to permit rotation at the foot of the columns, the bending moment is zero at these points. The

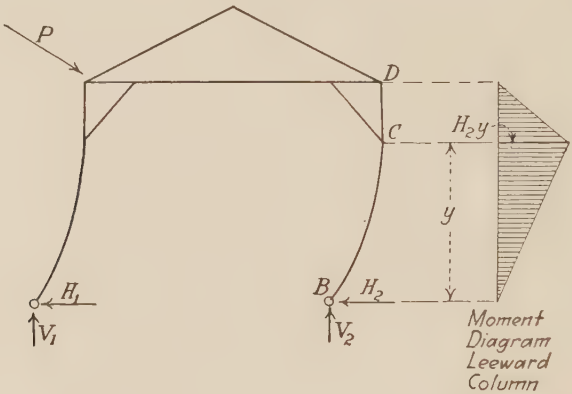


FIG. 71.

columns are riveted to the knee braces and trusses at points C and D; the portions CD will therefore remain practically vertical and the portions CB will distort as shown in the figure. The moment in the columns at point D will be zero because of the assumption of a hinged condition[1] at this point as stated in Art. 50. The moment will be a maximum at the foot of the knee brace and of an amount equal to H_2y or H_1y, in which H_2 and H_1 represent the horizontal components of the reactions at the

[1] It should be noted that in this and in the following discussion, the top of the column is assumed to be hinged when determining moments and stresses, whereas it is assumed as fixed when outlining the distorted curve, and, in the case of fixed bases to be discussed later, when determining the location of the point of inflection. The true condition lies somewhere between these two; the considerations mentioned above are both on the side of safety without marked effect on the economy, and are warranted for that reason.

leeward and windward sides, respectively, each equal to one-half of the horizontal component of P, and y is the vertical distance from the base of the column to the foot of the knee brace.

Figure 72 represents the distortion of the same bent with the same load when the bases of the columns are fixed rigidly to the foundations. As in the preceding case, DC may be assumed to remain vertical after distortion. Since B is fixed, the tangent to the distorted column at the base is approximately vertical. Between B and C the curvature reverses in order to fit the two tangents at B and C. Thus at some point E the curvature changes from concave left to convex left. The point E is the point of inflection, or the point at which the moment changes from positive to negative, and hence the point of zero moment. At this point an imaginary hinge may be assumed.

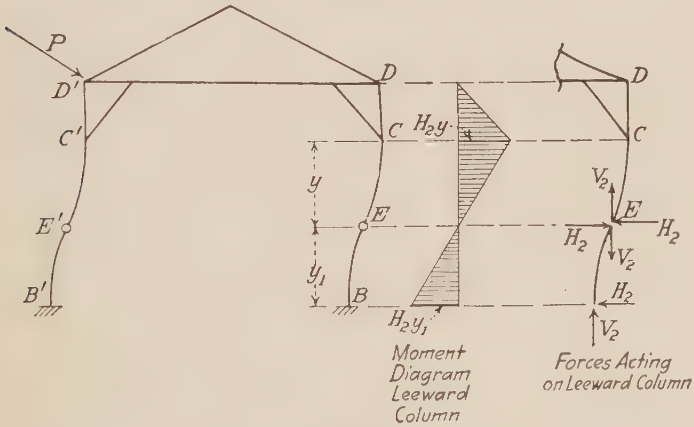


FIG. 72.

The portion of the bent above E can be treated as in Art. 52 to determine the values of V_1, V_2, H_1, and H_2 either analytically or graphically, since only that portion of the wind load above the assumed hinge produces stresses in the bent. The value of the moment at the foot of the knee brace is H_2y for the leeward column, in which y is the vertical distance from C to E. The moment at the foot of this column is H_2y_1, in which y_1 is the vertical distance from E to B. The direct stresses in the portions EC and EB are equal to V_2, while that in the portion CD depends upon the stress in the knee brace, which is unknown as yet. The value of the shear at any point below C is constant and

equal to H_2. When the complete loading is considered instead of one single load as in the above discussion, the presence of the loads at various points on the windward side will modify the shear and moment in the columns on that side. The leeward column only need be considered in the general case, since the stress in it is greater than in the windward column.

Fixing the column bases has the effect of decreasing the effective height of the bent—the hinge may be considered as moved upward to the point E. The less the effective height of the bent, the smaller are the lever arms b and a (Fig. 67) of the resultant load about the left and right hinges, respectively. Equations (1) and (2) show that a decrease in the lever arms b and a results in smaller values of V_1 and V_2 and hence smaller values of the reactions R_1 and R_2. In general, a decrease in the amounts of the reactions results in a decrease in the stresses in both columns and truss.

If the column bases could be held absolutely rigid so that the tangents to the distorted columns at the bases could be considered truly vertical, and the construction at D and C were such that the portions DC of both columns could be assumed as vertical in the distorted position, then the point of inflection E would be in its highest position, midway[2] between C and B. In the usual form of construction, the fixing of the columns is accomplished by anchoring the bases to the foundations by means of tightly drawn anchor bolts. As long as the anchor bolts remain tight, a fixed condition may be assumed to exist. There is, however, an ever-present tendency for the bolts to loosen and permit more or less rotation of the columns on the foundations. This condition causes the tangent at B to the curve $DCEB$ (Fig. 72) to incline upward toward the right—away from its vertical position. This deviation of the tangent causes point E to be lowered. The limit of the lowered position of E is the base of the column, the position that obtains when the base of the column is considered hinged.

On the other hand, even though the anchor bolts were to loosen considerably, the downward pressure of the dead load, acting on a fairly wide base plate, may be considered as partially

[2] If the construction at point D were such that a true hinge existed there, the point E would be somewhat above this midpoint; for reasons discussed in Art. 50, this condition should not be assumed in locating E unless particular attention is paid to producing a true hinge at D.

fixing the columns and thus keeping E from reaching the point B. It is, therefore, safe to assume that the point of inflection remains somewhat above B, and somewhat below the midpoint of the distance BC. A widely used assumption in cases where a fixed condition is supposed to exist, is that the point of inflection is located at a distance above B equal to one-third of BC. If reasonable care is taken in the construction to see that the anchor bolts are properly placed and tightly drawn, this assumption is safe and yet will not materially affect the economy of the design. It will be used in the following problems.

54. Reactions Due to Wind Loads. Bases of Columns Fixed. The reactions of a bent with fixed column bases are found by the same methods, analytical or graphical, as are used when the column bases are hinged. The assumed point of inflection is also an assumed hinge, and the analysis is made by considering only that portion of the bent above this imaginary hinge. The effective height of the bent is thus reduced by an amount equal to the distance from the base of the column to the assumed hinge.

The total wind pressure on the side walls above the point of inflection is assumed to be distributed to the three points E', C', and D' (Fig. 72), in proportion to the area contributing to each. Thus, the horizontal load on the windward column at the point E' is equal to one-half of that on the vertical panel $E'C'$. The force at C' is equal to one-half of the total pressure on panels $E'C'$ and $C'D'$, and the horizontal force at D' is equal to one-half the pressure against the panel $C'D'$.

The complete analysis required in the determination of reactions for a bent with fixed column bases is given in the following problem. The bent of Fig. 65 is used, and the wind loads on the truss are those computed in Art. 49. The bent is repeated in Fig. 73. The point of inflection E is assumed at a distance of 7.0 ft. from the base of the column in accordance with the recommendation made in Art. 53. In determining the reactions and stresses the horizontal wind pressure at B (Fig. 65) becomes $\frac{14 + 9}{2} \times 20 \times 16 = 3680$ lb., and at E $(^{14}\!/_{2}) \times 20 \times 16 = 2240$ lb. The horizontal pressure at C remains 1440 lb. as found in Art. 49 (see Fig. 74(a) for amounts of apex loads). The first graphical solution outlined in Art. 52 will be followed. As previously stated, the only difference between the solution of the bent with fixed column bases and the bent with hinged column

bases is that only that portion of the bent above the point of inflection is used in the former.

Lay off the given loads *FG*, *GH*, . . . *QS*, in order, to form the force polygon *fgh* . . . *s*. The closing line *sf* represents the amount and direction of the resultant load. Draw the equilibrium polygon *ly* . . . *wz* for any pole *O*, and extend the extreme sides *ly* and *zw* of this polygon to their intersection at *x*, thus locating a point in the line of action of the resultant wind load.

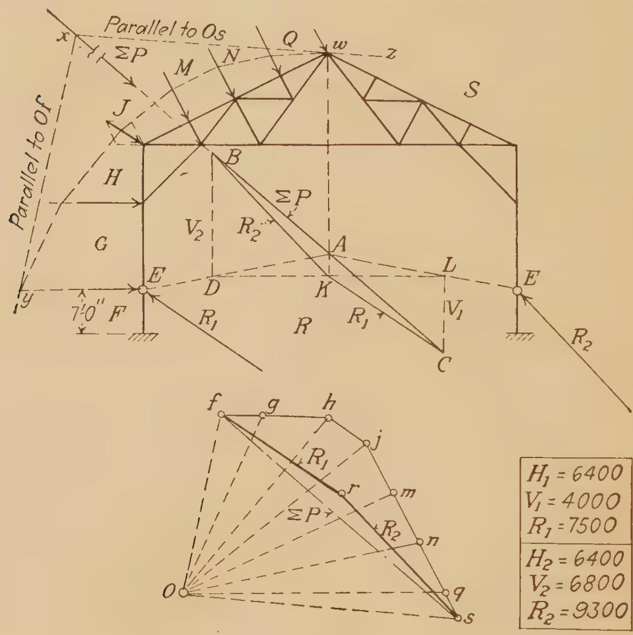


FIG. 73.

Prolong the line of action of the resultant wind load to intersect a vertical through the peak of the truss at *A*, and lay off this load, equal to *sf*, about *A* as a center. From the extremities *B* and *C* draw the verticals *BD* and *CL* to intersect the lines from *A* to the left and right hinges, respectively. Connect *D* and *L* with the horizontal line *DL*. Denote by *K* the intersection of *DL* with a vertical through *A*. Then *KB* represents the right reaction and *CK* the left reaction, as was shown in Art. 52.

To complete the force polygon for the external forces draw through *s* a line *sr* equal and parallel to *KB*, and through *f* a

line parallel and equal to CK. These lines should terminate at the common point r, to close the polygon fgh . . . srf.

55. Wind-load Stress Diagrams. Since, as stated in Art. 51, the columns of a framed bent are subjected to stresses other than direct tension and compression, the graphical solutions of Chapter III do not apply to these members. The graphical analysis of stresses in a framed bent must begin at the hinges; hence it is necessary that some special means be employed to eliminate the bending stresses in the columns from the stress diagram. Two general methods are used: (1) by temporarily replacing the columns with a series of forces which will have the same effect on the whole structure as the original columns; (2) by the use of an auxiliary framework of such a character as

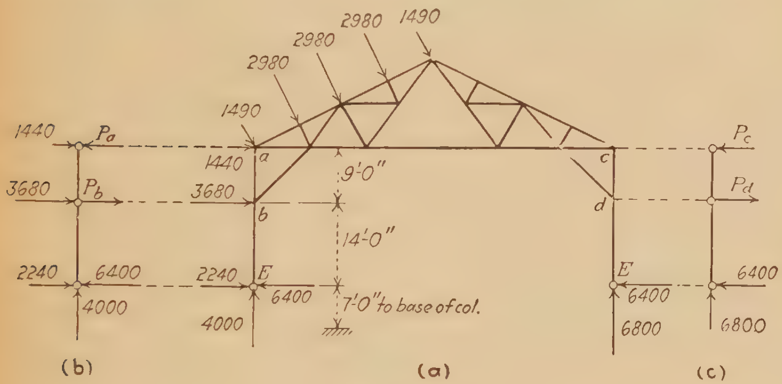


Fig. 74.

temporarily to make the columns "one-force pieces." In either case, after the stresses in the other members of the bent have been determined in the usual manner, the true stresses in the columns are found by comparing the changed and true conditions and modifying the above solutions accordingly.

The necessary procedure in each of the two methods mentioned above will be explained in detail in the determination of the wind-load stresses in the bent of Fig. 65 with fixed column bases, the loads and reactions for which have already been found (see Art. 49 and Fig. 73).

It is first proposed to find the stresses in the bent by temporarily removing the columns. Figure 74(a) shows that portion of the bent above the assumed hinges, with the loads and reactions

acting upon it. The vertical and horizontal components of the reactions have been scaled from Fig. 73.

In order to remove the columns without affecting the equilibrium of the bent, it is necessary to replace them with the forces which they would exert on the original structure if they were left in place. Figure 74(b) shows the windward column as a free body with the external forces acting on it. These external forces cause reactions at a and b equal to P_a and P_b, respectively; in the original bent these reacting pressures are furnished by the truss at a and the knee brace at b.

P_a is found by taking moments about b:

$$(6400 - 2240)14 - (P_a - 1440)9 = 0$$

from which $P_a = 7910$ lb.

Similarly, by taking moments about a,

$$(6400 - 2240)23 - (P_b + 3680)9 = 0$$

from which $P_b = 6950$ lb.

In the above equations, forces producing clockwise rotation have been considered positive, and those producing counterclockwise rotation, negative. The directions of P_a and P_b are found from these relations. As far as the effect on the truss is concerned, the reactions P_a and P_b of Fig. 74 cause equal pressures acting in the opposite directions, *i.e.*, from left to right at a and from right to left at b. At b, an upward pressure equal to 4000 lb. is exerted by the column.

In a similar manner (Fig. 74(c)), the reacting forces P_c and P_d for the leeward column are found.

$$P_c = 6400 \times \frac{14}{9} = 9960 \text{ lb.}$$

$$P_d = 6400 \times \frac{23}{9} = 16{,}360 \text{ lb.}$$

The effect of each of these two forces on the truss is equal in amount but opposite in direction to the corresponding force shown in Fig. 74(c). An upward force equal to the vertical reaction at the foot of the column (6800 lb.) also acts at d. Figure 75(a) shows the bent with the columns removed; equilibrium is maintained by the forces computed above and indicated on the figure.

In Fig. 75(b) the stress diagram is drawn for the truss of Fig. 75(a), by isolating each joint in succession from the rest of the

truss and drawing a force polygon for each joint as explained in Chapter III. The second upper chord joint is solved as explained in Art. 43. The stresses in the upper portions of the columns are given directly in the stress diagram. The stress in the lower portion of the windward column is equal to the vertical reaction at *b* (Fig. 74), or 4000 lb., and that in the corresponding portion of the leeward column is equal to the vertical reaction at *d*, 6800 lb.

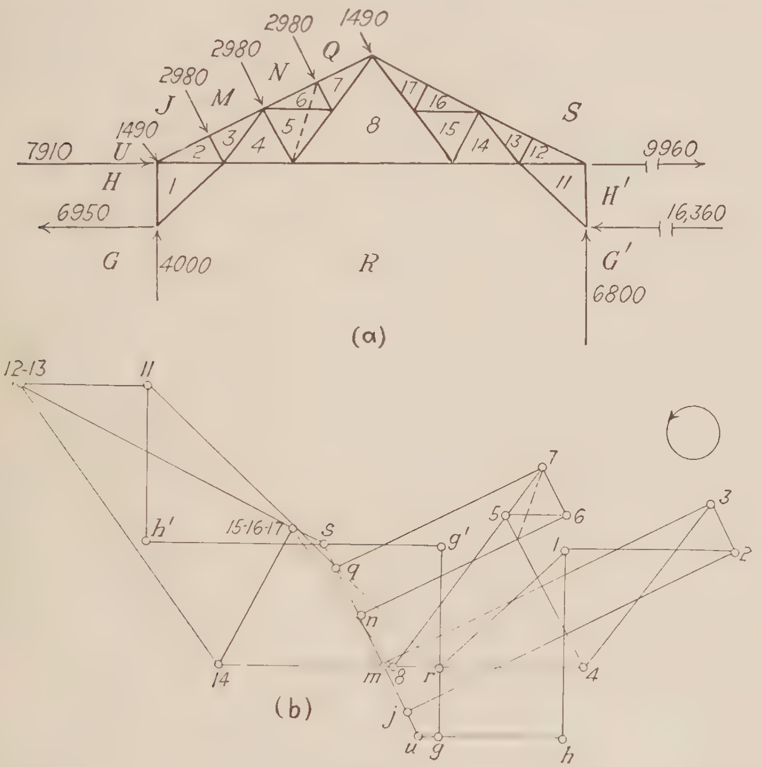


FIG. 75.

In applying the second method to the solution of this same bent, the auxiliary framework employed is as indicated by the broken lines to the side of each column in Fig. 76(*a*). Any convenient point of intersection for the extra members may be used. In the present case the upper chord has been produced to intersect a horizontal through the foot of the knee brace. The third member connects this point with the assumed hinge *E*.

The moment in the column at the foot of the knee brace causes direct stresses to be induced in the auxiliary members of the framework; these stresses are determinate and the framing details of the entire bent are of such a character as to permit of the solution of each joint. The external loads on the windward column are assumed as acting at the joints of the auxillary framework.

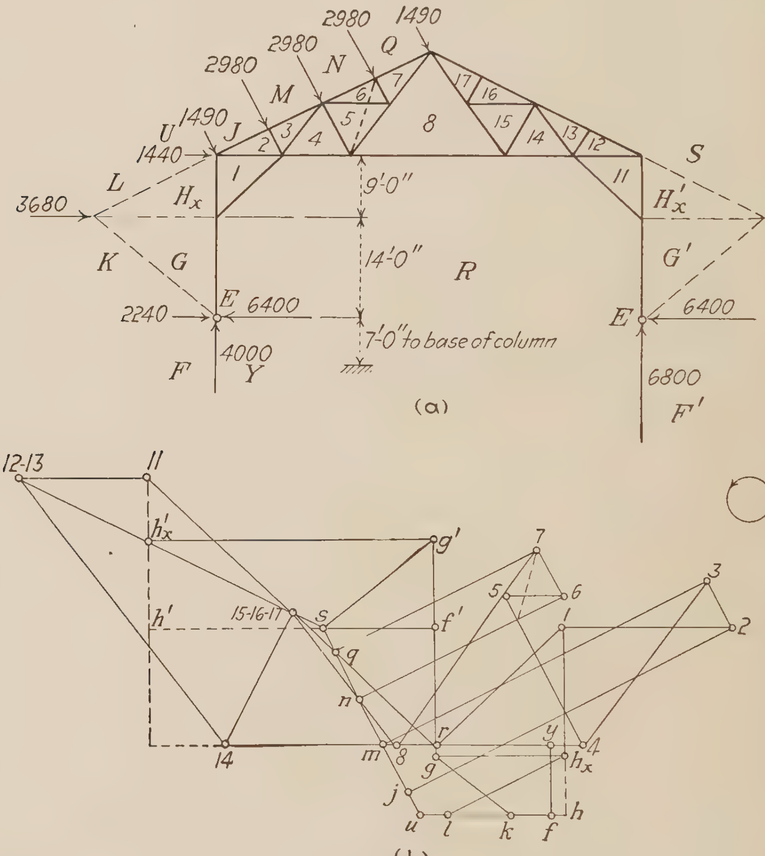


Fig. 76.

The stress diagram is completed in the usual manner. The complete diagram is shown in Fig. 76(*b*); the vertical and horizontal components of the reactions caused by the given loads have been scaled from Fig. 73. The stresses in the truss members

have not been affected by the addition of the extra members, hence the true stress in any member of the truss or in either knee brace is given in Fig. 76(*b*). The true stresses in the columns, however, must be found by noting the effect of the addition of the auxiliary frames, and modifying the stresses shown for these members in the diagram, accordingly.

When the temporary frames are removed, it can be seen by inspection that the compressive stress in the lower part of the windward column is equal to the vertical reaction at the foot of this column, 4000 lb., or *fy* in the stress diagram. The stress in the upper portion is equal to the stress in the lower portion plus the vertical component of the stress in the knee brace, or, in Fig. 76(*b*), *fy* plus the vertical component of 1–*r*. In order to be able to measure this summation, project 1–*r* on a vertical through 1, and project *fy* on this same vertical. The sum of these two projections, 1–*h*, is the required stress.

The true stress in the lower portion of the leeward column is equal to the vertical reaction at its foot, 6800 lb. The true stress in the upper portion, *i.e.*, the stress that exists after the removal of the auxiliary members, is equal to the stress in the lower portion minus the vertical component of the stress in the knee brace; minus because the latter is in compression and its stress acts downward when a circular section cut around the foot of the brace is considered. The stress in the lower portion is given by the line *f′r* in Fig. 76(*b*), and the stress in the knee brace by the line 11-*r*. Project 11-*r* on a vertical through 11, and project *f′r* on this same vertical. The difference between these two projections, 11-*h′*, gives the magnitude of the true stress in the upper portion of the column.

If the constructions are accurate, the stresses obtained from Fig. 76(*b*) will be equal to those obtained from Fig. 75(*b*). If the columns were hinged at the bottom instead of being fixed as in the present case, the stress diagram would be constructed in the same manner as outlined above; the hinge would be at the foot of the column instead of at some point above the foot; the reactions would be greater as previously explained; the stresses would also be greater.

56. Maximum Stresses. The maximum stress in any member is the algebraic sum of the individual stresses in the member. Since the wind stresses in most of the members reverse with a reversal in the direction of the wind, the stresses due to wind

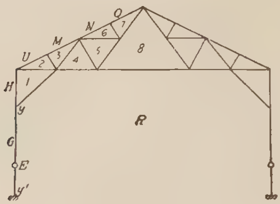


FIG. 77.

MAXIMUM STRESSES

Member	Dead load	Snow load	One-half snow load	Wind left to right	Wind right to left	Dead load plus snow load	Dead load plus wind load	Dead load plus wind load plus ½ snow load	Maximum stress
J-2	−10,000	−17,800	− 8,900	−20,900	+19,800	−27,800	−30,900 + 9,800	−39,800 + 900	−39,800 + 9,800
M-3	− 9,440	−16,800	− 8,400	−20,900	+19,600	−26,240	−30,340 +10,360	−38,740 + 1,960	−38,740 +10,360
N-6	− 8,870	−15,800	− 7,900	−13,200	+ 2,400	−24,670	−22,070	−29,970	−29.970
Q-7	− 8,300	−14,800	− 7,400	−13,200	+ 2,400	−23,100	−21,500	−28,900	−28,900
1-2	+ 8,960	+16,000	+ 8,000	+10,000	− 7,700	+24,960	+18,960	+26,960	+26,960
4-*R*	+ 7,680	+13,700	+ 6,850	+ 8,600	−12,400	+21,380	+16,280 − 4,720	+23,130	+23,130 − 4,720
8-*R*	+ 5,120	+ 9,120	+ 4,560	− 2,100	− 2,100	+14,240	+ 3,020	+ 7,580	+14,240
2-3 6-7	− 1,140	− 2,030	− 1,010	− 2,980	0	− 3,170	− 4,120	+ 5,130	− 5,130
3-4	+ 1,280	+ 2,280	+ 1,140	+11,700	−19,600	+ 3,560	+12,980 −18,320	+14,120 −17,180	+14,120 −18,320
4-5	− 2,290	− 4,080	− 2,040	− 9,600	+ 8,800	− 6,370	−11,890 + 6,510	−13,930 + 4,470	−13,930 + 6,510
5-6	+ 1,280	+ 2,280	+ 1,140	+ 3,100	0	+ 3,500	+ 4,380	+ 5,520	+ 5,520
5-8	+ 2,560	+ 4,500	+ 2,280	+10,700	− 9,800	+ 7,120	+13,260 − 7,240	+15,540 − 4,960	+15,540 − 7,240
7-8	+ 3,840	+ 6,840	+ 3,420	+14,100	− 9,800	−10,630	+17,940 − 5,960	+21,360 − 1,540	+21,360 − 5,960
1-*R*	0	0	0	+ 9,700	−22,700	0	+ 9,700 −22,500	+ 9,700 −22,700	+ 9,700 −22,700
H-1	− 5,120	− 9,120	− 4,560	−10,700	+ 8,900	−14,240	−15,820 + 3,780	.... −20,380	−20,380 + 3,730
G-*R*	− 5,120	− 9,120	− 4,560	− 4,000	− 6,800	−14,240	− 9,120	−16,480	−16,480

All stresses in pounds; + = tension, − = compression.

must be tabulated with the wind blowing from left to right and also from right to left. In symmetrical bents it is not necessary to construct separate wind-load stress diagrams for these two conditions, as was the case in the simple trusses of Chapter III, because the stress in any member with the wind blowing from right to left is the same as in the symmetrical member with the wind blowing from left to right. Either Fig. 75 or Fig. 76, therefore, gives all the necessary information for the two conditions.

As explained in Art. 46, it is customary to assume that full snow-load stresses and maximum wind-load stresses will not exist simultaneously. In determining maximum stresses, this fact should be recognized, and all of the probable simultaneous stress combinations should be figured. In preparing the table on page 104, three combinations were assumed: (1) dead load plus snow load; (2) dead load plus wind load; (3) dead load plus wind load plus one-half snow load. A fourth combination that might exist, and which in some cases might give greater stresses than any of the above is dead load plus snow load plus one-half wind load. It should be noted that a reversal of stress occurs in some of the members; the design of these members must be governed accordingly.

57. Stresses in Lateral Bracing. Since the lateral bracing between bents (see Figs. 63 and 63(a)) consists of diagonal and transverse members with parallel chords (in the plane of the bracing) the analytical solution is much simpler in application than the graphical. The methods of solution are outlined in Arts. 14 and 111.

CHAPTER V

BRIDGE TRUSSES UNDER DEAD LOAD

58. A bridge truss is composed of an upper chord, a lower chord, and web members. As in the roof truss each member is subject to stress only in the direction of its length, the elementary figures are triangles, and the loads are applied only at its joints or panel points. In a simple truss the upper chord is always in compression and the lower chord in tension, while some of the web members are in tension and some in compression.

The joints of a steel truss may be either pinned or riveted. In the former type of joint a steel pin, usually from 3 in. to 6 in. in diameter, passes through holes provided in each member meeting in the joint. In order to have sufficient bearing on the pin it is often necessary to thicken some of the members at the joint by riveting to the members additional short plates known as pin plates. A truss whose members are connected in this manner is called a *pin-connected truss.* The latter type of joint consists of one or two (two in all but the shortest and lightest of trusses) steel plates to which each of the members meeting in the joint are connected by rivets. A truss with such joint connections is called a *riveted truss.*

A bridge usually consists of two parallel trusses supporting the bridge floor or roadway. These two trusses are held rigidly together by bracing. The horizontal bracing between the two lower chords is the lower lateral bracing, and the similar bracing between the upper chords is known as the upper lateral bracing. The vertical bracing between the posts is called the sway bracing, while the bracing in the plane of the end posts is the portal bracing.

A *through* bridge is one in which the roadway is carried directly at the lower chord panel points, with lateral bracing overhead between the upper chord joints. A *deck* bridge is one in which the roadway is carried directly at the upper chord panel points, or on the upper chords themselves. A *pony truss* bridge is

usually of short span; the roadway is carried at the lower chord panel points, but there is not sufficient height of truss to allow of upper lateral bracing.

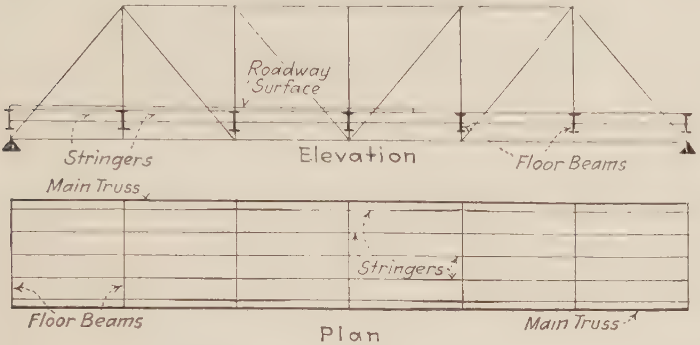


Fig. 78.

The floor of a highway truss bridge generally consists of floor beams, running at right angles to the trusses and connected to them at the panel points; stringers or joists which are supported

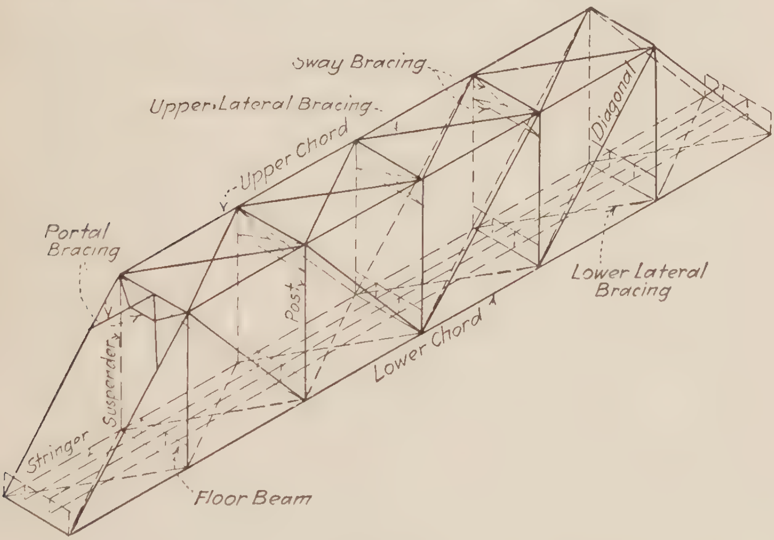


Fig. 79.

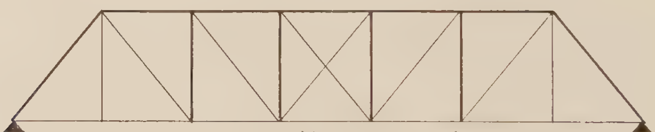
by the floor beams and are parallel to the trusses; and planks, steel plates, or reinforced concrete slabs which rest upon the stringers and support the load or the wearing surface of the roadway.

In some bridges, for the most part of the deck type, the roadway is supported directly on the floor beams, and the stringers are not necessary. In this type of construction it is usually more economical to have the floor beams resting directly on the upper chord at more frequent intervals than the panel points. Such an arrangement causes flexural stresses in the upper chord which must be provided for in the design. While not theoretically a true truss, if proper provision is made for the flexural stresses, such a structure is often to be preferred over any other type. The sidewalks, if any, are usually carried outside the trusses either by the projection of the floor beams or by cantilever brackets.

The floor of a railroad truss bridge is more generally of the open type which consists of floor beams and stringers, the latter supporting the cross-ties upon which the rails and guard rails rest. Usually there are two stringers for each track, but when their depth is limited four are sometimes used. Solid floors are now in general use where the bridge carries the track over a street or main highway. In this type a deck of steel or reinforced concrete is carried by the stringers and floor beams or on floor beams without stringers. On this deck ballast is placed and the track carried over the bridge without any interruption in the ballasted roadbed.

In the usual cases, in both highway and railroad bridges, the loads are brought more or less directly to the stringers, transferred from them to the floor beams, which in turn deliver them to the trusses at the panel points. In most simple truss design, therefore, all of the load is applied to the truss only at these panel points.

59. Types of Trusses. The types of bridge trusses most commonly used in modern practice are illustrated in Figs. 80 to 90.



Pratt Truss

FIG. 80.

The Pratt truss is the most common of all types, and is almost a standard for spans of moderate length. It may be either a deck

or a through structure. Its characteristic features are that it has vertical members at each panel point and its main diagonals slope *downward* toward the middle of the span, except sometimes

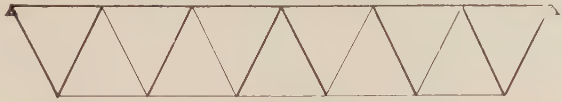


FIG. 81.

the end ones, which are known as the end posts. In the deck form the end diagonals are sometimes parallel to the other

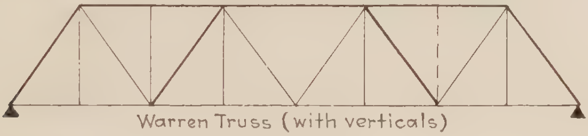


FIG. 82.

diagonals and sometimes slope in the opposite direction. In a through truss they always slope in the opposite direction.

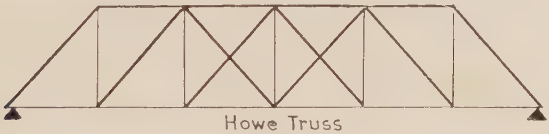


FIG. 83.

The Warren truss is used for short-span deck bridges, and occasionally for through bridges as an alternative to the Pratt

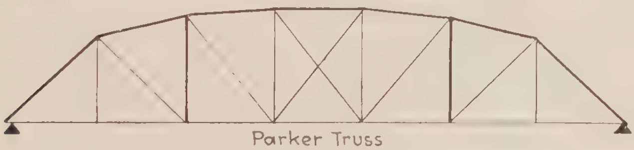
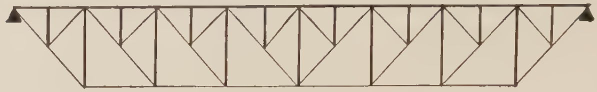


FIG. 84.

truss. It may be built either with or without verticals at the panel points. Its alternate diagonals slope in opposite directions.

The Howe truss is used where good timber is plentiful and much cheaper than other materials, the verticals usually being the only members built of steel or iron. Its main diagonals slope upward toward the center.

For longer spans, *i.e.*, from 150 to 200 ft. upwards, it is more economical to use a truss of varying depth. The most common of these is the Curved Chord Pratt or Parker truss. It has the



Baltimore Truss

Fig. 85.

characteristics of the ordinary Pratt and is used in either the deck or the through form.

For still longer spans it is desirable to reduce the panel length while still maintaining about the same inclination of the

Pettit Truss

Fig. 86.

diagonals. As the economical depth of the truss increases with the span it becomes necessary, in order to have an economical panel length and diagonals not too nearly vertical, either to subdivide the panel or to use a double system of webbing.

K Truss

Fig. 87.

The two types of trusses with subdivided panels in general use are the Baltimore truss and the Pettit or Pennsylvania truss, either one of which may be used in the deck or through form.

Whipple Truss

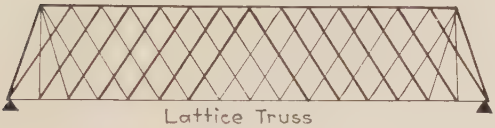
Fig. 88.

The *K*-truss is a desirable form of a multiple web system. It has the advantage of lower secondary stresses and is particularly well adapted for long riveted spans. The Whipple truss is in reality two Pratt trusses superimposed upon one another. It

is sometimes analyzed in this way, but such an analysis is not strictly accurate. The double triangular or double Warren truss and the Lattice truss are other forms sometimes used, but the indeterminate character of the stresses, and the uncertainty involved, prevent their extensive use in modern construction.

Double Triangular Truss

FIG. 89.



Lattice Truss

FIG. 90.

60. Weights of Truss Bridges. The dead or fixed load consists of the weight of the trusses and their bracing together with the weight of the floor and the floor bracing. The panel length, the distance between trusses, and the live loading being known, the floor and floor system may be designed and their exact weight determined. The weight of the trusses and their bracing cannot be exactly determined until the design is completed. It varies with the load to be carried, the unit stresses employed, the type of truss, the character of its details, the panel length, the distance between trusses, the type of floor, and the length of span. The principal factors affecting the weight are the live load to be carried and the length of span. Any empirical formula used for approximating the weight of the trusses and their bracing will usualy contain functions of these two variables.

The weights of highway bridge trusses are extremely variable on account of the great variety of types of floor in use, ranging from light planking supported on wooden stringers and steel floor beams to heavy reinforced concrete slabs on a system of steel stringers and floor beams. The formula given below allows for this variation in the weight of the floor.

Let w = weight per foot of one truss and one-half of the bracing

w_1 = total superimposed load per foot in pounds brought to each truss; this includes the weight of the floor, the live load, and the impact

l = span in feet.

Then

$$w = l + \frac{l(w_1 - 500)}{2000} \qquad (a)$$

The weights of single-track railroad bridges with open floors can usually be more closely approximated. The distance between trusses is nearly a constant and the principal variations are the live load and the length of span. For spans from 150 to 300 ft., the weight per foot of one truss and one-half of the bracing and floor system may be taken as

$$w = 2\,(2l + 5E) \qquad (b)$$

where l = span in feet and E = the number representing the class of Cooper's live loading to be carried (see Art. 89).

The use of a solid floor with ballast materially increases the weight of floor and adds somewhat to the weight of the trusses as determined by the preceding formula.

The weight of a double-track bridge is from 80 to 90 per cent. greater than that of the corresponding single-track structure.

It is impossible to determine exactly what proportion of the weight of the trusses and their bracing is carried at the upper and lower panel points, respectively. Sometimes it is all assumed as being carried by the panel points of the chord sustaining the roadway, but this is far from the true distribution. It is recommended that two-thirds of the dead weight of trusses and bracing be assumed as distributed to the loaded chord, and the remainder to the unloaded chord.

Trusses with Horizontal Chords and Single Web Systems

61. Stresses in Web Members. There are, as far as the analysis of stresses is concerned, two classes of web members. The first and simplest class is illustrated by member *Bb* of Fig. 91. Such members are known as "suspenders" or subverticals. No section can be passed through *Bb* without cutting at least two other members of the truss. The only section that can be passed which does not cut another member having a vertical component is the section *MM*. From an inspection of the section it is seen that the stress in *Bb* is dependent only upon, and is equal to, the load P_1.

The other class of web members is illustrated by the member *cD*. The section *NN* is the only section that may be passed through *cD* and not cut another member having a vertical component. Since the internal stresses in any section of a truss hold

in equilibrium the external forces on either side of the section, and since BD and cd are horizontal and hence have no vertical components, cD must hold in equilibrium the vertical forces on the left of the section; *i.e.*, the sum of the vertical component of cD, the reaction R, and the loads P_1 and P_2 must equal zero. If θ be the acute angle between the member cD and a vertical

$$R_1 - P_1 - P_2 + S \cos \theta = 0 \text{ (Fig. 92)}$$

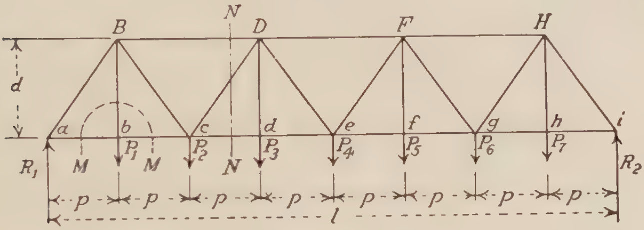


FIG. 91.

But $R_1 - P_1 - P_2$ is the algebraic sum of all the vertical forces on the left of the section and is called the "vertical shear," and may be designated by V. Hence

$$V + S \cos \theta = 0 \text{ or } S = -V \sec \theta$$

For the member De, the unknown stress, assumed to act away from the section, would point downward so that the equation would become

$$R_1 - P_1 - P_2 - P_3 - S \cos \theta = 0 \text{ or } S = +V \sec \theta \quad \text{(Fig. 93)}$$

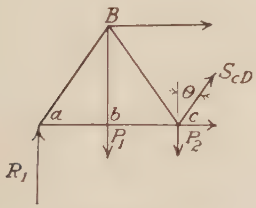


FIG. 92.

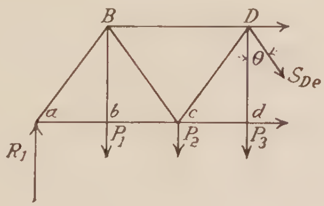


FIG. 93.

From the above it follows that for trusses with horizontal chords, the stress in any web member, other than subverticals, is equal to the vertical shear multiplied by the secant of the angle which the member makes with the vertical.

It is also evident that the sign of stress is dependent on the sign of the shear and the direction of the slope of the diagonal. For any section on the left of the middle of the truss the dead-load

shear is always positive. For a diagonal sloping upward toward the middle of the truss, such as cD, the dead-load stress is always negative or compressive. The practical significance of the sign of the shear may be seen if the portion of the truss to the right of the section NN be considered stationary, and that to the left free to move. Since the resultant of the vertical forces, the vertical shear, acts upward (*i.e.* is positive) the panel point c will tend to rise and approach the panel point D, thus indicating compression in cD. For a diagonal sloping downward toward the middle of the truss, such as De, the reverse is true, and tension is indicated.

In a truss of many panels it is easier to compute the shear in any section by making a summation of loads between the section and the middle of the truss. Considering the truss of Fig. 91, with the loads P_1, P_2, etc., of equal value, the net reaction R_1 is $3\frac{1}{2}P$. The shear in cD is

$$3\tfrac{1}{2}P - P_1 - P_2 \text{ or } 1\tfrac{1}{2}P$$

The summation of the loads between the section NN and the middle of the truss is

$$P_3 + \tfrac{1}{2}P_4 \text{ or } 1\tfrac{1}{2}P$$

as previously determined.

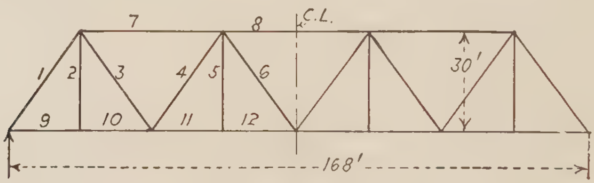


FIG. 94.

Illustrative Problem. A through, single-track, railroad bridge with an open floor has a span of 168 ft. The trusses are of riveted Warren type with verticals at each alternate panel point. There are eight panels, the depth of the truss is 30 ft., and the live load to be carried is Cooper's class E-60. The skeleton diagram of the truss is shown in Fig. 94. By equation (*b*) of Art. 60

$$w = 2(2 \times 168 + 5 \times 60) = 1272 \text{ lb.}$$

Considering one-third of this weight as carried by the upper panel points, the upper panel loads are $\frac{1272}{3} \times 42 = 17.8$ kips.[1]

[1] One kip equals 1000 lb.

Assuming the weight of track as 500 lb. per foot the lower panel loads are

$$\left[\frac{1272 \times 2}{3} + \frac{500}{2}\right]21 = 23.0 \text{ kips.}$$

A half-panel load of 11.5 kips is carried to the panel points at each end of the truss, but as these loads do not produce stresses in any of the truss members it is not necessary to consider them. The secant of the angle between any diagonal and the vertical is

$$\frac{\sqrt{21^2 + 30^2}}{30} = 1.22$$

$$\begin{array}{llll}
V_1 = 2 \times 17.8 + 3\tfrac{1}{2} \times 23.0 = 116.1 & S_1 = 116.1 \times 1.22 = -141.6 \\
V_3 = 1 \times 17.8 + 2\tfrac{1}{2} \times 23.0 = 75.3 & S_3 = 75.3 \times 1.22 = +\ 91.9 \\
V_4 = 1 \times 17.8 + 1\tfrac{1}{2} \times 23.0 = 52.3 & S_4 = 52.3 \times 1.22 = -\ 63.8 \\
V_6 = \tfrac{1}{2} \times 23.0 \qquad\qquad\quad = 11.5 & S_6 = 11.5 \times 1.22 = +\ 14.0
\end{array}$$

$$S_2 = S_5 = +23.0$$

62. Stresses in Chord Members. *Method of Moments.* The method of moments may be used in determining the chord stresses in any truss. The center of moments is located as follows: A section is passed through the truss cutting the chord member whose stress is desired. This section should also cut the opposite chord and a web member. The center of moments is the intersection of the web member and the opposite chord. For example, in Fig. 91, to find the stress in cd a section NN is passed through cd, cD, and BD. The intersection of cD and BD, that is, D, is selected as the center of moments for the chord cd; for the lines of action of the other two members cut by the section pass through this point, hence their lever arms are zero, and they do not appear in the moment equation. The equation of moments is $R_1 \times 3p - P_1 \times 2p - P_2 \times p - cd \times d = 0$. Since the loads, reactions, and panel lengths are known the stress in cd may be computed.

Considering the truss of Fig. 94 the center of moments for the member 8 is the intersection of the members 6 and 12, and the equation of moments is

$$116.1 \times 84 - 23.0(1 + 2 + 3) \times 21 - 17.8(1 + 3) \times 21 + S_8 \times 30 = 0$$

whence

$$S_8 = -178.6 \text{ kips.}$$

Similarly for the member 12, the center of moments is at the intersection of members 6 and 8, the moment equation is

$$116.1 \times 63 - 23.0(1 + 2) \times 21 - 17.8 \times 42 - S_{12} \times 30 = 0$$

and

$$S_{12} = +170.6 \text{ kips.}$$

The center of moments for S_{11} is the same as for S_{12}, the moment equation is the same, therefore

$$S_{11} = S_{12} = +170.6 \text{ kips.}$$

In a similar manner S_7 is found to be -134.0 kips and $S_9 = S_{10} = +81.3$ kips.

63. Stresses in Chord Members. *Resolution of Forces.* For trusses with parallel chords the stresses in the chord members can be obtained more easily by the resolution of forces than by the method of moments. In Fig. 95 which represents the left end of the truss of Fig. 91, the stresses S_1 and S_9 are in equilibrium

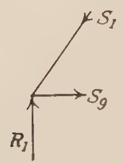


Fig. 95.

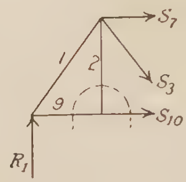


Fig. 96.

with the reaction R_1. R_1 has no horizontal component, hence S_9 is in equilibrium with the horizontal component of S_1. Since S_1 is a diagonal sloping upward toward the middle of the truss it is in compression, and the arrow representing the stress points toward the section or to the left. S_9 is assumed to act away from the section or to the right. Then

$$-S_1 \sin \theta + S_9 = 0$$

and since

$$S_1 = V_1 \sec \theta, \quad S_9 = +V_1 \tan \theta.$$

Passing a circular section through members 9, 2, and 10 as shown in Fig. 96

$$-S_9 + S_{10} = 0$$

or

$$S_9 = S_{10}$$

In Fig. 96 the forces S_7, S_3, and S_{10} are in equilibrium with the reaction R_1 and the load P_1, and, as before, assuming the unknown stress S_7 to act away from the section

$$S_7 + S_3 \sin \theta + S_{10} = 0$$
$$S_7 = -(S_{10} + V_3 \tan \theta)$$

indicating that S_7 is in compression.

In a similar manner, by passing successive sections, the stress in each chord member may be found by *adding* the horizontal component ($V \tan \theta$) of the diagonal cut by the section to the chord stress previously obtained.

The chord stresses of the truss of Fig. 94 are computed and tabulated below. The value of $\tan \theta$ is $\frac{21}{30} = 0.70$.

Diagonal (cut by section)	Shear (in diagonal)	Horizontal component ($V \tan \theta$)	Stress	Chord
1	116.1	81.3	81.3	9, 10
3	75.3	52.7	134.0	7
4	52.3	36.6	170.6	11, 12
6	11.5	8.0	178.6	8

No attention need be paid to signs in the construction of this table, for, provided a section cuts only one diagonal, all the horizontal components are positive and are added arithmetically to the preceding chord stress. In a simple truss the upper chord is always in compression and the lower chord always in tension.

If it is desired to find the stress in only one chord member, such as member 8, it may be computed more quickly by adding the column of shears and multiplying this sum by *tan θ, i.e.*,

$$116.1 + 75.3 + 52.3 + 11.5 = 255.2$$
$$255.2 \times 0.70 = 178.6 \text{ kips.}$$

as above. Any intermediate chord stress may be found in a similar manner.

The work involved in this method of computation by resolution of forces is less than that required where the method of moments is used, but it is applicable only to trusses with parallel chords.

64. The Pratt Truss. This type of truss is more widely used than any other for spans under 250 ft. For the shorter spans it is usually built as a riveted truss without counters (see Art. 71).

The longer spans formerly were and sometimes are built as pin connected, and counters used to prevent reversal of stress in the diagonals. The dead-load stresses in the main members are determined by the same method in either case. The following example gives the necessary computations.

The through Pratt truss, whose skeleton diagram is given in Fig. 97, has a span of 150 ft.-0 in., and a depth of 30 ft.-0 in. It is to be designed for Cooper's E-60 loading. The weight of the track is assumed as 500 lb. per foot of bridge.

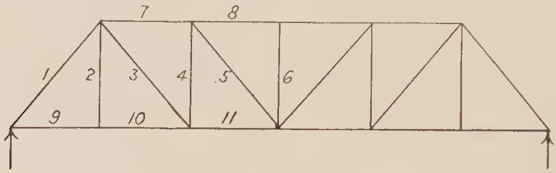


Fig. 97.

From equation (b) of Art. 60

$$w = 2(2 \times 150 + 5 \times 60) = 1200 \text{ lb.}$$

The upper panel load is

$$\frac{25 \times 1200}{3} = 10.0 \text{ kips.}$$

The lower panel load is

$$\frac{(25 \times 1200)2}{3} + 250 \times 25 = 26.2 \text{ kips.}$$

$$\sec \theta = \frac{\sqrt{25^2 + 30^2}}{30} = 1.30$$

$$\tan \theta = \frac{25}{30} = 0.83$$

Web Stresses

Member	Shear	Stress
1	90.5	−117.7
3	54.3	+ 70.6
4	28.1	− 28.1
5	18.1	+ 23.5
2	....	+ 26.2
6	....	− 10.0

CHORD STRESSES

Diagonal	Shear	Horizontal component	Stress	Chord
1	90.5	75.1	75.1	9, 10
3	54.3	45.1	120.2	7, 11
5	18.1	15.0	135.2	8

TRUSSES WITH INCLINED CHORDS AND SINGLE WEB SYSTEMS

65. The method of resolution of forces is not directly applicable to trusses with inclined chords, and the method of moments will in general involve less computation.

Fig. 98 represents a portion of a Parker truss. To determine the stress in *Bc* the section is cut as shown. The chord *BC* produced intersects the chord *bc* produced at the point *O*, which

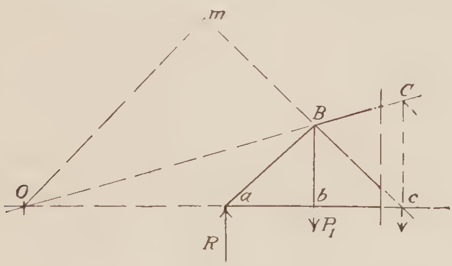


FIG. 98.

is, therefore, the center of moments for the member *Bc*. The lever arm is *Om*, the perpendicular distance from *O* to *Bc* produced. From the similar triangles *Omc* and *Bbc*

$$Om = \frac{Oc \times Bb}{Bc}$$

With the net reaction at *a*, and the panel loads at *B* and *b* known, taking moments about *O*

$$S_{Bc} \times Om - R \times Oa + P_1 \times Ob = 0$$

from which the stress in *Bc* may be determined.

While there is no difficulty involved in this method, it can be somewhat simplified by determining first the vertical component of the diagonal, and from it the stress. The lever arm is then the same as that for the adjacent vertical and often a multiple of the panel length.

In Fig. 99, consider the unknown stress in Bc resolved into two components applied at c, one horizontal, Bc_H, and the other vertical, Bc_V. The line of action of the horizontal component

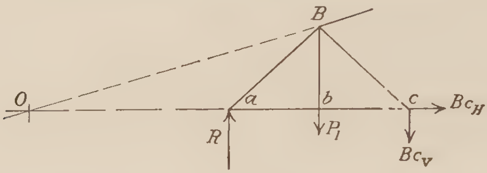


FIG. 99.

passes through the center of moments O. Taking moments about O, as before

$$Bc_V \times Oc - R \times Oa + P_1 \times Ob = 0$$

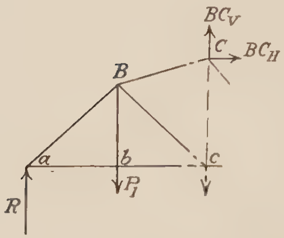


FIG. 100.

from which Bc_V may be determined. The actual stress in Bc is Bc_V multiplied by the secant of the angle that Bc makes with the vertical.

Similarly, for the chord BC, consider its stress resolved into two components, applied at C (see Fig. 100). The center of moments is at c, and the moment equation is

$$BC_H \times Cc + R \times ac - P_1 \times bc = 0$$

The stress in BC is BC_H multiplied by the length of the chord itself BC, divided by its horizontal projection bc.

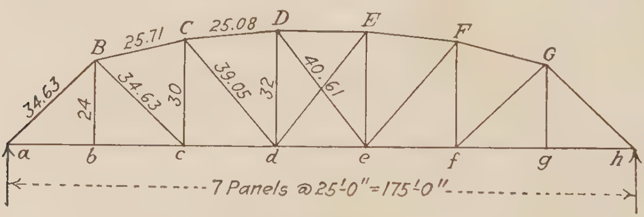


FIG. 101.

The stresses in verticals, such as Cc, are determined in a manner similar to that used for the diagonals, except that the stress is determined directly since there is no horizontal component to be considered. Similarly, the lower chord stresses are

determined directly, since they have no vertical components. The stresses in the first diagonal aB, and in the vertical Bb, may be obtained by the methods of Art. 64.

As an example of the above, the dead-load stresses in the Parker truss of Fig. 101 will be computed. The lengths of the truss members are given on the diagram. The upper panel load is 10,000 lb. or 10 kips, and the lower panel load 30,000 lb. or 30 kips. The net reaction at a is 120 kips.

Web Stresses

$aB = 120 \times \frac{34.63}{24} = -173.2$ kips.

$Bb = +30.0$ kips.

Bc BC produced intersects the lower chord produced at a distance of three panel lengths to the left of a

$$Bc_V \times 5 \times 25 - 120 \times 3 \times 25 + 40 \times 4 \times 25 = 0$$

$$Bc_V = +40.0 \qquad Bc = +40 \times \frac{34.63}{24} = +57.7 \text{ kips.}$$

Cc $$-Cc \times 5 \times 25 - 120 \times 3 \times 25 + 40 \times 4 \times 25 + 30 \times 5 \times 25 = 0$$

$Cc = -10.0$ kips.

Cd CD produced intersects the lower chord produced at a distance of 13 panel lengths to the left of a

$$Cd_V \times 16 \times 25 - 120 \times 13 \times 25 + 40 \times 14 \times 25 + 40 \times 15 \times 25 = 0$$

$$Cd_V = +25.0 \qquad Cd = +25.0 \times \frac{39.05}{30} = +32.6 \text{ kips.}$$

Dd $$-Dd \times 16 \times 25 - 120 \times 13 \times 25 + 40 \times 14 \times 25 + 40 \times 15 \times 25 + 30 \times 16 \times 25 = 0$$

$Dd = +5.0$ kips.

$De = dE = 0$

Chord Stresses

$ab = bc$ $$-ab \times 24 + 120 \times 25 = 0$$

$ab = +125.0$ kips.

cd $$-cd \times 30 + 120 \times 50 - 40 \times 25 = 0$$

$cd = +166.7$ kips.

BC $$BC_H = -166.7 \quad BC = -166.7 \times \frac{25.71}{25} = -171.4 \text{ kips.}$$

de $$-de \times 32 + 120 \times 75 - 40 \times 50 - 40 \times 25 = 0$$

$de = +187.5$ kips.

$CD \quad CD_H = -187.5 \qquad CD = -187.5 \times \frac{25.08}{25} = -188.1$ kips.

$DE \quad DE = -187.5$ kips.

TRUSSES WITH SUBDIVIDED PANELS

66. The Baltimore Truss. The Baltimore truss (Fig. 102) is used very generally for long spans. The usual form may be described as a Pratt truss with subdivided panels. The stresses are easily determined, the panel lengths are short, and the diagonals have an economical inclination. The upper diagram of Fig. 102 shows a type of through truss in which all the short diagonals or subdiagonals are in compression, while the lower diagram shows another type in which the subdiagonals are in tension, except the one at each end of the span. Since this type

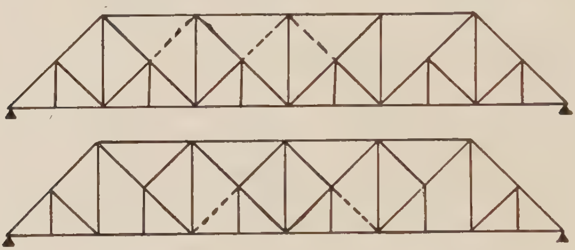


FIG. 102.

of truss is used for long spans, and is often built as a pin-connected truss, counter diagonals, as indicated by the broken lines, are often used (see Art. 75). These members are not stressed by the dead load alone. The usual form of deck truss is shown in Fig. 107, although the type shown in Fig. 85 is also used.

In Fig. 103(a) which represents the left end of a Baltimore truss, the stress in the member aB may be found by the methods of Art. 64. If the members Bb and Bc are removed, the end of the truss takes the form shown in Fig. 103(b), one-half the load normally applied at b being considered applied at a, and the other half at c.

Consider now the frame aBc, set in Fig. 103(b), with the member Bb attached rigidly to it at B, and the frame attached rigidly to the points a and c of the truss. This condition is shown in

Fig. 103(c). The load applied at B[2] in the small frame, transferred to B from b through the member Bb causes two equal stresses in the members aB and Bc of the frame, each one of which is compression and equal to one-half the load at b multiplied by *sec* θ. Also the load at B is transferred through B and the members aB and Bc to a and c, one-half to each panel point so that the net reaction at a is the same whether the auxiliary frame is used or not. Since the sum of the vertical components at the joint a in either case must equal zero, it follows that the stress in aC of Fig. 103(b) must be equal to the sum of the stresses in the separate members aB and aC of Fig. 103(c). Now consider that at the point B the two separate members are rigidly fastened together. This gives the condition of Fig. 103(a), and the stress in aB is equal to that in aC of Fig. 103(b), and to the sum of the

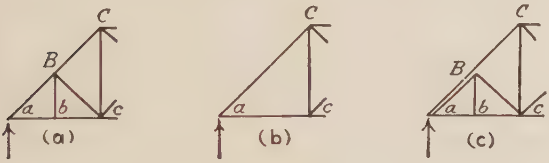


FIG. 103.

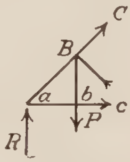


FIG. 104.

stresses in the two separate members of Fig. 103(c), which is R sec θ or V_1 *sec* θ. The stress in the subdiagonal Bc is, as before, one-half of the load applied at b multiplied by *sec* θ.

Next consider the member BC. Cutting a vertical section through BC, Bc, and bc as in Fig. 104, and taking the sum of the vertical components equal to zero

$$R - P + BC \cos\theta + Bc \cos\theta = 0$$

BC being unknown, is assumed to act away from the section, or *upward*. Bc is known, having a value of $\frac{P}{2}$ sec θ in compression, and hence the arrow is pointed toward the section, or upward.

Therefore

$$BC = -\left[(R - P) + \frac{P}{2}\right] \sec\theta$$

[2] In this analysis all load is considered applied on the lower chord. If a load is also applied at B the sum of the two loads (at B and b) is equally distributed through aB and Bc.

or

$$BC = -\left(V_{bc} + \frac{P}{2}\right) \sec \theta$$

A similar analysis can be made for the panels *cd* and *de* (Fig. 105) treating $Cd'E$ as a separate frame supporting at d' through $d'd$ the load applied at d. By similar reasoning it is seen that the stress in Cd' is not affected, while the stress in $d'E$ is one-half the load applied at d multiplied by $\sec \theta$, but in this case the stress is tension.

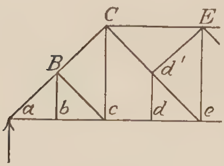


FIG. 105.

Therefore it follows that the stress in the subdiagonal of a Baltimore truss is one-half of all the load applied at its intersection with the main diagonal, multiplied by the secant of the angle which it makes with the vertical. Furthermore, if the point of application of the load is at the upper end of the diagonal its stress is compression, if at the lower end, tension.

Another method of analysis of the subdivided panel is as follows: Cut a circular section around the joint d' of Fig. 105. Then in Fig. 106 the stresses Cd' and $d'e$ are known to be tension, and the arrows are pointed away from the section. Similarly the stress in $d'd$ is tension and equal to the load applied at d. The stress $d'E$ may be resolved into components at any point in its line of action. Let this be done as indicated at E. Taking moments about C, the lever arms of Cd', $d'e$, and $d'E_H$ are zero, and the equation is

$$d'd \times p - d'E_V \times 2p = 0$$

or

$$d'\,E_V = \frac{d'\,d}{2}$$

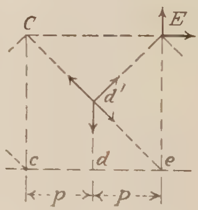


FIG. 106.

The stresses in all the verticals can be obtained by resolution of forces. Since the vertical components of the subdiagonals are easily obtained, if it becomes necessary in computing the stress in a vertical to cut a section through a diagonal as well, it will simplify the computations to choose the section so that a subdiagonal is cut.

Similarly the stresses in the chords can be obtained more easily by the method of resolution of forces. The horizontal component of a subdiagonal, however, may act in a direction opposite to that

of the chord stress previously obtained, and attention must be given to the direction of its stress. It is sometimes necessary to cut a section through two diagonals, in which case they should both be subdiagonals, since the stresses and components of the latter are easily determined.

Figure 107 represents the skeleton diagram of a deck Baltimore truss of 12 panels of 24 ft. each. The depth of the truss is 50 ft. The dead panel load is 44 kips at the upper panel points, and at the lower panel points 32 kips. No loads are considered to be applied at the joints b, d, f, etc.

$$\sec \theta = 1.39 \qquad \tan \theta = 0.96$$

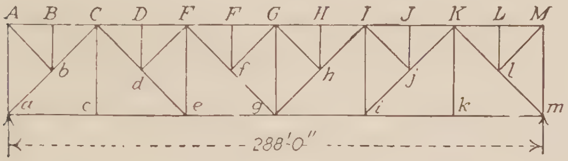


Fig. 107.

Web Stresses

The stress $Bb = Dd = Ff = -44$ kips. The stress in $Cc = +32$ kips. By the previous analysis the stress in $Ab = dE = fG = \frac{44}{2} \times 1.39 = +30.6$ kips. Considering one-half of a panel load applied at A, by cutting a section through Aa, Ab, and AB, and placing the summation of vertical components equal to zero, the stress in Aa is found to be -44.0 kips.

ab The shear in section (Fig. 108) is

$$5\tfrac{1}{2} \times 44 + 2\tfrac{1}{2} \times 32 = 322$$

Placing the summation of vertical components equal to zero

$$+V - Ab \cos \theta + ab \cos \theta = 0$$

Substituting, $+322 - \frac{44}{2} + ab \cos \theta = 0$

from which $ab = -300 \sec \theta = -417.0$ kips.

bC $V = 4\tfrac{1}{2} \times 44 + 2\tfrac{1}{2} \times 32 = 278 \quad bC = -278 \sec \theta = -386.4$ kips.

Cd $\quad V = 202 \qquad Cd = +280.8$ kips.

de $\quad V$ in section $= 158$

$$158 + \frac{44}{2} - de \cos\theta = 0 \qquad de = +250.2 \text{ kips.}$$

Ef $\quad V = 82 \qquad Ef = +114.0$ kips.

fg $\quad V$ in section $= 38$

$$38 + \frac{44}{2} - fg \cos\theta = 0 \qquad fg = +83.4 \text{ kips.}$$

Ee $\quad V$ in section (Fig. 109) is

$$2\tfrac{1}{2} \times 44 + \tfrac{1}{2} \times 32 = 126$$

$$\text{Vertical components} = 0$$

$$126 + \frac{44}{2} + Ee = 0 \qquad Ee = -148.0 \text{ kips.}$$

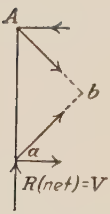


FIG. 108.

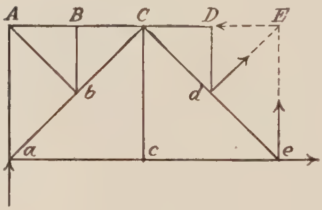


FIG. 109.

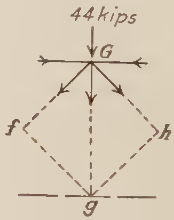


FIG. 110.

Gg $\quad$ Circular section as shown in Fig. 110

$$\text{Vertical components} = 0$$

$$-44 - \frac{44}{2} - \frac{44}{2} - Gg = 0 \qquad Gg = -88.0 \text{ kips.}$$

CHORD STRESSES

The chord stresses are obtained by the resolution of forces, the successive sections being taken as shown in Fig. 111.

In section (4) *DE*, the known stress, is in compression, while the subdiagonal *dE* is in tension. The arrows show the relative directions. If *eg*, the unknown stress, is represented as acting away from the section, then placing the summation of the horizontal components equal to zero

$$eg + dE \sin\theta - DE = 0$$

or

$$eg = DE - dE \sin \theta$$

From this equation it is evident that the stress in eg is smaller than that in DE, although the former chord is nearer the center. This condition very often occurs in a truss with subdivided panels, so care must be taken in ascertaining the direction of each stress when making a summation of the horizontal components in a section.

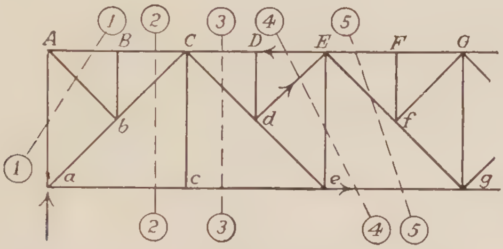


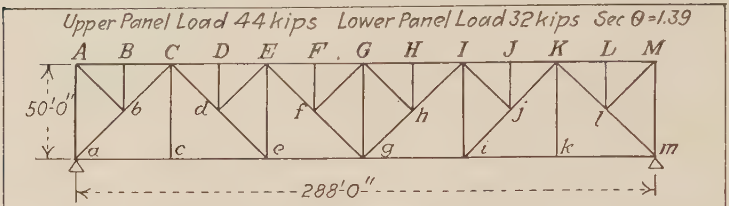
Fig. 111.

An analysis of the other sections will show that all the other horizontal components of diagonals tend to increase the chord stresses. In the table no signs of stresses are indicated in order that the computations may not be confused. Since the Baltimore truss is a simple truss, all the upper chord members are in compression, and the lower chord members in tension.

Diagonal	Vertical component	Horizontal component	Stress	Chord
Ab	22	21.1	21.1	AC
bC	278	266.9	288.0	ae
Cd	202	193.9	481.9	cE
dE	22	−21.1	460.8	eg
Ef	82	78.7	539.5	Eg

Figure 112 gives in compact form the necessary computations to determine the stresses in the principal web members. Illustrated calculations such as are there shown will be found to be a desirable method of solving this type of truss, and will be especially useful in connection with live-load stress determination.

Figure 113 shows the skeleton diagram of the truss with the value of the stresses in terms of sec θ and tan θ. A thorough study of this diagram will show the interrelation between the dead-load stresses in any section cut.



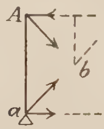
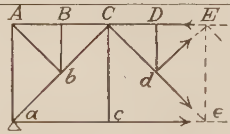
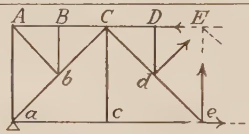
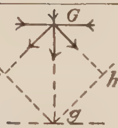
Member	Section	Shear in Section	Σ Vertical Components	Stress
ab		322	$+322-\frac{44}{2}+ab\cos\theta=0$	-417.0
bC	Thru BC, bC and ac	278	$+278+bC\cos\theta=0$	-386.4
Cd	Thru CD, Cd and ce	202	$+202-Cd\cos\theta=0$	$+280.8$
de		158	$+158+\frac{44}{2}-de\cos\theta=0$	$+250.2$
Ee		126	$+126+\frac{44}{2}+Ee=0$	-148.0
Ef	Thru EF, Ef and eg	82	$+82-Ef\cos\theta=0$	$+114.0$
fg	Thru FG, fG, fg and eg	38	$+38+\frac{44}{2}-fg\cos\theta=0$	$+83.4$
Gg			$-44-\frac{44}{2}-\frac{44}{2}-Gg=0$	-88.0

FIG. 112.

67. The Pettit Truss. The Pettit truss bears the same relation to the Parker truss as the Baltimore truss bears to the Pratt truss. A typical form is illustrated in Fig. 114. The two members indicated by heavy broken lines are counters, while the other

members shown by broken lines serve merely to stiffen the structure. The vertical members, so indicated, are designed to support the weight of the upper chord, while the horizontal

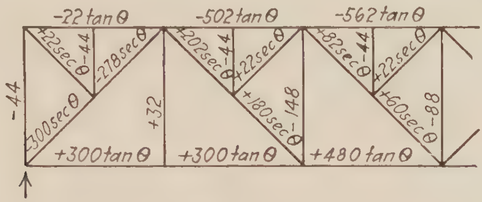


FIG. 113.

members carry no definite load. They form no part of the truss proper and have little effect upon the stresses, and may be considered omitted in the analysis except that the verticals bring a portion of the upper chord load to the intermediate joints.

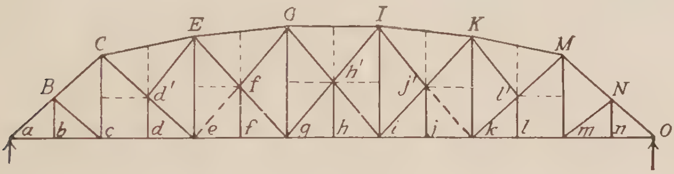


FIG. 114.

The analysis by the method of moments presents but little variation from that given for the Parker and Baltimore trusses. It is, however, somewhat longer and more tedious.

The stresses in the subdiagonals are not so easily obtainable as in the Baltimore truss, for since the slopes of the main and subdiagonals in a double panel are not the same, the vertical components of the stresses in the latter are not equal to one-half of the panel loads.

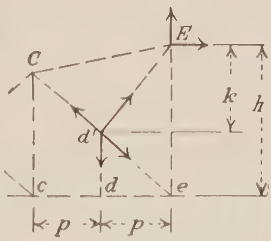


FIG. 115.

In Fig. 115 consider the stress in $d'E$ broken up into two components, one horizontal and one vertical, applied at E. Taking moments about e

$$d'E_H \times h - d'd \times p = 0$$

or

$$d'E_H = d'd \times \frac{p}{h} \text{ and similarly } d'\,E_V = d'd \times \frac{k}{h}.$$

and

$$d'E = d'd \times \frac{p}{h} \times \frac{l}{p} = d'd \times \frac{l}{h}$$

where l is the length of $d'E$.

Figure 116 is the skeleton diagram of a 14-panel Pettit truss of 336 ft. span. The lengths of the vertical and inclined members are given on the diagram. The dead panel load is 36 kips at the

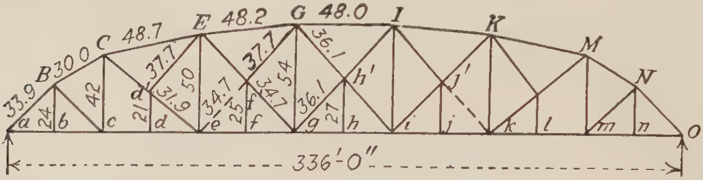


FIG. 116.

upper panel points and 52 kips at the lower panel points. The net reaction at a is 482 kips. The stresses in most of the members can be found in a manner similar to that used for the Parker truss. The stress in $d'E$ is by the previous analysis

$$52 \times \frac{37.7}{50} = 39.2 \text{ kips.}$$

The stress in $d'e$ is determined as follows: With a section cut as indicated in Fig. 117, the center of moments is taken at the

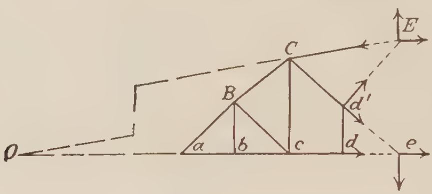


FIG. 117.

intersection of the two chords CE and de, the point O, which is 204 ft. or $8\frac{1}{2}$ panel lengths from the left support. The stress in $d'E$ is resolved into two components, one horizontal and one vertical, considered applied at the point E. The horizontal component is

$$52 \times \frac{24}{50} = 25.0 \text{ kips.}$$

and the vertical component

$$52 \times \frac{50 - 21}{50} = 30.2 \text{ kips.}$$

Taking moments about O and solving for the vertical component of $d'e$

$$d'e_v = \frac{482 \times 204 - 25.0 \times 50 + 30.2 \times 300 - 88(228 + 252) - 52 \times 276}{300} = +165.3$$

and

$$d'e = 165.3 \times \frac{31.9}{21} = +251.1 \text{ kips.}$$

The stress in the vertical Ee is obtained directly by writing a moment equation with O as the center of moments, the section being cut through CE, $d'E$, Ee, ef', and ef, there being no stress in ef' with dead load only. The stresses in Gh' and $h'I$ are equal, their vertical components being one-half of the panel load applied at h. The members gh' and $h'i$ are not stressed under dead load.

The stresses in the remaining members are determined by similar analyses, the necessary computations being given below.

$Bb = dd' = ff' = +52$ kips.

Center of moments at intersection of BC produced and ac produced, *i.e.*, 8 ft. to the left of a.

$$Bc = \frac{(482 \times 8 - 88 \times 32)}{56}\,\frac{33.9}{24} = +26.3 \text{ kips.}$$

Summation of vertical components equal zero.

$$aB = -\left(4 \times 36 + 6\tfrac{1}{2} \times 52\right)\frac{33.9}{24} = -680.8 \text{ kips.}$$

Center of moments at c.

$$BC = \frac{-(482 \times 48 - 88 \times 24)}{42}\,\frac{30.0}{24} = -625.7 \text{ kips.}$$

Center of moments 8 ft. to the left of a.

$$Cc = \frac{-482 \times 8 + 88 \times 32 + 52 \times 56}{56} = +33.4 \text{ kips.}$$

Center of moments at B.

$$ab = bc = \frac{482 \times 24}{24} = +482.0 \text{ kips.}$$

Center of moments at C.

$$cd = de = \frac{482 \times 48 - 88 \times 24}{42} = +500.6 \text{ kips.}$$

Center of moments at O.

$$Cd' = \frac{482 \times 204 - 88(228 + 252)}{300} \cdot \frac{31.9}{21} = +284.0 \text{ kips.}$$

Center of moments at e.

$$CE = \frac{88(48 + 72) - 482 \times 96}{50} \cdot \frac{48.7}{48} = -724.6 \text{ kips.}$$

Center of moments at O.

$$Ee = \frac{88(288 + 252) + 52(276 + 300) - 482 \times 204 - 30.2 \times}{300}$$
$$\frac{300 + 25.0 \times 50}{} = -113.2 \text{ kips.}$$

Center of moments at intersection of EG produced and lower chord produced, *i.e.*, 504 ft. to the left of a.

$$Ef' = \frac{482 \times 504 - 88(528 + 552 + 600) - 52 \times 576}{648} \cdot \frac{34.7}{25} =$$
$$+139.5 \text{ kips.}$$

Center of moments at g

$$EG = \frac{88(48 + 96 + 120) + 52 \times 72 - 482 \times 144}{54} \cdot \frac{48.2}{48}$$
$$= -789.1 \text{ kips.}$$

As for $d'E$

$$f'G = \frac{52 \times 37.7}{54} = +36.3 \text{ kips.}$$

Center of moments 504 ft. to the left of a.

$$f'g = \frac{482 \times 504 - 88(528 + 552 + 600) - (52)(576 + 624)}{648}$$
$$\frac{- 23.1 \times 54 + 27.9 \times 648}{} \cdot \frac{34.7}{25} = +106.0 \text{ kips.}$$

Center of moments at G.

$$ef = fg = \frac{482 \times 96 - 88(72 + 48) - 52 \times 24}{50}$$
$$= +689.3 \text{ kips.}$$

Summation of vertical components equal zero.

$$Gh' = \frac{52 \times 36.1}{2 \quad 27} = +34.8 \text{ kips} \qquad (\text{Then } gh' = 0)$$

Center of moments 504 ft. to the left of a.

$$Gg = \frac{88(528 + 552 + 600) + 52(576 + 624 + 648) - 482}{648}$$
$$\frac{\times 504 + 23.1 \times 54 - 27.9 \times 648}{} = -24.4 \text{ kips.}$$

Center of moments at G.

$$gh \neq hi = \frac{482 \times 144 - 88(120 + 96 + 48) - 52(72 + 24)}{54} =$$
$$+762.7 \text{ kips.}$$

Center of moments at g.

$$GI = \frac{\dfrac{88(120 + 96 + 48) + 52(72 + 24) - 482 \times 144}{54} - \dfrac{24}{27} \times 26 \times 54}{} = -785.8 \text{ kips.}$$

In the above computations each stress has been calculated independently of those previously determined. In most cases the work can be made less laborious by using previously determined stresses and utilizing either the first or second condition of static equilibrium. The stresses in the center panel due to dead load alone are not important. Gh' and gh' may be assumed to divide the shear between them or one of them may be assumed to carry the total, as was done above. In either case the live-load stresses will determine the required sections of the members.

CHAPTER VI

STRESSES IN TRUSSES DUE TO UNIFORM LIVE LOADS

68. Live Loads. The live or moving loads that actually pass over a highway bridge are of great variety. The types of loading producing the maximum stresses in the various portions of the structure are not so varied. A crowd of people densely packed on the roadway and on the sidewalks of a bridge will often produce the maximum stresses in the main truss members. The floor system, on the other hand, is more heavily stressed by the passage of an overloaded truck, a traction engine, or an electric car. With two types of loadings so different, one from the other, it follows that a specification must provide for both contingencies. The usual loading specified is a loaded truck (or trucks) or traction engine followed by a uniform load. However, either the concentrated load or the uniform load may be alone on the bridge and may cover a part or the whole of the bridge.

Traction engines weighing 20 tons are not unusual in some localities. The heaviest motor truck in common use has a total weight of 13 tons when loaded. With an overload of 50 per cent, which frequently occurs, the total weight is nearly 20 tons. The highway commissions of several states have adopted, in their specifications, concentrated live loads varying from 10-ton trucks or traction engines to trucks of 25 tons. In most cases at least two-thirds of the load is considered to be carried by the rear axle. The axles are from 10 to 14 ft. center to center and the gage of the wheels is from $5\frac{1}{2}$ to 8 ft. The uniform load usually specified varies from 50 to 100 lb. per square foot.

For railroad bridges the live load consists of the moving locomotives and the train, and acts as a system of concentrated rolling loads. The maximum load usually specified consists of two locomotives coupled together followed by a uniform load representing the train load. A great variety of engine loadings has been used by the various railroad companies, each road, as a rule, selecting a loading based on the weights of the heaviest locomotive in service or to be anticipated during the life of the structures

under consideration. With such great divergence in specifications the calculation of stresses by exact methods produced variable and questionable results. This resulted in the proposal of several typical compromise loadings in which the wheel weights and spacings were modified to secure greater simplicity. The system of loads proposed by Theodore Cooper in 1894, consisting of two locomotives followed by a uniform train load, is the only one which has come into any extensive use. This system is now more universally used by the railroads of this country than any other type of load. Cooper's E-60 loading is shown on page 175. Any other class of Cooper's loading may be obtained directly from this one by multiplying each axle weight of the E-60 loading by the ratio of the class desired, to E-60. For example, each axle load in the E-50 class is equal to $^{50}\!/_{60}$ times the corresponding axle load in the E-60 class. The wheel spacing is the same for each class. The more recently proposed M system of loading is believed by its proponent, D. B. Steinman, and by many other bridge engineers, to be more nearly typical of the present-day heavy locomotive loads than Cooper's system. As in the Cooper system, the wheel spacings are the same for each class and the loads in direct ratio to the class number. It is given as an alternative loading in the "Tentative Specifications for Steel Railway Bridge Superstructures," of the A. S. C. E. (1923) (see page 177).

To some extent "equivalent" uniform loads are used in stress calculations. They are somewhat simpler in their application, and if properly chosen, give results agreeing closely with those obtained by the use of concentrated loads. The refinements of calculation by the wheel-load method are unwarranted, for the stresses arising from impact and vibration due to unbalanced locomotive drivers, inequalities of track, etc., are a very large and variable percentage of these carefully determined stresses. For ordinary bridges, however, the calculation of stresses, using any one of the conventional wheel-load systems, is not difficult, so little, if anything, is gained by the use of an equivalent uniform load. On the other hand, for arches, cantilevers, and suspension bridges, especially those of long span, the equivalent load is desirable. The proper selection of uniform loads is discussed in Art. 106.

The general effect on a bridge structure produced by a live or moving load is the same whether the load is considered as uniform

or as a system of concentrated loads. The former type presents fewer complications to the beginner and furnishes an approach to the apparently more exact method of calculation. It seems advisable, therefore, to consider first, bridge trusses under uniform live loads. The remainder of this chapter will deal with uniform live loads only.

Trusses with Horizontal Chords and Single Web Systems

69. Web Members. The conventional method of calculation of the stresses in the web members of a truss due to uniform load is to assume all panel points on one side of the panel cut by the section through the member fully loaded, while no load is considered on the other side.

The maximum and minimum shears are obtained under these two possible loadings. For instance, in Fig. 118, if all panel

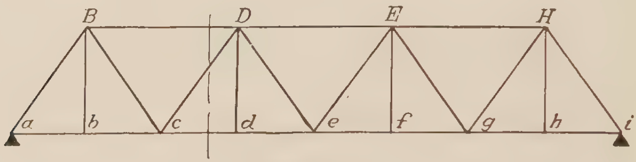


Fig. 118.

points to the right of the section are loaded with a load P, taking moments about the right support, the reaction at a is $\frac{15P}{8}$, and since there is no load between a and the section, the shear in the section is $^{15}\!/_8 P$. If a load were added at c the reaction would become $^{21}\!/_8$, but the shear in the section would be

$$\frac{21}{8}P - \frac{8}{8}P = \frac{13}{8}P,$$

which is less than the value obtained with only the panel points to the right of the section loaded. It follows, therefore, that the largest positive shear occurs when the live load extends from the section to the right support.

This assumed condition of loading is actually an impossible one, for in order to have a full panel load at the panel point d the panels cd and de must be fully loaded. Since the load on a panel is transferred through the stringers and floor beams to the

panel points of the truss (see Art. 84) the load on the panel cd would cause a partial panel load at the point c. This would, in case of loading for maximum positive shear bring a half-panel load to the point c on the left of the section, provided a full load were desired at d.

The true maximum shear in the panel cd will evidently occur when the uniform load extends from the right beyond d some distance into the panel cd.

Let a single load P be applied on the stringer in the panel cd (Fig. 119) at a distance x from the right end of the panel. Let

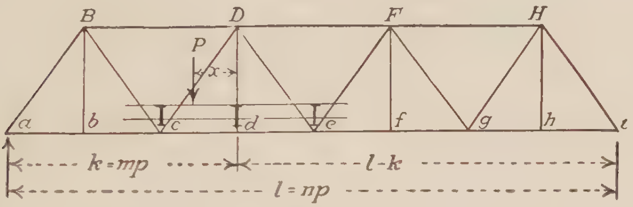


FIG. 119.

k be the distance from the right end of the panel to the left support, $l - k$ the corresponding distance to the right support, and p the panel length. Let R_a be the left reaction of the truss, and r_c the proportion of the load P brought to the panel point c by the stringer and floor beam.

Then

$$V_{cd} = R_a - r_c = \frac{P(l - k + x)}{l} - \frac{Px}{p} = P\left(\frac{l - k + x}{l} - \frac{x}{p}\right)$$

If the position of the load is such that

$$\frac{l - k + x}{l} = \frac{x}{p}$$

or

$$x = \frac{p(l - k)}{l - p}$$

the shear in the panel is 0. Any load in the panel to the right of this position causes a positive shear and any load to the left a negative shear. Therefore, the uniform load should extend into the panel a distance

$$x = \frac{p(l - k)}{l - p}$$

for the true maximum shear (Fig. 120). If $k = mp$ and $l = np$ the expression reduces to $x = \dfrac{p(n-m)}{n-1}$ and the true maximum live-load shear in any panel is

$$\frac{w\left[p(n-m) + \dfrac{p(n-m)}{n-1}\right]^2}{2np} - \frac{w\left[\dfrac{p(n-m)}{n-1}\right]^2}{2p}$$

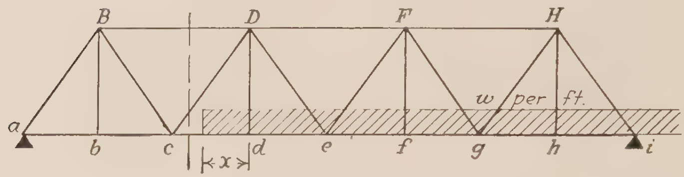


FIG. 120.

which reduces to

$$V = \frac{wp(n-m)^2}{2(n-1)}$$

This true maximum shear is slightly smaller than the shear obtained by the conventional method, as is shown by the tabulation at the end of this article. The conventional system will be used hereafter throughout this chapter.

Let a live panel load of 84 kips be assumed for the Warren truss of Fig. 121, whose span is 168 ft. The depth of the truss

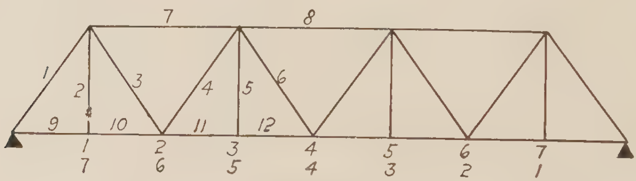


FIG. 121.

is 30 ft. and $\sec\theta = 1.22$ and $\tan\theta = 0.70$. The maximum positive shear in the third panel is

$$\frac{84(5 \times 21 + 4 \times 21 + 3 \times 21 + 2 \times 21 + 1 \times 21)}{8 \times 21} =$$

$$\frac{84}{8}(5 + 4 + 3 + 2 + 1) = 157.5 \text{ kips.}$$

The maximum negative shear in this same panel is

$$\frac{84(7 \times 21 + 6 \times 21)}{8 \times 21} - 2 \times 84 = \frac{84}{8}(7 + 6) - 84 \times 2 =$$

$$-\frac{84}{8}(1 + 2) = -31.5 \text{ kips.}$$

Referring to Fig. 121 with the panel points of the *loaded* chord numbered from right to left and again from left to right, it is seen that the equation for the maximum positive shear is simply *the panel load divided by the number of panels and multiplied by the summation of the panel point numbers to the right of the section referred to the right support.* Similarly, the equation for the maximum negative shear is *the panel load divided by the number of panels and multiplied by the summation of the panel point numbers to the left of the section referred to the left support.*

Once the live load shears are obtained they are treated in the same manner in computing the stresses in web members as were the dead-load shears. Hence the maximum live-load stress in 4 is

$$157.5 \times 1.22 = -192.2 \text{ kips.}$$

and the minimum live-load stress

$$31.5 \times 1.22 = +38.4 \text{ kips.}$$

The stresses in the other diagonals are found in a similar manner. Since the stresses in members 2 and 5 are dependent only on the load applied at their extremities, the maximum live-load stress in each of these members is +84.0 kips.

The greatest live-load stress having the same sign as the dead-load stress is known as the maximum live-load stress. The largest stress of the opposite sign is known as the minimum live-load stress. The maximum positive and negative live load shears in the diagonals of the truss of Fig. 121 as computed by the conventional method, together with the corresponding stresses are tabulated below.

Diagonal	Maximum positive live-load shear	Maximum negative live-load shear	Maximum live-load stress	Minimum live-load stress
1	294.0	0	−358.7	0
3	220.5	−10.5	+269.0	−12.8
4	157.5	−31.5	−192.2	+38.4
6	105.0	−63.0	+128.1	−79.9

The true live-load shears are:

Diagonal	Maximum positive live-load shear	Maximum negative live-load shear
1	294.0	0
3	216.0	− 6.0
4	150.0	−24.0
6	96.0	−54.0

70. Chord Members. The maximum live-load stresses in the chord members of a truss occur when the live load covers the whole span.

In Fig. 122 the section passed through the chord member *de* shows that the center of moments is at D, distant x from the left support. With a load on the right of the section the stress in *de*

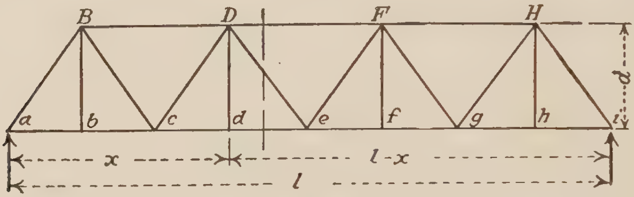


FIG. 122.

is $\frac{R_a x}{d}$ and the stress increases with R_a. But R_a increases as loads are added to the right of the section. Similarly, with a load on the left of the section the stress in *de* is $\frac{R_i(l - x)}{d}$ and this also increases with R_i which in turn is increased with additional loads on the left of the section. Hence, every load added on either side of the section increases the stress, and the maximum stress is obtained when every panel point is loaded.

It follows, therefore, that live-load stresses in chord members are obtained in exactly the same manner as the dead-load stresses. Either the method of moments or that of resolution of forces may be used. If all the dead load is assumed concentrated on the loaded chord, the live-load stresses may be obtained directly from the dead-load stresses by multiplying the latter by the ratio of live panel load to dead panel load.

The chord stresses in the truss of the last article, due to a live panel load of 84 kips, are tabulated below.

Diagonal	Shear	Horizontal component	Stress	Chord
1	294.0	205.8	205.8	9, 10
3	210.0	147.0	352.8	7
4	126.0	88.2	441.0	11, 12
6	42.0	29.4	470.4	8

Note that the shears tabulated above are not the maximum live-load shears used in the last article, but are the shears occurring when each and every panel point is sustaining a full live panel load.

71. Counters. In a long-span bridge with comparatively long diagonals, it is often more economical to design the diagonals for one kind of stress only. When this is done it becomes necessary to place in some of the panels another diagonal, which crosses

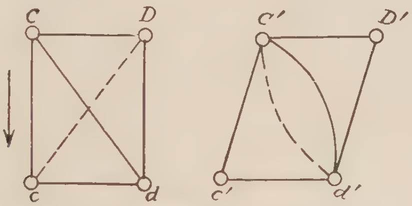


Fig. 123.

the main diagonal, for the purpose of taking care of the resultant negative shear due to the combined dead and live loads. The main diagonals are those which resist the dead-load stresses when there is no live load on the bridge. The other diagonals are known as *counter diagonals*, or, simply *counters*.

Let Fig. 123 represent any panel of a truss whose diagonals can sustain tensile stress only. When the resultant of all the external forces on the left of the section is negative, *i.e.*, acts downward, the quadrilateral $CDdc$ tends to assume the shape $C'D'd'c'$. The diagonal Cd, being incapable of resisting the compressive stress developed, tends to buckle into the shape $C'd'$. This distortion may be prevented by placing another diagonal cD, capable of resisting tensile stress in the panel, for in order that C and d may come closer together, c and D must be

pushed further apart. If this is prevented by the tension developed in cD, the original form of the quadrilateral is maintained. The member cD is a counter and is only stressed when the main diagonal Cd has ceased to act.

The action of compressive diagonals in a similar situation may be understood by reference to Fig. 124. The compressive diagonal bearing against the horizontal members, but not having tensile joints at those intersections, fails to take the tension developed by a negative shear in the panel, and pulls loose from its bearings. In this case a similar compression member Cd is necessary to prevent distortion.

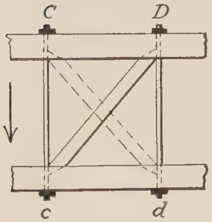


Fig. 124.

In the Howe truss, in which usually as many members as possible are constructed of timber, the stress most economically resisted by the diagonals is compression, and the main diagonals slope upward toward the center, so as to take compression under dead load.

In the Pratt truss with counters, the diagonals are made of comparatively thin bars, called eyebars, which yield laterally under slight compression. Eyebar diagonals lend themselves readily to pin-connected construction, and formerly the Pratt pin-connected truss with counters was used for both railroad and highway bridges of comparatively short span. Present-day practice does not favor the use of pin connections or counters for short spans, but in the longer spans this type of construction is often found.

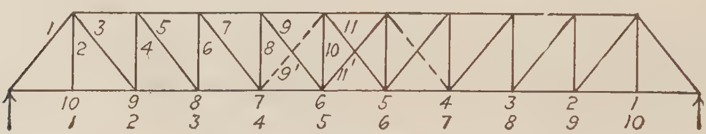


Fig. 125.

Figure 125 is the skeleton diagram of a single-track through Pratt railroad bridge truss. The joints are pin connected, and counters are to be used where necessary. The span is 286 ft. and the depth of truss 33 ft.-0 in. The dead load is estimated at 2000 lb. per foot of truss, and a live load of 7200 lb. per foot of track is assumed. The dead upper and lower panel loads are

taken as 16 and 36 kips, respectively, while the live panel load is 93½ kips.

$$(\sec\theta = 1.27 \text{ and } \tan\theta = 0.79)$$

The maximum stresses in the web members are computed in exactly the same manner as if counters were not to be used, and are tabulated below.

Web member	Dead-load shear	Dead-load stress	Number of loaded panel points for maximum stress	Multiplier for $\frac{93.5}{11}$	Positive live-load shear	Live-load stress	Maximum stress
1	260	−330.2	10	55	467.5	−593.7	−923.9
2	. . .	+ 36.0	. .	. .	. . . . .	+ 93.5	+129.5
3	208	+264.2	9	45	382.5	+485.8	+750.0
4	172	−172.0	8	36	306.0	−306.0	−478.0
5	156	+198.1	8	36	306.0	+388.6	+586.7
6	120	−120.0	7	28	238.0	−238.0	−358.0
7	104	+132.1	7	28	238.0	+302.3	+434.4
8	68	− 68.0	6	21	178.5	−178.5	−246.5
9	52	+ 66.0	6	21	178.5	+226.7	+292.7
10	16	− 16.0	5	15	127.5	−127.5	−143.5
11	0	0	5	15	127.5	+161.9	+161.9

The minimum stress in the diagonal 1 is the dead-load stress, since no live load can be brought upon the bridge which will produce a negative shear in the first panel. Similarly, the minimum stress in the suspender 2 is the dead-load stress.

With a live panel load at the first panel point from the left support, the live-load shear in diagonal 3 is $-\frac{93.5}{11} \times 1 = -8.5$ kips, the sum of the dead- and live-load shears $+199.5$ kips, and the minimum stress $+253.4$ kips.

For the vertical 4, the maximum negative shear is -25.5 kips with the first two panel points loaded, the sum of the live-load and dead-load shears $+146.5$ kips, and the minimum stress -146.5 kips.

Similar analyses give minimum stresses in members 5, 6, and 7 of $+165.7$, -69.0, and $+67.3$ kips, respectively.

Proceeding in the same manner with the vertical 8, a tensile stress is apparent. This is impossible when counters are used, as will be shown later.

The dead-load shear in diagonal 9 is +52 kips. With three live panel loads on from the left, the live-load shear is $-\frac{93.5}{11}(1 + 2 + 3) = -51.0$ kips. The resultant shear is +1.0 kip, and the stress in the diagonal +1.3 kips. With four live panel loads on from the left, the live-load shear is $-\frac{93.5}{11}(1 + 2 + 3 + 4) = -85.0$ kips. The resultant shear is −33.0 kips, which would mean a stress of −41.9 kips in diagonal 9 if it were capable of taking compression. Since it can take only tensile stresses, the counter 9′ now comes into action and its stress is +41.9 kips, while the stress in 9 is 0. The minimum stress in the diagonal 9 is, therefore, 0. The maximum stress in the counter 9′ is +41.9 kips, and the minimum stress 0.

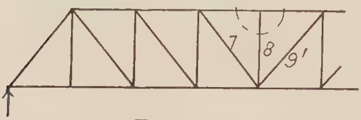


FIG. 126.

With 9′ acting, there is no diagonal intersecting the upper chord at the point where the vertical 8 intersects this chord. Therefore, by cutting a circular section as indicated in Fig. 126, the stress is seen to be equal to the dead upper panel load, or −16 kips.

That the compressive stress in any vertical of a Pratt truss with counters can never be less than the dead upper panel load can be proved as follows:

In Fig. 127 *de* and *ef* represent any two panels between the left support and the center of a through Pratt truss,

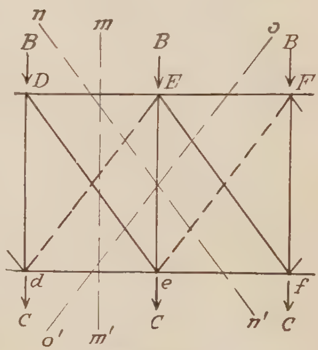


FIG. 127.

Let the dead load shear in section n-n' be A

the dead upper panel load B

the dead lower panel load C

Then the dead load shear in the section m-m' = $A + C$

For the main diagonal De to cease acting and the counter diagonal dE to come into action, the negative live-load shear in the section m-m' must be greater than $A + C$. With the counter dE acting, the counter eF in the next panel toward the center will also be acting, since the negative live-load shear will be equal

to, or greater than, that in the section m-m', while the dead-load shear is less by the amount $B + C$. Therefore the section must be cut as o-o', where the dead-load shear $= A - B + C$, so that the total negative shear is greater than $A + C - (A - B + C) = B$, the upper panel load. In the section n-n', with the adjacent main diagonals, De and Ef acting, a compressive stress greater than B will be produced in the vertical. Therefore, with the main diagonals acting in the two panels adjacent to a

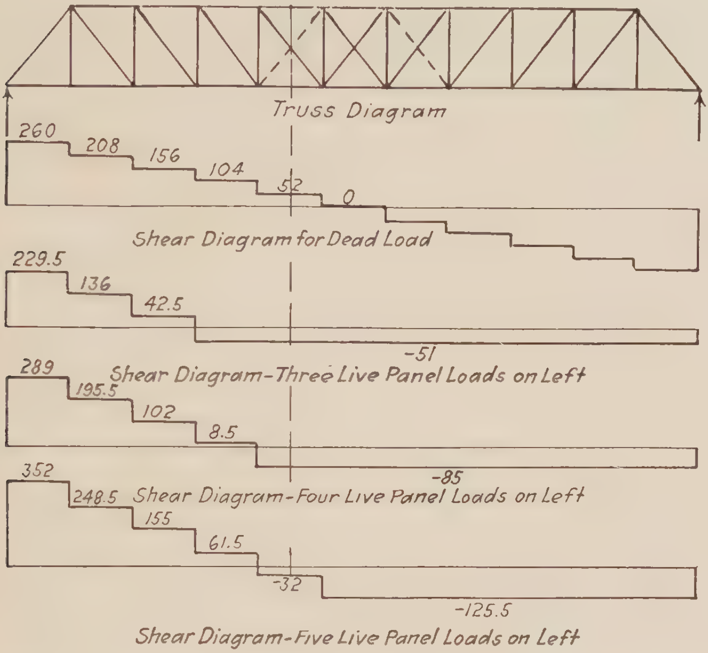


FIG. 128.

vertical, the stress in the vertical is greater than the dead upper panel load. Also, when the two counter diagonals act, the stress in the vertical is greater than the dead upper panel load. With one main diagonal and one counter diagonal acting as in Fig. 126 the stress is equal to the dead upper panel load, and this is, therefore, the minimum stress.

As the live load proceeds from left to right across the bridge, the counter 9′ will continue to act as long as the shear in that panel is negative, and cease to act when the shear again becomes

positive. With five loads on from the left, the live-load shear in 9 is $-\frac{93.5}{11}(1 + 2 + 3 + 4 + 5) + 93.5 = -32.0$ kips, and the resultant shear $+20.0$ kips. Therefore, the counter 9′ commences to act with four loads on from the left and ceases to act when five loads are on from the left. This is shown graphically in Fig. 128.

A similar analysis of diagonal 11 shows that the minimum stress is 0, while the maximum stress in 11′ is the same as that in the main diagonal, the one occurring with the five panel points to the left being loaded, and the other when the five panel points to the right are loaded. As the live load proceeds from left to right, 11′ commences to act when the load reaches the first panel point, and ceases to act when the load covers the whole structure. When the first panel point on the left becomes unloaded, the diagonal 11 commences to take stress, and continues to do so until the load has passed from the bridge.

The minimum stress in vertical 10 is, as in 8, equal to the dead upper panel load, or -16.0 kips.

The chord stresses are determined with the main diagonals acting, the maximum stresses occurring with full live load on the bridge, the minimum stresses with no live load.

Trusses with Inclined Chords and Single Web Systems

72. The Parker Truss without Counters. In general, the same conditions of loading are applied to trusses with inclined chords to obtain the maximum and minimum stresses, as were used for trusses with horizontal chords. As in the dead-load stress calculation, the method of moments is used.

As an example let the truss whose dead-load stresses were computed in Art. 65 be taken, with a live panel load of 98 kips (Fig. 129).

	Kips
With live load on every panel point of the truss, the left reaction is......	294
With 5 panel loads brought on from the right, $R_L = \frac{98}{7}(1 + 2 + 3 + 4 + 5)$..........	210
With 4 loads..........	140
With 3 loads..........	84
With 2 loads..........	42
With 1 load..........	14

The stresses in the web members on the right of the center are computed with the load brought on from the right. The values thus obtained correspond to the stresses in the symmetrical members with the load brought on from the left. Note that the left reaction is used for all computations, but that for members to the right of the center, the center of moments is on the right of the truss.

The two diagonals in the center panel are to be designed to resist either tension or compression, and it is sufficiently accurate to assume that under all loading conditions, each will resist one-half of the shear in the panel, one diagonal being in tension and the other in compression.

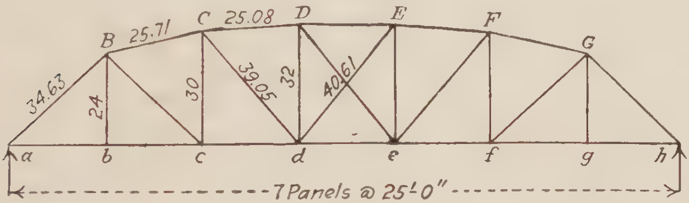


FIG. 129.

The stresses in aB and Bb are determined as for a Pratt truss;

$$aB = -294 \times \frac{34.63}{24} = -424.2 \text{ kips, and } Bb = +98.0 \text{ kips.}$$

BC produced intersects the lower chord produced three panel lengths to the left of a. With this intersection as the center of moments and the live load extending from c to h,

$$Bc = \frac{210 \times 3 \times 25}{5 \times 25} \times \frac{34.63}{24} = +181.6 \text{ kips.}$$

The center of moments for Cc is the same as for Bc, but the load extends only from d to h,

$$Cc = -\frac{140 \times 3 \times 25}{5 \times 25} = -84.0 \text{ kips.}$$

CD produced intersects the lower chord produced 13 panel lengths to the left of a. With this intersection as the center of moments, and the load extending from d to h,

$$Cd = \frac{140 \times 13 \times 25}{16 \times 25} \times \frac{39.05}{30} = +148.1 \text{ kips.}$$

In the center panel the maximum shear occurs with the panel points e, f, and g, loaded, and has a value of 84 kips. Under the assumed conditions of equally divided shear

$$De = \frac{84 \times 40.61}{32} \div 2 = +53.3 \text{ kips.}$$

Since, with any unsymmetrical live loading, both of the center diagonals are stressed, it is impossible to cut a section through the member Dd without also cutting a diagonal. The section shown in Fig. 130 offers as little difficulty as any, and with the center of moments the same as for Cd, and the panel points e, f, and g, loaded,

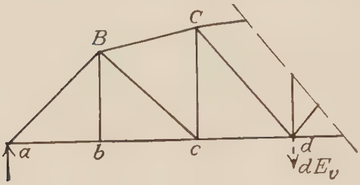


FIG. 130.

$$Dd = \frac{42 \times 16 \times 25 - 84 \times 13 \times 25}{16 \times 25} = -26.3 \text{ kips.}$$

The maximum tension in Dd is obtained by computing that for the symmetrical member Ee. Here again, a diagonal must also be cut by the section, and the section of Fig. 131 may be

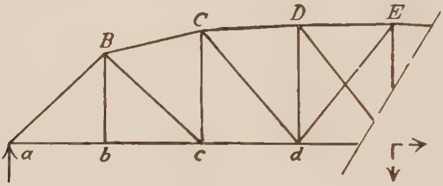


FIG. 131.

used with the center of moments 13 panel lengths to the right of h, and the loading the same as for Dd,

$$Ee = \frac{84 \times 20 \times 25 - 42 \times 16 \times 25}{16 \times 25} = +63.0 \text{ kips.}$$

The maximum stresses in eF, Ff, and fG, computed in order to furnish the minimum stresses for the symmetrical members, Cd, Cc, and Bc, respectively, are all obtained by taking the center of moments on the right side of the truss and are

$$eF = -\frac{42 \times 20 \times 25}{16 \times 25} \times \frac{39.05}{30} = -68.3 \text{ kips.}$$

$$Ff = \frac{42 \times 10 \times 25}{5 \times 25} = +84.0 \text{ kips.}$$

and

$$fG = -\frac{14 \times 10 \times 25}{5 \times 25} \times \frac{34.63}{24} = -40.4 \text{ kips.}$$

The maximum and minimum stresses due to dead and live load are as follows:

Member	Dead-load stress	Maximum live-load stress	Minimum live-load stress	Maximum	Minimum
aB	−173.2	−424.2	0	−597.4	−173.2
Bb	+ 30.0	+ 98.0	0	+128.0	+ 30.0
Bc	+ 57.0	+181.8	−40.4	+238.8	+ 16.6
Cc	− 10.0	− 84.0	+84.0	− 94.0	+ 74.0
Cd	+ 32.6	+148.1	−68.3	+180.7	− 35.7
Dd	+ 5.0	+ 63.0	−26.3	+ 68.0	− 21.3
De	0	+ 53.3	−53.3	+ 53.3	− 53.3

73. The Parker Truss with Counters. From the previous computations it is seen that there is no reversal of stress in the diagonal Bc, and, therefore, no counter is required in the panel bc. There is, however, a reversal in the diagonal Cd, so that counters are required in the three center panels as shown in Fig. 132.

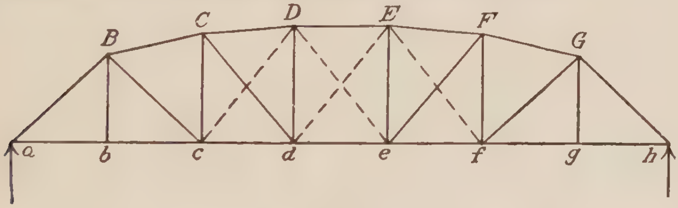


FIG. 132.

The stresses are calculated for a truss similar to Fig. 133, the load being brought on from the right and the left reaction used for all computations as before.

The maximum live-load stresses in the members aB, Bb, Bc, Cc, and Cd are the same as in the previous calculations.

Through Dd a section may be passed parallel to Cd and De, which cuts no other web members. With the load extending from e to h and the center of moments 13 panel lengths to the left of a, the maximum live-load stress in

$$Dd = -\frac{84 \times 13 \times 25}{16 \times 25} = -68.3 \text{ kips.}$$

The maximum stress in De is twice as great as with two diagonals in the panel, since it must now resist all or none of the shear, hence its value is 106.6 kips.

The stress in Ef with live load at panel points f and g is equal to the maximum stress in the counter cD; the center of moments is 13 panel lengths to the right of h, and

$$Ef = \frac{42 \times 20 \times 25}{15 \times 25} \times \frac{40.61}{32} = \frac{56 \times 40.61}{32} = +71.0 \text{ kips.}$$

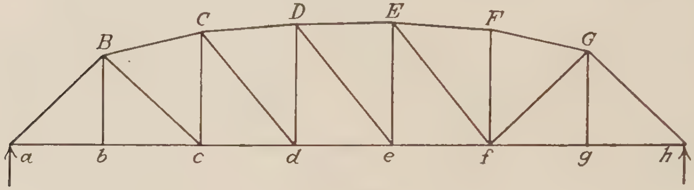


FIG. 133.

The dead-load stress in this member is not equal and opposite to that in Cd, since their lengths and inclinations are not the same. Its value is

$$\frac{40 \times 14 \times 25 + 40 \times 15 \times 25 - 120 \times 13 \times 25}{15 \times 25} \times \frac{40.61}{32} = -33.8 \text{ kips.}$$

In Fig. 134, if a section m-m be passed as shown, it is evident that the live-load tension in the member Ff is equal to the dif-

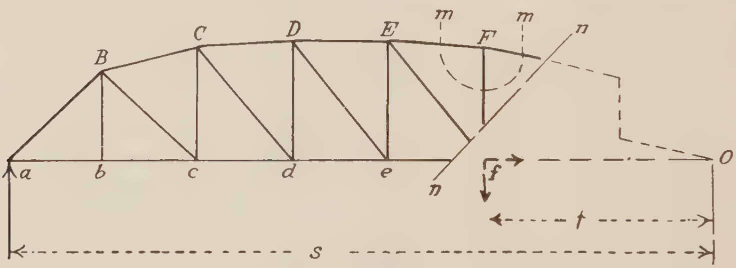


FIG. 134.

ference between the vertical components of the stresses in the chords EF and FG. This difference is the greatest when the live load covers the entire span, and the least when there is no live load on the span. In the section n-n, the stress in Ff may be determined by taking the moment center at the intersection of FG produced with the lower chord produced (O). If P be the

value of the panel load, p the panel length, and R_a the left reaction

$$Ff = [R_a s - Ef_v \times t - P(t + p) - P(t + 2p) \text{ etc.}] \div t$$

From the above it is evident that the maximum tension in Ff will occur when Ef is zero, while from the consideration of section m-m it is seen that the stress in Ff increases as the live load advances on the span. The live load, however, must not cause the member eF to come into action, for then in the section m-m, a portion of the difference in the stress between the vertical components of the two chord stresses would be resisted by this diagonal and the stress in Ff would not be so great as when eF was not acting.

With the conventional system of loading (*i.e.*, full panel loads) it is usually impossible so to place the live load that the stresses in both of the diagonals of the panel in question are zero. The load, therefore, will be placed so as to obtain the smallest possible value for the tension in Ef. The dead-load stress previously computed for Ef is -33.8 kips. With full live panel loads at f and g, and the center of moments at the intersection of EF produced with the lower chord produced, the live-load stress in Ef is $+71.0$ kips as previously computed. With an additional panel load at e, the tension in Ef is not sufficient to overcome the compression due to dead load.

The maximum live-load tension in Ff may now be computed, using the equation based on Fig. 134.

$$Ff = \frac{42 \times 10 \times 25 - 56 \times 5 \times 25}{5 \times 25} = +28.0 \text{ kips.}$$

The dead-load stress in $Ff = Cc$, when the adjacent counter is in action, is (Fig. 135)

$$\frac{120 \times 13 \times 25 - 40 \times 14 \times 25 - 40 \times 15 \times 25 - 10 \times 15 \times 25}{15 \times 25} = +16.7 \text{ kips.}$$

Similarly, the maximum tension in $Ee = Dd$ occurs when neither of the center diagonals is stressed, and the vertical component of the stress in EF is as large as possible. This condition obtains when the live load covers the entire span, under which condition the live-load stress in

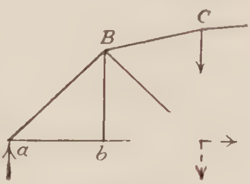


FIG. 135.

$$Dd = \frac{294 \times 13 \times 25 - 98(14 + 15 + 16)25}{16 \times 25} = +36.8 \text{ kips.}$$

The maximum and minimum stresses in the web members are as follows:

Member	Dead-load stress	Maximum live-load stress	Minimum live-load stress	Maximum	Minimum
aB	−173.2	−200.4	0	−373.6	−173.2
Bb	+ 30.0	+ 98.0	0	+128.0	+ 30.0
Bc	+ 57.0	+181.8	−40.4	+238.8	+ 16.7
Cc	− 10.0	− 84.0	......	− 94.0	
	+ 16.7	....	+28.0	....	+ 44.7
Cd	+ 32.6	+148.1	0	+180.7	0
cD	− 33.8	+ 71.0	0	+ 37.2	0
Dd	+ 5.0	− 68.3	+36.8	− 63.3	+ 41.8
De	0	+106.6	0	+106.6	0

The live-load stresses in the chord members are the same for both systems of web members, since they are obtained with full live load on the bridge, under which condition of loading neither the counters, if any, nor the center diagonals are stressed. They may be obtained directly from the dead-load stresses by multiplying by the constant ratio of the live and dead panel loads, *i.e.*, $^{98}/_{40}$.

Trusses with Subdivided Panels

74. The Baltimore Truss without Counters. With a live panel load of 84 kips, the live-load stresses in the truss of Fig. 136 are

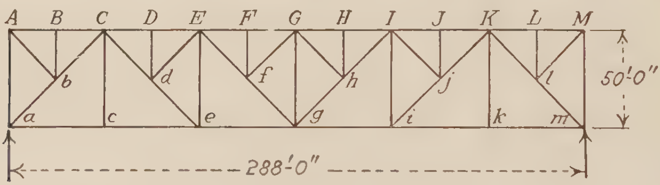


Fig. 136.

computed below. Equations similar to those used in computing the dead-load stresses (see Art. 66 and Fig. 112) may be written for each member. On account of the influence of the subdiagonals on the stresses in the adjacent web members, it is sometimes necessary to bring the load beyond the section to obtain the maximum stresses.

Figure 137(a) represents a portion of the truss under consideration. With the live loading extending only to the section, there is no live-load stress in the vertical Dd or the subdiagonal dE. The figure shows only the members stressed by the live load. The stress in the member de is then equal to the shear times sec θ, as de is the only member in the section which has a vertical component. Under these conditions the live-load shear in the section is $^{84}\!/_{12}$ (1+ . . 8) = 252 kips, and the stress in de = +350.3 kips.

Figure 137(b) represents the same portion of the same truss with the loading extending one panel beyond the section. There is now a live-load stress in the vertical Dd and also in the subdiagonal dE. The value of the vertical component of the latter is $^{84}\!/_{2}$ = 42 kips. The shear in the section is $^{84}\!/_{12}$ (1+ . . 9) −84 = 231 kips, and this shear is balanced by the vertical components of de and dE. The equation for the Σ vertical components = 0 in the section is +231 + 42 − de cos θ = 0, from which the stress is de = +379.5 kips. This value is seen to be greater than that obtained with the load extending only to the section as shown in Fig. 137(a).

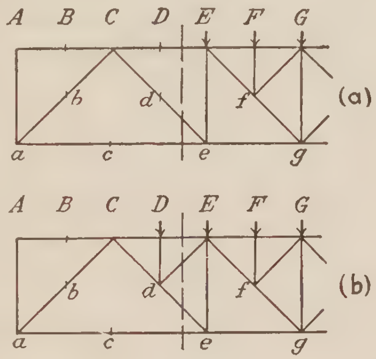


FIG. 137.

The loading, sections, equations, and stresses for the web members are given in the following tables:

MAXIMUM LIVE-LOAD STRESSES—LOADS FROM RIGHT

Member	Loads	Section through members	Shear in section	ΣVertical components	Stress
ab	1–11	AB, Ab, ab, ac	462	$+462 - {}^{84}\!/_{2} + ab \cos\theta = 0$	−583.8
bC	1–10	BC, bc, ac	385	$+385 + bC \cos\theta = 0$	−535.2
Cd	1–9	CD, Cd, ce	315	$+315 - Cd \cos\theta = 0$	+437.9
de	1–8	DE, dE, de, ce	252	$+252 - de \cos\theta = 0$	
	1–9	DE, dE, de, ce	231	$+231 + {}^{84}\!/_{2} - de \cos\theta = 0$	+379.5
Ee	1–8	DE, dE, Ee, eg	252	$+252 + Ee = 0$	
	1–9	DE, dE, Ee, eg	231	$+231 + {}^{84}\!/_{2} + Ee = 0$	−273.0
Ef	1–7	EF, Ef, eg	196	$+196 - Ef \cos\theta = 0$	+272.4
fg	1–6	FG, fG, fg, eg	147	$+147 - fg \cos\theta = 0$	
	1–7	FG, fG, fg, eg	112	$+112 + {}^{84}\!/_{2} - fg \cos\theta = 0$	+214.1
Gg	F, G, H	FG, fG, Gg, Gh, GH	. . .	$-84 - {}^{84}\!/_{2} - {}^{84}\!/_{2} - Gg = 0$	−168.0

MINIMUM LIVE-LOAD STRESSES—LOADS FROM LEFT

Member	Loads	Section through members	Shear in section	Σ Vertical components	Stress
ab	none	....	.....	....	0
bC	1	BC, bC, ac	− 7	$-7 + bC \cos\theta = 0$	+ 9.7
Cd	1–2	CD, Cd, ce	− 21	$-21 - Cd \cos\theta = 0$	−29.2
de	1–3	DE, dE, de, ce	− 42	$-42 + {}^{84}\!/_{2} - de \cos\theta = 0$	
	1–2	DE, dE, de, ce	− 21	$-21\ d'e \cos\theta = 0$	−29.2
Ee	1–3	DE, dE, Ee, eg	− 42	$-42 + {}^{84}\!/_{2} - de \cos\theta = 0$	0
	1–2	DE, dE, Ee, eg	− 21	$-21 + Ee = 0$	+21.0
Ef	1–4	EF, Ef, eg	− 70	$-70 - Ef \cos\theta = 0$	−97.3
fg	1–5	FG, fG, fg, eg	−105	$-105 + {}^{84}\!/_{2} - fg \cos\theta = 0$	
	1–4	FG, fG, fg, eg	− 70	$-70 - fg \cos\theta = 0$	−97.3
Gg	none	....	.....	....	0

The maximum live-load stress in $Bb = Dd = Ff = -84$ kips. Cc has no live-load stress. $Ab = dE = fG = {}^{84}\!/_{2} \times 1.39 = +58.4$ kips. With loads at A and B, the live-load stress in Aa is $-(84 + {}^{84}\!/_{2}) = -126$ kips.

In the above tables it is seen that the maximum live-load stresses in members de, Ee, and fg are obtained by loading beyond the section, while on the other hand, the minimum stresses occur when the loading is one panel point away from the section.

The maximum and minimum stresses in the web members are tabulated below, the dead-load stresses having been obtained in Art. 66.

Member	Dead-load stress	Live-load stress for maximum	Live-load stress for minimum	Maximum	Minimum
Aa	− 44.0	−126.0	0.0	− 172.0	− 44.0
$Ab = dE = fG$	+ 30.6	+ 58.4	0.0	+ 89.0	+ 30.6
$Bb = Dd = Ff$	− 44.0	− 84.0	0.0	− 128.0	− 44.0
Cc	+ 32.0	0.0	0.0	+ 32.0	+ 32.0
ab	−417.0	−583.8	0.0	−1000.8	−417.0
bC	−386.4	−535.2	+ 9.7	− 921.6	−376.7
Cd	+280.8	+437.9	−29.2	+ 718.7	+251.6
de	+250.2	+379.5	−29.2	+ 629.7	+221.0
Ee	−148.0	−273.0	+21.0	− 421.0	−127.0
Ef	+114.0	+272.4	−97.3	+ 386.4	+ 16.7
fg	+ 83.4	+214.1	−97.3	+ 297.5	− 13.9
Gg	− 88.0	−168.0	0.0	− 256.0	− 88.0

In a truss of the type illustrated in Fig. 138 the number of panels determines the loading which produces the maximum and

minimum stresses in certain of the members. Considering a 12-panel truss and the member *Ee*, the live loading producing maximum compression extends from the right support to the panel point *f*. With this loading there is no live-load stress in the diagonal *d'E* so that the live-load stress in *Ee* is equal, but of opposite sign, to the live-load shear. In order to have a live-load stress in the diagonal *d'E*, it would be necessary for the live load to extend to the panel point *d*, which would increase the left reaction by ($8/12 + 9/12$) of one panel load, but decrease the shear in the section by ($8/12 + 9/12 - 12/12 - 12/12) = 7/12$ of a panel load. Since the vertical component of the stress in the diagonal *d'E* is $1/2$ or $6/12$ of a panel load, the net result as far as the member *Ee* is concerned is a decrease in stress over that obtained with the load extending only to the panel point *f* of $1/12$ of a live panel load.

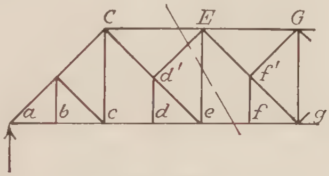


Fig. 138.

In a 16-panel truss the reverse is true, since, by extending the load to the panel point *d*, the shear in the section is decreased by an amount equal to ($12/16 + 13/16 - 16/16 - 16/16) = 7/16$ of a panel load, and the net result as far as the member *Ee* is concerned is an increase of stress over that obtained with the load extending only to the panel point *f* of $1/16$ of a live panel load.

75. The Baltimore Truss with Counters. In the truss of the preceding article, there is only one member, *fg*, that has maximum and minimum stresses of opposite signs. If the truss were pin connected, and eyebars (capable of resisting tension only) were used for the tension diagonals and the lower chord, it would be necessary to have a counter in the double panel *EFG*, since the members *Ef* and *fg* would be incapable of withstanding compressive stresses.

The member *ef* is the additional web member required. With the live loading so placed on the structure that compression is indicated in *fg*, *fg* ceases to take any stress, and the member *ef* is brought into action. When this occurs, *fG* is no longer a sub-

diagonal, but acts with *fe* as the main diagonal of the double panel. *Ef*, on the other hand, becomes the subdiagonal of the double panel.

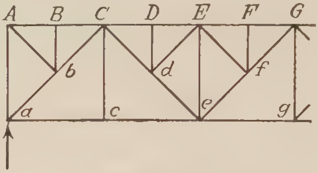


Fig. 139.

With *fg* no longer taking any stress the active members of the truss are shown in Fig. 139. The stresses in the members affected by the introduction of counters are computed below.

Member	Stress designation	Shear in section	Vertical components	Stress
ef	Dead load	82	$+82 - 44 + ef \cos\theta = 0$	-83.4
	Live load (1–4)	-70	$-70 + ef \cos\theta = 0$	$+97.3$
	Maximum	.....	....	$+13.9$
	Minimum	.....	....	0
fG	Dead load	38	$+38 + fG \cos\theta = 0$	-52.8
	Live load (1–5)	-105	$-105 + fG \cos\theta = 0$	$+146.0$
	Maximum	.....	....	$+93.2$
	Minimum	.....	....	$+30.6$

The members *Ee*, *Ef*, and *Gg* are stressed, while the counter *ef* is acting, but the stresses are of the same sign as the maximum and minimum stresses of the tables on pages 153 and 154, and their values are neither as great as the maximum nor as small as the minimum values there given. Hence they are not important.

The maximum and minimum stresses of the members of the truss are the same as for the truss without counters except

fG, whose maximum stress is +93.2 kips instead of +89.0 kips and

fg, whose minimum stress is 0 instead of −13.9 kips

while the counter *ef* has a maximum stress of +13.9 kips and a minimum stress of 0.

The maximum chord stresses are in either case obtained with full live load on the truss, while the minimum chord stresses are due to dead load alone.

CHAPTER VII

CONCENTRATED MOVING LOADS ON BEAMS AND GIRDERS

76. The methods of the determination of the maximum bending moments, shears, and reactions in simple beams and girders sustaining uniform live loads differ but little from those used for dead load alone. The maximum moment at any point on the span and the maximum reaction are obtained with live load covering the entire span, while reference to Art. 69 shows that the maximum shear at any point is obtained with all that portion of the span to the right of the section under consideration sustaining full live load.

The exact position of a system of concentrated loads which will produce the maximum moment, shear or reaction is not so easily determined on account of the partial or total lack of symmetry of the component loads. In order to avoid the tedious labor of making several trial calculations for each maximum value desired, it is advisable to develop general criteria for the positions of the loads which produce the maximum values of moment, shear, and reaction. Once these are understood, their application is simple, and since usually only one or possibly two positions of the loads satisfy the criterion, only one or two complete calculations are necessary.

77. Influence Lines. Although influence lines do not offer any particular advantage in the actual computation of the stresses in simple beams and trusses, they do serve as a means of graphically representing the variation in shear, moment, etc., and of developing the criteria for the position of concentrated loads which produces the maximum value of the function desired. In structures more difficult of analysis, they offer, in many instances, an alternative method of stress calculation considerably less tedious than a complete analytical treatment.

An influence line may be defined as a line which represents the variation of a reaction, shear, moment, stress, or other function, when a single concentrated load of unit value moves across the

structure. The variation of the reaction of one of the supports of a beam, as a single load of unity crosses the span, may be represented by a line called a reaction influence line. In Fig. 140, there is shown the reaction influence line for the left reaction, R_L, of a simple beam. This is a straight line, for with unit load placed any distance k from the left support

$$R_L = \frac{l - k}{l}$$

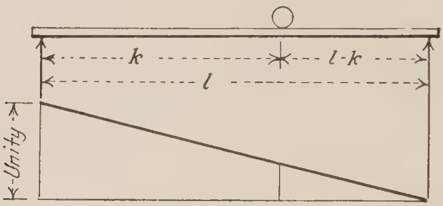


FIG. 140.

As the load moves from right to left across the span, the reaction varies uniformly from 0 when $k = l$ to unity when $k = 0$. The value of the reaction for any position of the load is represented by the ordinate between the horizontal reference line and the inclined line (the influence line) directly beneath the load.

The shear influence line for a simple beam at any section C is illustrated in Fig. 141. With the load on the right of the section

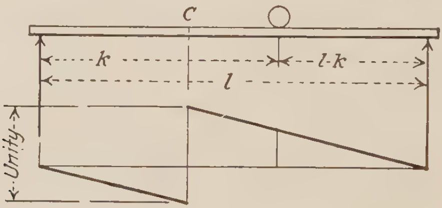


FIG. 141.

the shear is equal to $\frac{l - k}{l}$. With the load on the left it is $\frac{l - k}{l} - 1 = \frac{l - k}{l} - \frac{l}{l} = -\frac{k}{l}$. As the load travels from right to left the shear at C increases, until when the load is just to the right of C, it reaches its maximum value. When the load is just to the left of C, the maximum negative shear at C occurs, which decreases uniformly as the load approaches the left support.

The moment influence line for any section C is shown in Fig. 142. The value of the moment at C is measured by the ordinate directly beneath the load.

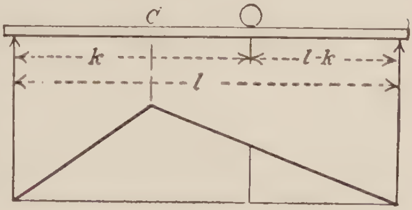


Fig. 142.

In a structure where the load is transferred by means of a system of floor beams (or stringers and floor beams) to certain definite load points, the influence line between these consecutive load points is always a straight line. Let y_1 and y_2 (Fig. 143) be the ordinates of the influence line when the load is placed at points A and B, respectively. For any intermediate position, the proportion of load carried to the adjacent points A and B is $\frac{l-k}{l}$ and $\frac{k}{l}$, respectively, and these proportions vary uniformly as the load moves between A and B (see Fig. 140). The ordinate y equals $\frac{l-k}{l}y_1 + \frac{k}{l}y_2$, and also varies uniformly. Hence the influence line for the portion of the whole span, A–B, is a straight line. The influence line for any such structure can, therefore, be completely determined by calculating the value of the function for a load unity placed successively at each load or panel point.

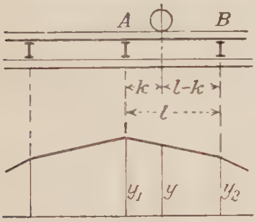


Fig. 143.

Where only a single load, a uniformly distributed load, or an infinite number of equal concentrated loads equally spaced are to be considered, Figs. 140, 141, and 142 give sufficient information to show the positions of the load which produce the maximum reaction, shear, or moment, respectively. Most concentrated load systems, however, which are actually used as design loads, consist of unequal loads unequally spaced so that the position for the maximum value of any function is also dependent upon the ratio and spacing of the component loads.

78. Position of Loads for Maximum Reaction. From Fig. 140 it is seen that the maximum reaction for a single load of unity occurs with the load directly over the support. For a system of concentrated loads moving from right to left, the left reaction

increases as the loads approach it. When one of the loads passes over the support and off the structure, the reaction suddenly decreases and then gradually increases again as the remaining loads continue to approach. For the maximum reaction, therefore, the heaviest of the concentrations should be placed directly over the support, the other loads of the system being so placed that there are as many loads as possible on the span. The actual value is readily obtained by the usual moment equation about the right support. In some systems of concentrated loads, more than one position may need to be tried and the greatest value taken.

79. Position of Loads for Maximum Shear. The lower portion of Fig. 144 shows the shear influence line for section C of the simple beam $A–B$. The moving loads P_1, P_2, P_3, and P_4 on the beam are represented by the repective numerals enclosed in circles to differentiate them from fixed loads. While the loads may move, they must move as a unit, and the distances between

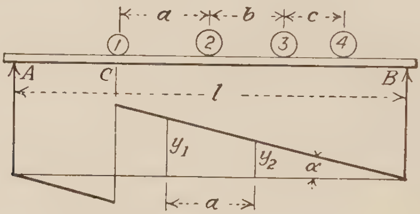


FIG. 144.

them remain fixed. It is evident that all loads to the right of C cause positive shear at C, while all loads to the left of C cause negative shear. As the loads move from right to left, the positive shear due to the loads on the right of C increases, while the negative shear due to the loads on the left decreases. The resultant positive shear is thus increased by such a movement until some one of the loads passes the point C, when there is a sharp decrease by an amount equal to the value of the load passing C.

In Fig. 144, let W represent the sum of all the loads on the span. Let P_1 be placed just to the right of C. If the loads are moved to the left a distance a, P_2 is brought just to the right of

C. During the movement of the loads the positive shear at C suddenly decreases by an amount equal to P_1 as P_1 passes C. It then gradually increases as P_2 approaches C. This increase may be expressed as

$$(W - P_1)(y_1 - y_2) = (W - P_1)\, a \tan \alpha$$

in which y_2 represents the ordinate of the influence line under the center of gravity of the load $W - P_1$ in the first position and y_1 the corresponding value with the load in the second position. The decrease in negative shear may also be expressed as

$$P_1\, a \tan \alpha$$

so that the total increase in the positive shear is

$$Wa \tan \alpha = \frac{Wa}{l}$$

The total change in shear due to the loads moving from their original position a distance a to the left is

$$\frac{Wa}{l} - P_1$$

provided no additional loads come onto the span from the right. If this occurs and W' represents the total load on the span with P_2 just to the right of C, the increase in shear is somewhere between

$$\frac{Wa}{l} - P_1 \text{ and } \frac{W'a}{l} - P_1$$

If both of these expressions are positive, the greatest shear occurs with P_2 at C. If both are negative, P_1 at C gives the maximum shear, while if the first is negative and the second positive, it is necessary to make calculations for both positions.

In many systems of concentrated loads the first load is considerably smaller than most of the others. It follows from the above discussion, that if the total load on the span divided by the length of the span is greater than the first load divided by the distance between the first and second loads, that is, if $\frac{W}{l} > \frac{P_1}{a}$, the second load at the section is the position for the maximum shear.[1]

For any system of concentrated loads, the maximum shear at any section is obtained with the loads so placed on the span that

$$\frac{W}{l} > \frac{P}{a}$$

[1] For a system of two loads only, such as a steam roller or a truck, it is evident that the maximum shear occurs with the heavier load at the section and the lighter load to the right of the section.

in which P is the value of the first load (which may be either on or off the span) to the left of the section and a is the distance from this load to the load at the section. If $\frac{W}{l}$ is not greater than $\frac{P}{a}$, the loads should be moved to the right until that condition is realized.

As an example of the foregoing, the maximum shears at the quarter points of the 40-ft. span shown in Fig. 145 are computed below. In the upper portion of the figure, the first horizontal line indicates the amount of each load in thousands of pounds

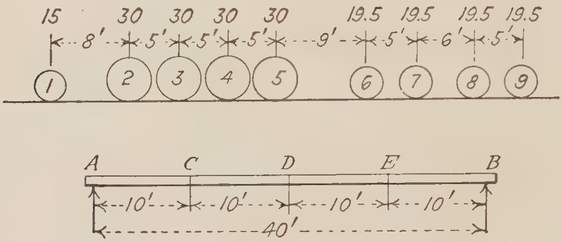


Fig. 145.

or kips, and the second line the fixed horizontal distances between loads.

Section A:

② at A, since $\frac{W}{l}$ lies between $\frac{178.5}{40}$ and $\frac{198}{40} = 4.46$ and 4.95 while

$$\frac{P}{a} = \frac{15}{8} = 1.87$$

$$R_A = \frac{30(40 + 35 + 30 + 25) + 19.5(16 + 11 + 5)}{40} = 113.0 \text{ kips}$$

If ③ is placed at A, $\frac{W}{l} = \frac{168}{40} = 4.20$ while $\frac{P}{a} = \frac{30}{5} = 6.00$

Section C:

② at C since $\frac{W}{l} = \frac{174}{40} = 4.35$ and $\frac{P}{a} = 1.87$

$$R_A = \frac{15 \times 38 + 30(30 + 25 + 20 + 15) + 19.5(6 + 1)}{40} = 85.1$$

$$V_C = 85.1 - 15 = 70.1 \text{ kips.}$$

Section D:

② at D since $\frac{W}{l} = \frac{135}{40} = 3.37$ and $\frac{P}{a} = 1.87$

$$R_A = \frac{15 \times 28 + 30(20 + 15 + 10 + 5)}{40} = 48.1$$

$$V_D = 48.1 - 15 = 33.1 \text{ kips.}$$

80. Position of Loads for Maximum Moment. The moment influence line for section C of the simple beam A–B is shown in Fig. 146. Let P_2 be placed just to the right of C. The same effect on the beam at C is produced by considering the loads on either side of C concentrated at their respective centers of gravity, as when the various single loads are considered separately. Therefore, the moment at C may be expressed as

$$Py_1 + (W - P)y_2$$

in which P is the sum of all the loads to the left of the section, W is the total load on the span, and y_1 and y_2 are the ordinates to the

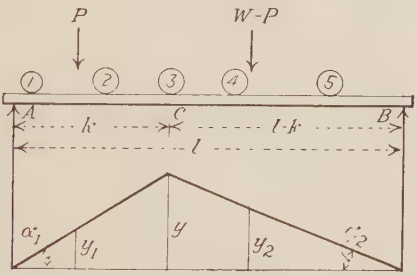


FIG. 146.

moment influence line under P and $W - P$, respectively. If the whole system of loads moves a distance x to the left, the change in the value of the moment is

$$(W - P)x \tan \alpha_2 - Px \tan \alpha_1$$

The rate of change per unit of length is

$$(W - P) \tan \alpha_2 - P \tan \alpha_1 = \left(\frac{W - P}{l - k} - \frac{P}{k}\right)y$$

As the loads are moved to the left, this expression either increases or decreases. The position of the loads for the maximum moment is the position where movement in either direction causes a decrease. A load passing B, in moving to the left increases $W - P$, while a load passing A decreases P. In either case the

value of $\frac{W - P}{l - k} - \frac{P}{k}$ is increased. A load passing C, however, increases P and at the same time decreases $W - P$. Therefore, with some single load at C, movement in either direction causes a decrease in the value of the moment. The loads should be so placed, therefore, that $\frac{W - P}{l - k} = \frac{P}{k}$ or by composition $\frac{W}{l} = \frac{P}{k}$ which may be written

$$\frac{k}{l} W = P$$

This is the criterion for determining the position of a system of concentrated loads which produces the maximum moment at any point in a simply supported beam. In most systems of concentrations, it will be found that more than one position of the loads will satisfy the above condition, so that the moment for each position must be computed and the greatest value taken.[2]

81. Position of Loads for Maximum Moment (*without reference to influence lines*). In Fig. 147, let P_3 be placed

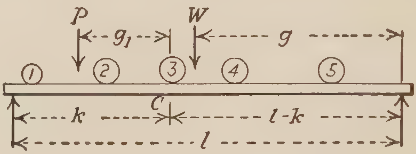


FIG. 147.

just to the right of C. Let g_1 be the distance from the center of gravity of the loads to the left of C from C, and g the distance from the center of gravity of all the loads on the span from the right support. Then

$$M_C = \frac{Wg}{l} k - Pg_1$$

If the loads are moved to the left a short distance x

$$M_C = \frac{W(g + x)}{l} k - P(g_1 + x)$$

and the change in the value of M_C is

$$\frac{Wkx}{l} - Px$$

[2] For span lengths between 70 and 165 ft. the maximum moment at sections near the center will often be found to be greater on the right of the center than for the corresponding symmetrical sections on the left, due to the conventional systems of loading, in general use.

and the rate of change per unit of length is

$$\frac{Wk}{l} - P$$

The position for the maximum moment at C is reached when a further movement of the load causes this expression to change sign. Hence the maximum is obtained when

$$\frac{k}{l} W = P$$

82. Absolute Maximum Moment. With a system of concentrated loads moving across a beam, the bending moment which exists under any single load will vary as the loads move, and will evidently be a maximum with the load at or near the center of the beam. In Fig. 148, P_3 is placed at a distance z

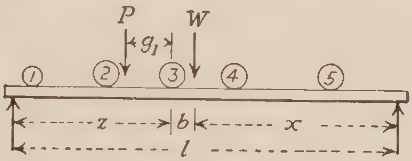


FIG. 148.

from the left support. The distance of the center of gravity of all the loads from the right support is x, while the distance of the center of gravity of all the loads to the left of, and including P_3 from P_3, is g_1. Then the moment under P_3 is

$$\frac{Wx}{l}z - Pg_1$$

But W, P, l, and g_1 are all constants, so in order that this expression may be a maximum, xz must be a maximum. Also $x + z = l - b$, and since $l - b$ is a constant, $x + z$ is also a constant. Therefore xz is a maximum[3] when $x = z$.

Hence the maximum moment under any single load of a system occurs when that load and the center of gravity of all the loads are equidistant from the supports of the beam, or the maximum moment under any single load of a system occurs when the center of the span bisects the distance between that load and the center of gravity of all the loads. Since the moment is

[3] This may be shown by the calculus or by a practical demonstration similar to the following. $x + z = 50$ $x = 25$ $z = 25$ $xz = 625$
$x = 26$ $z = 24$ $xz = 624$
$x = 27$ $z = 23$ $xz = 621$, etc.

greatest near the center of the span, it follows that the absolute maximum bending moment in the beam will occur under the load nearest the center of gravity of all the loads, when they are placed so as to fulfill the above requirement.[4]

83. Calculation of Moments in a Beam. The maximum moments at the quarter point and center of the beam of Fig. 145 due to the system of loads there given will be computed. Also the point of absolute maximum moment will be determined and its value obtained.

Section C:

$$\text{② at } C \quad W = 174 \qquad \frac{k}{l}W = \frac{10}{40} \times 174 = 43.5 \qquad P = 15-45^{5}$$

$$\text{③ at } C \; W = 159 - 178.5 \qquad \frac{k}{l}W = 39.7 - 44.6 \qquad P = 30-60$$

$$\text{② at } C \quad M_B = 15 \times 38 + 30(30 + 25 + 20 + 15) + 19.5(6 + 1) = 3407$$

$$M_C = \frac{3407}{40} \times 10 - 8 \times 15 = 732 \text{ kip ft.}$$

[4] In short spans with some systems of concentrations, the position as determined above does not always give the absolute maximum moment; *i.e.*, for two equal loads distant a from one another

$$M = \frac{2P\left(\frac{l}{2} - \frac{a}{4}\right)^2}{l}$$

while with a span of length less than $2a$, if one load is placed at the center, the other load is not on the span and the moment is $\frac{Pl}{4}$. Equating the two moments and reducing, $a = .586l$ $\therefore$ if $a > .586l$ the maximum moment occurs with one load at the center of the span. Similarly for three equal loads equally spaced, if $a > .450l$, two loads placed in the usual manner produce the maximum moment, while with four such loads if $a > .268l$ three loads give the maximum.

With two unequal loads, such as truck or steam-roller loads, the maximum moment may occur under the heavier load at the center of the span or with it placed equidistant with the center of gravity from the center. The former will be the case except when $l > \dfrac{a}{1 + r - \sqrt{r(1 + r)}}$ where r is the ratio of the heavier to the lighter load.

[5] Consider ② placed just to the right of C, a distance x, x being so small that $k + x$ is practically equal to k. $P = 15$. Next consider all the loads moved to the left a distance $2x$. $P = 45$. It must be recognized that x is taken so small that the definite value of the calculations is not affected. As P varies from 15 to 45 (which for convenience is expressed as $P = 15 - 45$) at some point during its movement through the distance $2x$ it passes through the value of 43.5 and hence the criterion for maximum moment is satisfied.

③ at C $\quad M_B = 30(35 + 30 + 25 + 20) + 19.5(11 + 6) = 3632$

$$M_C = \frac{3632}{4} - 5 \times 30 = 758 \text{ kip ft.}$$

Section D:

④ at D $\quad W = 174 \quad \frac{k}{l}W = 87 \quad P = 75 - 105$

⑤ at D $\quad W = 159 - 178.5 \quad \frac{k}{l}W = 79.5 - 89.2 \quad P = 90 - 120$

Does not satisfy.

④ at D $\quad M_B = 15 \times 38 + 30(30 + 25 + 20 + 15) + 19.5(6 + 1) = 3407$

$$M_D = \frac{3407}{2} - [15 \times 18 + 30(10 + 5)] = 983.2 \text{ kip ft.}$$

Absolute Maximum Moment:

The center of gravity of loads ① to ⑦ is

$$\frac{15 \times 37 + 30\ (29 + 24 + 19 + 14) + 19.5 \times 5}{174}$$

= 18.58 ft. from ⑦ or .42 ft. to the right of ④. ∴ ④ is placed .21 ft. to the left of the center of the span. With the loads in this position, the originally assumed loads ① to ⑦ are the only loads on the span.

$$M_B = 15 \times 38.21 + 30(30.21 + 25.21 + 20.21 + 15.21, + 19.5(6.21 + 1.21) = 3443$$

$$M_{Max} = \frac{3443 \times 19.79}{40} - [15 \times 18 + 30(10 + 5)] = 983.4 \text{ kip ft.}$$

It is often necessary to make computations for more than one position, for example, using the same system of concentrations on a span of 30 ft.

③ at center $\quad W = 135 \quad \frac{k}{l}W = 67.5 \quad P = 45 - 75$

Center of gravity ① to ⑤ $= \frac{1245}{135} = 9.22$ ft. to the left of ⑤

or .78 ft. to the right of ③. ③ is placed .39 ft. to the left of the center and the moment under ③ is

$$\left[\frac{15 \times 28.39 + 30(20.39 + 15.39 + 10.39 + 5.39)}{30}\right] \times 14.61 - (15 \times 13 + 30 \times 5) = 615.8 \text{ kip ft.}$$

④ at center $\quad W = 139.5 \quad \frac{k}{l}W = 69.8 \quad P = 60 - 90$

Center of gravity ② to ⑥ $= \frac{1980}{139.5} = 14.20$ ft. to the left of ⑥

or .20 ft. to the left of ④. ④ is placed .10 ft. to the right of the center and the moment under ④ is

$$\left[\frac{30(24.90 + 19.90 + 14.90 + 9.90) + 19.5 \times .90}{30}\right] \times 15.10 - 30(10 + 5) = 609.8 \text{ kip ft.}$$

There is frequently a greater difference between the absolute maximum moment and the maximum moment at the center from that indicated by the foregoing examples. In a 20-ft. span, loads ② to ⑤ of the system here used produce the maximum moments. With either ③ or ④ at the center of the span the moment at that point is 300 kip ft. while the absolute maximum moment under either ③ or ④ occurs 1.25 ft. from the center and has a value of 309.4 kip ft.

Girders with Floor Beams

84. Distribution of Loads. It is often impossible or uneconomical to carry the roadway or the ties directly on a beam or girder. In such cases a floor system of floor beams and stringers

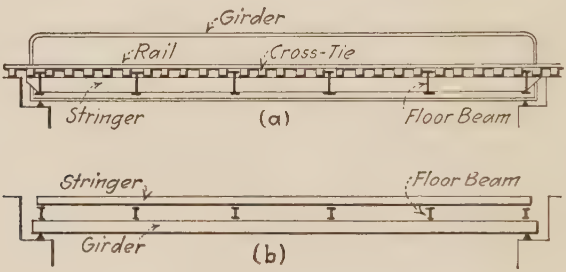


Fig. 149.

or floor beams alone is used to transfer the loads from the roadway to the girders. The effect of the live load thus reaches the girders only at certain definite points where the floor beams are supported by the girders. Figure 149(*a*) shows a longitudinal section through a typical railroad girder bridge. The rails transfer the load to the cross-ties, from them it is taken by the stringers and transferred to the floor beams and thence to the girders. In a highway bridge the rails and cross-ties are replaced by a reinforced concrete slab, planks, or steel plates. In some cases the stringers are also omitted and the floor is designed to carry the load directly to the floor beams. In either case the load reaches

the girders only at the points where the floor beams are supported by them. The usual manner of framing stringers into floor beams and floor beams into girders is illustrated in Fig. 149(*a*). The same effect on the girder, as far as bending moments and shears are concerned, is produced by the arrangement shown in Fig. 149(*b*). No matter how many loads are sustained by the stringers, apart from its own weight the girder is subjected only to four concentrated loads.

85. Position of Loads for Maximum Shear. Since the effect of the live load is distributed to the girder only at the points where floor beams are supported, it is evident that between these points the live-load shear is a constant. The shear between the points A and C of Fig. 150 is $R_L - r_A$ where R_L is the left reaction of the girder and r_A the reaction of the floor beam A due to the loads between A and C.

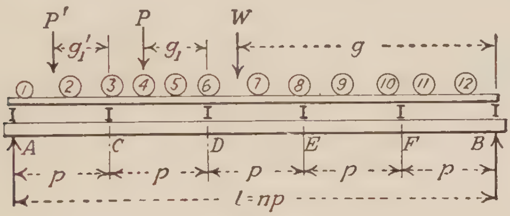


Fig. 150.

The shear between C and D is

$$R_L - P_1 - P_2 - r_C,$$

where r_C is the reaction of the floor beam C due to the loads between C and D.

In Fig. 150, P_6 is placed just to the right of D. Let P represent the loads between C and D, and g_1 represent the distance of their center of gravity from D. Let W represent the sum of all the loads on the span and g represent the distance of its center of gravity from the right support.

Then

$$V_{CD} = \frac{Wg}{l} - (P_1 + P_2) - \frac{Pg_1}{p}$$

If the load is moved to the left a distance x, the change in shear is

$$\frac{Wx}{l} - \frac{Px}{p}$$

and the rate of change per unit of length

$$\frac{W}{l} - \frac{P}{p}$$

When the distances between floor beams are equal, as is usually the case, $l = np$ and the criterion for maximum shear between any two floor beams becomes

$$W = nP$$

From Arts. 69 and 77 it is seen that loads such as P_1 and P_2 cause a negative shear between C and D. Therefore, for the maximum positive shear between C and D, no load should ordinarily be placed to the left of C. This does not affect the above analysis, which is the general case, and covers the condition of short panels and unusual systems of concentrations where the maximum shear is obtained with the loading extending beyond the left end of the section.

86. Position of Loads for Maximum Moment. Referring to Fig. 150,

$$M_D = \frac{Wg}{l} \times 2p - r_A \times 2p - r_C \times p$$

but

$$r_A = \frac{P'g_1'}{p} \text{ and } r_C = \frac{P'(p - g_1')}{p} + \frac{Pg_1}{p}$$

Substituting the values of r_A and r_C

$$M_D = \frac{Wg}{l} \times 2p - 2P'g_1' - P'p + P'g_1' - Pg_1$$

$$= \frac{Wg}{l} \times 2p - P'(p + g_1') - Pg_1$$

which is the same as for a girder without floor beams.

Therefore, the position of a system of concentrated loads, that produces the maximum moment in a girder at any floor beam, is the same as would be produced in a girder without floor beams at a corresponding point, so that, as in Art. 80, the criterion is

$$\frac{k}{l} W = P$$

If the distances between floor beams are equal, $l = np$, and mp may be substituted for k, where m is the number of divisions or panels between the section and the left support. Then

$$\frac{m}{n} W = P$$

87. Position of Loads for Maximum Floor-beam Reaction. The load sustained by any one floor beam is dependent only upon the amount of load in the two adjacent divisions or panels. The maximum value of this load, which is delivered to the girders as floor-beam reactions, occurs when as many loads as possible are placed on the two panels with the heavier loads near to, or directly over, the floor beam in question. In Fig. 151 let P be the resultant of all the loads between A and C and g' the distance from P to C. Let W_1 be the resultant of all the loads between A and D, and g the distance from W_1 to D. Taking moments about C

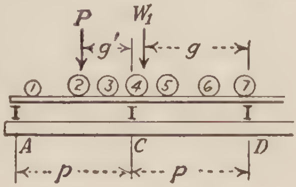


FIG. 151.

$$r_A p - Pg' = 0$$

from which

$$r_A p = Pg'$$

Taking moments about D

$$r_A 2p + r_C p - W_1 g = 0$$

Substituting Pg' for $r_A p$

$$r_C = \frac{W_1 g - 2Pg'}{p}$$

If the loads are moved to the left a distance x, the change in r_C is

$$\frac{W_1 x - 2Px}{p}$$

and the rate of change per unit of length is

$$\frac{W_1 - 2P}{p}$$

Since p is a constant, W_1 and P are the only quantities which can change the value of this expression, and it follows from previous similar analyses that *the maximum floor-beam reaction occurs when*

$$W_1 = 2P$$

and is equal to *the moment of the loads in the two panels about the right end of the two panels minus twice the moment of the loads in the left-hand panel about the right end of the left-hand panel divided by the panel length.*

88. Calculations of Shears, Moments, and Floor-beam Reactions. A through plate girder bridge has a span of 60 ft. (Fig.

152). The distance between girders is 16 ft.-0 in. The distance between floor beams is 12 ft.-0 in. The loading shown is to be supported by a line of stringers framing into the floor beams.

Maximum Shear:

AC ② at C $W = 213$ $P = 15 - 45$[6] $nP = 75 - 225$

$$V = \frac{15 \times 56 + 30(48 + 43 + 38 + 33) + 19.5(24 + 19 + 13 + 8)}{60} - \frac{15 \times 8}{12} = 105.8 \text{ kips}$$

③ at C $W = 198$ $P = 30 - 60$ $nP = 150 - 300$

$$V = \frac{30(53 + 48 + 43 + 38) + 19.5(29 + 24 + 18 + 13)}{60} - \frac{30 \times 5}{12} = 105.8 \text{ kips}$$

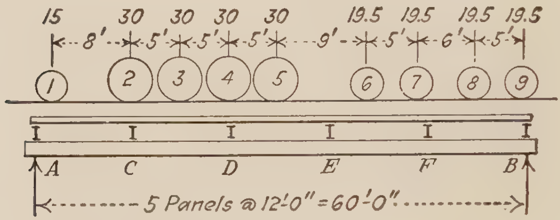


FIG. 152.

CD ② at D $W = 193.5$ $P = 15 - 45$ $nP = 75 - 225$

$$V = \frac{15 \times 44 + 30(36 + 31 + 26 + 21) + 19.5(12 + 7 + 1)}{60} - \frac{15 \times 8}{12} = 64.5 \text{ kips}$$

DE ② at E $W = 135 - 154.5$ $P = 15 - 45$ $nP = 75 - 225$

$$V = \frac{15 \times 32 + 30(24 + 19 + 14 + 9)}{60} - \frac{15 \times 8}{12} = 31.0 \text{ kips}$$

Moments:

C ② at C $W = 213$ $\frac{m}{n}W = 42.6$ $P = 15 - 45$

$$M = \frac{6948}{60} \times 12 - 15 \times 8 = 1269.6 \text{ kip ft.}$$

③ at C $W = 198$ $\frac{m}{n}W = 39.6$ $P = 30 - 60$

[6] See footnote, Art. 83.

$$M = \frac{7098}{60} \times 12 - 30 \times 5 = 1269.6 \text{ kip ft.}$$

D ④ at D $W = 213$ $\frac{m}{n} W = 85.2$ $P = 75 - 105$

$$M = \frac{6522}{60} \times 24 - 720 = 1888.8 \text{ kip ft.}$$

Maximum Floor-beam Reaction:

③ at C $W_1 = 120$ $P = 30 - 60$ $2P = 60 - 120$

$$r_C = \frac{30(17 + 12 + 7 + 2) - 2 \times 30 \times 5}{12} = 70.0 \text{ kips}$$

Moment Tables

89. In order to lessen the numerical work involved in the computations of stresses due to a specified system of concentrated loads, it is desirable to prepare a table or diagram giving weights, distances, and moments. Many forms for these tables have been devised by different designers to suit their respective needs. Two such tables, for Cooper's E-60 loading, are given on pages 175 and 176, while one for the M-50 loading is given on page 177.

In Table I, the horizontal line a gives the spacing between wheels in feet. Line b gives the distance from any wheel to wheel 1, and line c the distance from any wheel to the head of the uniform load. In line d the load on each wheel is given, and in line e, the successive wheels are designated by the numbers in circles. Line f gives the summation of all the loads to the right from wheel 1 to any given wheel, and line g the summation to the left from wheel 18 to any given wheel. Line h gives the summation of the moments of all the loads to the left of a given wheel about that wheel. Line 19 contains the summation of the moments of all the loads between a given wheel and the head of the uniform load, including the given wheel load, about the head of the uniform load. Thus the moment of the wheel loads 6 to 18 inclusive, about the head of the train is given in line 19 under wheel 6 as 11,690 kip ft. The remaining horizontal lines are similar to line 19, the moment center being taken under the wheel load corresponding to the number appearing in the first vertical column. For instance, the moment of wheel loads 5 to 13, inclusive, about wheel 13 is given under wheel 5 in line 13 as 4900 kip ft.

Table II gives the same information as Table I although the arrangement is somewhat different. The first three horizontal lines give, respectively, the amount of each wheel load, the spacing between wheels, and the number designation of each wheel. The next three lines contain, respectively, the summations of distances, loads, and moments from wheel 1 to any given wheel. Thus, under wheel 5, the distance of wheel 5 from wheel 1 is given as 23 $(= 8 + 5 + 5 + 5)$, the summation of wheel loads 1 to 5 as 135 $(= 15 + 30 + 30 + 30 + 30)$, and the summation of the moments of all the wheel loads to the left of wheel 5 about wheel 5 as 1245 $[= 15 \times 23 + 30(15 + 10 + 5 + 0)]$. Under certain conditions wheels may pass off the structure as the loads are moved to the left, so that the summations of distances, loads, and moments are given in the table with each wheel of the first locomotive successively omitted. For instance, under wheel 12 in the horizontal triple line 6, the distance from wheel 6 to wheel 12 is given as 37, the summation of wheel loads 6 to 12 as 153, and the summation of the moments of wheel loads 6 to 12 about wheel 12 as 2607.

Tables similar to Table I are the more widely used, but Table II is the more convenient since it gives directly the summations of distances and loads in addition to the summation of moments for the conditions when one or more wheels are off the structure, or to the left of the panel under consideration. In Table I the moments are, in all but the obvious instances calculated to the nearest 10 kip ft.

Table III, constructed for M-50 loading, has the same arrangement as Table II. The last column on the right is added because, for certain intermediate span lengths, the maximum shears and moments are obtained with the load reversed, so that the second and heavier set of driving wheels is brought closer to the section. Commencing at the bottom of the table, this column gives successively the summation of moments, loads, and distances between the wheel in question and wheel 11, from wheel 10 to wheel 1.

Table IV is really a continuation of Table II for use where there is a length of uniform load on the structure. It can also be used in conjunction with Table I. For instance, it is desired to obtain the maximum reaction of a 151-ft. span, due to Cooper's E-60 loading. With wheel 2 at the left support, from either Table I or Table II, it is found to be 101 ft. from wheel 2 to the head of

TABLE I

a	8'	5'	5'	5'	9'	5'	6'	5'	8'	8'	5'	5'	5'	9'	5'	6'	5'	5'	
b		8'	13'	18'	23'	32'	37'	43'	48'	56'	64'	69'	74'	79'	88'	93'	99'	104'	109'
c	109'	101'	96'	91'	86'	77'	72'	66'	61'	53'	45'	40'	35'	30'	21'	16'	10'	5'	
d	15	30	30	30	30	19.5	19.5	19.5	19.5	15	30	30	30	30	19.5	19.5	19.5	19.5	Uniform Load 3000 lb. per lin. ft.
e	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
f	15	45	75	105	135	154.5	174	193.5	213	228	258	288	318	348	367.5	387	406.5	426	1
g	426	411	381	351	321	291	271.5	252	232.5	213	198	168	138	108	78	58.5	39	19.5	
h	0	120	345	720	1245	2460	3230	4280	5240	6950	8770	10060	11500	13090	16220	18060	20380	22420	24550
19	24550	22910	19880	17000	14270	11690	10190	8790	7500	6310	5510	4160	2970	1910	1010	605	290	97.5	
18	22420	20860	17980	15250	12670	10240	8830	7530	6340	5240	4520	3320	2280	1370	625	310	97.5		
17	20380	18900	16170	13590	11160	8880	7570	6360	5270	4280	3630	2580	1680	930	330	117			
16	18060	16670	14120	11720	9470	7370	6180	5090	4110	3230	2680	1810	1090	520	97.5				
15	16220	14900	12500	10260	8150	6200	5110	4120	3240	2460	1980	1260	690	270					
14	13090	11910	9780	7800	5970	4290	3370	2550	1850	1250	900	450	150						
13	11500	10390	8410	6580	4900	3370	2550	1830	1230	720	450	150							
12	10060	9030	7200	5520	3990	2610	1890	1260	755	345	150								
11	8770	7810	6130	4600	3220	1990	1370	840	430	120									
10	6950	6110	4670	3380	2240	1250	780	410	156										
9	5240	4520	3320	2270	1370	625	310	97.5											
8	4280	3630	2580	1680	930	330	117												
7	3230	2680	1810	1090	520	97.5													
6	2460	1980	1260	690	270														
5	1250	900	450	150															
4	720	450	150																
3	345	150																	
2	120																		

With a length of Uniform Load $= x$
$M = 24550 + 426x + 1.5x^2$
or not including wheel 1
$M = 22920 + 411x + 1.5x^2$

MOMENT TABLE
COOPER'S E-60 LOADING
TWO 213 TON ENGINES + 6000 LB. PER FOOT.
MOMENTS IN THOUSAND FOOT POUNDS FOR ONE RAIL.
LOADS IN THOUSANDS OF POUNDS FOR ONE RAIL.

Table II

MOMENT TABLE FOR COOPER'S E-60 LOADING.

SPACING: 8 — 5 — 5 — 5 — 9 — 5 — 6 — 5 — 8 — 8 — 5 — 5 — 5 — 9 — 5 — 6 — 5 — 5 — UNIFORM LOAD 3000 LB. PER FT.

WHEEL NO.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
LOADS	15	30	30	30	30	19.5	19.5	19.5	19.5	15	30	30	30	30	19.5	19.5	19.5	19.5	UNIFORM LOAD 3000 LB. PER FT.
DISTANCES	0	8	13	18	23	32	37	43	48	56	64	69	74	79	88	93	99	104	109
LOADS (1)	15	45	75	105	135	154.5	174	193.5	213	228	258	288	318	348	367.5	387	406.5	426	426
MOMENTS	0	120	345	720	1245	2460	3233	4277	5244	6948	8772	10062	11502	13092	16224	18062	20384	22416	24546
DISTANCES		0	5	10	15	24	29	35	40	48	56	61	66	71	80	85	91	96	101
LOADS (2)		30	60	90	120	139.5	159	178.5	198	213	243	273	303	333	352.5	372	391.5	411	411
MOMENTS		0	150	450	900	1980	2678	3632	4524	6108	7812	9027	10392	11907	14904	16667	18899	20856	22911
DISTANCES			0	5	10	19	24	30	35	43	51	56	61	66	75	80	86	91	96
LOADS (3)			30	60	90	109.5	129	148.5	168	183	213	243	273	303	322.5	342	361.5	381	381
MOMENTS			0	150	450	1260	1808	2582	3324	4668	6132	7197	8412	9777	12504	14117	16169	17976	19881
DISTANCES				0	5	14	19	25	30	38	46	51	56	61	70	75	81	86	91
LOADS (4)				30	60	79.5	99	118.5	138	153	183	213	243	273	292.5	312	331.5	351	351
MOMENTS				0	150	690	1088	1682	2274	3378	4602	5517	6582	7797	10254	11717	13589	15246	17001
DISTANCES					0	9	14	20	25	33	41	46	51	56	65	70	76	81	86
LOADS (5)					30	49.5	69	88.5	108	123	153	183	213	243	262.5	282	301.5	321	321
MOMENTS					0	270	518	932	1374	2238	3222	3987	4902	5967	8154	9467	11159	12666	14271
DISTANCES						0	5	11	16	24	32	37	42	47	56	61	67	72	77
LOADS (6)						19.5	39	58.5	78	93	123	153	183	213	232.5	252	271.5	291	291
MOMENTS						0	97.5	332	624	1248	1992	2607	3372	4287	6204	7367	8879	10236	11691
DISTANCES							0	6	11	19	27	32	37	42	51	56	62	67	72
LOADS (7)							19.5	39	58.5	73.5	103.5	133.5	163.5	193.5	213	232.5	252	271.5	271.5
MOMENTS							0	117	312	780	1368	1886	2553	3371	5112	6177	7572	8832	10190
DISTANCES								0	5	13	21	26	31	36	45	50	56	61	66
LOADS (8)								19.5	39	54	84	114	144	174	193.5	213	232.5	252	252
MOMENTS								0	97.5	410	842	1262	1832	2552	4118	5085	6363	7526	8786
DISTANCES									0	8	16	21	26	31	40	45	51	56	61
LOADS (9)									19.5	34.5	64.5	94.5	124.5	154.5	174	193.5	213	232,5	232.5
MOMENTS									0	156	432	755	1227	1850	3240	4110	5271	6336	7499
DISTANCES										0	8	13	18	23	32	37	43	48	53
LOADS (10)										15	45	75	105	135	154.5	174	193.5	213	213
MOMENTS										0	120	345	720	1245	2460	3233	4277	5244	6309

TWO 213 TON ENGINES + 6000 LB. PER FT.

MOMENTS IN THOUSAND FOOT POUNDS FOR ONE RAIL

LOADS IN THOUSANDS OF POUNDS FOR ONE RAIL

TABLE III

MOMENT TABLE FOR M-50 LOADING.

LOADS		12.5	25	25	25	25	25	31.25	31.25	31.25	31.25	31.25	UNIFORM LOAD 2500 LB. PER FT.									DISTANCES ○-⑪ LOADS ○-⑪ MOMENTS ○-⑪ ABOUT ○
SPACING			10	5	5	5	5	15	5	5	5	5	5									
WHEEL NO.		1	2	3	4	5	6	7	8	9	10	11										
DISTANCES		0	10	15	20	25	30	45	50	55	60	65	70	75	80	85	90	95	100	105	110	65
LOADS	1	12.5	37.5	62.5	87.5	112.5	137.5	168.75	200	231.25	262.5	293.75	294	306	319	331	344	356	369	381	394	293.75
MOMENTS		0	125	313	625	1063	1625	3688	4531	5531	6688	8000	9469	10969	12531	14156	15844	17594	19406	21281	23219	11094
DISTANCES			0	5	10	15	20	35	40	45	50	55	60	65	70	75	80	85	90	95	100	55
LOADS	2		25	50	75	100	125	156.25	187.5	218.75	250	281.25	281	294	306	319	331	344	356	369	381	281.25
MOMENTS			0	125	375	750	1250	3125	3906	4844	5938	7188	8594	10031	11531	13094	14719	16406	18156	19969	21844	8281
DISTANCES				0	5	10	15	30	35	40	45	50	55	60	65	70	75	80	85	90	95	50
LOADS	3			25	50	75	100	131.25	162.5	193.75	225	256.25	256	269	281	294	306	319	331	344	356	256.25
MOMENTS				0	125	375	750	2250	2906	3719	4688	5813	7094	8406	9781	11219	12719	14281	15906	17594	19344	7000
DISTANCES					0	5	10	25	30	35	40	45	50	55	60	65	70	75	80	85	90	45
LOADS	4				25	50	75	106.25	137.5	168.75	200	231.25	231	244	256	269	281	294	306	319	331	231.25
MOMENTS					0	125	375	1500	2031	2719	3563	4563	5719	6906	8156	9469	10844	12281	13781	15344	16969	5844
DISTANCES						0	5	20	25	30	35	40	45	50	55	60	65	70	75	80	85	40
LOADS	5					25	50	81.25	112.5	143.75	175	206.25	206	219	231	244	256	269	281	294	306	206.25
MOMENTS						0	125	875	1281	1844	2563	3438	4469	5531	6656	7844	9094	10406	11781	13219	14719	4813
DISTANCES							0	15	20	25	30	35	40	45	50	55	60	65	70	75	80	35
LOADS	6						25	56.25	87.5	118.75	150	181.25	181	194	206	219	231	244	256	269	281	181.25
MOMENTS							0	375	656	1094	1688	2438	3344	4281	5281	6344	7469	8656	9906	11219	12594	3906
DISTANCES								0	5	10	15	20	25	30	35	40	45	50	55	60	65	20
LOADS	7							31.25	62.5	93.75	125	156.25	156	169	181	194	206	219	231	244	256	156.25
MOMENTS								0	156	469	938	1563	2344	3156	4031	4969	5969	7031	8156	9344	10594	1563
DISTANCES									0	5	10	15	20	25	30	35	40	45	50	55	60	15
LOADS	8								31.25	62.5	93.75	125	125	138	150	163	175	88	200	213	225	125
MOMENTS									0	156	469	938	1563	2219	2938	3719	4563	5469	6438	7469	8563	938
DISTANCES										0	5	10	15	20	25	30	35	40	45	50	55	10
LOADS	9									31.25	62.5	93.75	94	106	119	131	144	156	169	181	194	93.75
MOMENTS										0	156	469	938	1438	2000	2625	3313	4063	4875	5750	6688	469
DISTANCES											0	5	10	15	20	25	30	35	40	45	50	5
LOADS	10										31.25	62.5	62.5	75	88	100	113	125	138	150	163	62.5
MOMENTS											0	156	469	813	1219	1688	2219	2813	3469	4188	4969	156
DISTANCES																						
LOADS																						
MOMENTS																						

ONE $293\frac{3}{4}$ TON ENGINE + 5000 LB. PER FT.

MOMENTS IN THOUSAND FOOT POUNDS FOR ONE RAIL

LOADS IN THOUSANDS OF POUNDS FOR ONE RAIL

TABLE IV.—COOPER'S E 60 LOADING

Length of uniform load	Summations of Loads and Moments about the Right End of the Uniform Load, the Last Load to the Left Being Wheel Load																			
	1		2		3		4		5		6		7		8		9		10	
	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment
5	441	26,714	426	25,004	396	21,824	366	18,794	336	15,914	306	13,184	286.5	11,585	267	10,083	247.5	8,699	228	7,412
10	456	28,956	441	27,171	411	23,841	381	20,661	351	17,631	321	14,751	301.5	13,055	282	11,456	262.5	9,974	243	8,589
15	471	31,274	456	29,414	426	25,934	396	22,604	366	19,424	336	16,394	316.5	14,600	297	12,903	277.5	11,324	258	9,842
20	486	33,666	471	31,731	441	28,101	411	24,621	381	21,291	351	18,111	331.5	16,220	312	14,426	292.5	12,749	273	11,169
25	501	36,134	486	34,124	456	30,344	426	26,714	396	23,234	366	19,904	346.5	17,915	327	16,023	307.5	14,249	288	12,572
30	516	38,676	501	36,591	471	32,661	441	28,881	411	25,251	381	21,881	361.5	19,685	342	17,696	322.5	15,824	303	14,049
35	531	41,294	516	39,134	486	35,054	456	31,124	423	27,344	396	23,714	376.5	21,530	357	19,443	337.5	17,474	318	15,602
40	546	43,986	531	41,751	501	37,521	471	33,441	441	29,511	411	25,731	391.5	23,450	372	21,266	352.5	19,199	333	17,229
45	561	46,754	546	44,444	516	40,064	486	35,834	456	31,754	426	27,824	406.5	25,445	387	23,163	367.5	20,999	348	18,932
50	576	49,596	561	47,211	531	42,681	501	38,301	471	34,071	441	29,991	421.5	27,515	402	25,136	382.5	22,874	363	20,709
55	591	52,514	576	50,054	546	45,374	516	40,844	486	36,464	456	32,234	436.5	29,660	417	27,183	397.5	24,824	378	22,562
60	606	55,506	591	52,971	561	48,141	531	43,461	501	38,931	471	34,551	451.5	31,880	432	29,306	412.5	26,849	393	24,489
65	621	58,574	606	55,964	576	50,984	546	46,154	516	41,474	486	36,944	466.5	34,175	447	31,503	427.5	28,949	408	26,492
70	636	61,716	621	59,031	591	53,901	561	48,921	531	44,091	501	39,411	481.5	36,545	462	33,776	442.5	31,124	423	28,569
75	651	64,934	636	62,174	606	56,894	576	51,764	546	46,784	516	41,954	496.5	38,990	477	36,123	457.5	33,374	438	30,722
80	666	68,226	651	65,391	621	59,961	591	54,681	561	49,551	531	44,571	511.5	41,510	492	38,545	472.5	35,699	453	32,949
85	681	71,594	666	68,684	636	63,104	606	57,674	576	52,394	546	47,264	526.5	44,105	507	41,043	487.5	38,099	468	35,252
90	696	75,036	681	72,051	651	66,321	621	60,741	591	55,311	561	50,031	541.5	46,775	522	43,616	502.5	40,574	483	37,629
95	711	78,554	696	75,494	666	69,614	636	63,884	606	58,304	576	52,874	556.5	49,520	537	46,263	517.5	43,124	498	40,082
100	726	82,146	711	79,011	681	72,981	651	67,101	621	61,371	591	55,791	571.5	52,340	552	48,986	532.5	45,749	513	42,609
105	741	85,814	726	82,604	696	76,424	666	70,394	636	63,884	606	58,784	586.5	55,235	567	51,783	547.5	48,449	528	45,212
110	756	89,556	741	86,271	711	79,941	681	73,761	651	67,731	621	61,851	601.5	58,205	582	54,656	562.5	51,224	543	47,889
115	771	93,374	756	90,014	726	83,534	696	77,204	666	71,024	636	64,994	616.5	61,250	597	57,603	577.5	54,074	558	50,642
120	786	97,266	771	93,831	741	87,201	711	80,721	681	74,391	651	68,211	631.5	64,370	612	60,626	592.5	56,999	573	53,469
125	801	101,234	786	97,724	756	90,944	726	84,314	696	77,834	666	71,504	646.5	67,565	627	63,723	607.5	59,999	588	56,372
130	816	105,276	801	101,691	771	94,761	741	87,981	711	81,351	681	74,871	661.5	70,835	642	66,896	622.5	63,074	603	59,349
135	831	109,394	816	105,734	786	98,654	756	91,724	726	84,944	696	78,314	676.5	74,180	657	70,143	637.5	66,224	618	62,402
140	846	113,586	831	109,851	801	102,621	771	95,541	741	88,611	711	81,831	691.5	77,600	672	73,466	652.5	69,449	633	65,529
145	861	117,854	846	114,044	816	106,664	786	99,434	756	92,354	726	85,424	706.5	81,095	687	76,863	667.5	72,749	648	68,732
150	876	122,196	861	118,311	831	110,781	801	103,401	771	96,171	741	89,091	721.5	84,665	702	80,336	682.5	76,214	663	72,003

the uniform load, indicating that 50 ft. of uniform load are on the span. From Table IV in column 2, the moment of all the loads to the left is given as 47,211, and the reaction is 312.7 kips. With a span of 153 ft., the length of uniform load is 52 ft. The moment about the right support is

$$47{,}211 + 561 \times 2 + \frac{3 \times 2^2}{2} = 48{,}339 \text{ kip ft.}$$

This moment can be obtained without the use of Table IV, from the values given in Tables I or II, but the numerical work involved is slightly greater.

90. Illustrative Use of Moment Tables.[7] With the loads placed as indicated in Fig. 153, *i.e.*, wheel 2 at the left support,

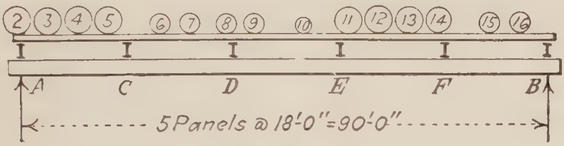


FIG. 153.

it is desired to compute: (*a*) the left reaction of the girder; (*b*) the shear in the panel CD; (*c*) the bending moment at E.

a. Using Table I:[8]

$$\underset{(b\text{-}2)}{90} + 8 = 98 \therefore \text{ right support is } 98 - \underset{(b\text{-}16)}{93} = 5 \text{ ft.}$$

to the right of wheel 16

$$M_B = \underset{(16\text{-}2)}{16{,}670} + (\underset{(f\text{-}16)}{387} - \underset{(f\text{-}1)}{15})5 = 18{,}530 \qquad R_A = 205.9 \text{ kips.}$$

Using Table II:

$$90 - \underset{(2\text{-}16)}{85} = 5$$

$$M_B = \underset{(2\text{-}16)}{16{,}667} + \underset{(2\text{-}16)}{372} \times 5 = 18{,}527 \qquad R_A = 205.9 \text{ kips.}$$

b. Using Table I:

D is 36 ft. from A or wheel 2 or $36 + 8 = 44$ ft. from wheel 1.

[7] The calculations of Arts. 79, 83, and 88 can be shortened by the use of the moment tables, since the load used in these calculations is Cooper's E-60 loading.

[8] The small figures or letters in parentheses under the values taken from the tables for use in the calculations of this article are the reference coordinates of the positions of the values in the tables. Thus (16-2) indicates that the moment 16,670 appears in horizontal line 16 under wheel 2 in Table 1.

$$\underset{(b\text{-}8)}{44 - 43} = 1 \text{ ft.} = \text{distance of } D \text{ to the right of wheel } 8$$

$$44 - 18 = 26 \text{ which is greater than } \underset{(b\text{-}5)}{23}, \text{ showing that wheels } 6$$

7, and 8 are on the panel CD

$$r_C = [\underset{(8\text{-}6)}{330} + (\underset{(d\text{-}6)}{19.5} + \underset{(d\text{-}7)}{19.5} + \underset{(d\text{-}8)}{19.5}) \times 1] \div 18 = 21.6$$

$$V_{CD} = 205.9 - \underset{(d\text{-}2)}{30} - \underset{(d\text{-}3)}{30} - \underset{(d\text{-}4)}{30} - \underset{(d\text{-}5)}{30} - 21.6 = 64.3 \text{ kips.}$$

Using Table II:

D is 36 ft. from A or wheel 2

$$\underset{(2\text{-}8)}{36 - 35} = 1 \text{ ft.} = \text{distance of } D \text{ to the right of wheel } 8$$

$$36 - 18 = 18 \text{ which is greater than } \underset{(2\text{-}5)}{15}, \text{ showing that wheels } 6,$$

7, and 8 are all on the panel CD

$$r_C = (\underset{(6\text{-}8)}{332} + \underset{(6\text{-}8)}{58.5} \times 1) \div 18 = 21.7$$

$$V_{CD} = 205.9 - \underset{(2\text{-}5)}{120} - 21.7 = 64.2 \text{ kips.}$$

c. Using Table I:

E is 54 ft. from A or wheel 2 or $54 + 8 = 62$ ft. from wheel 1.

$$\underset{(b\text{-}10)}{62 - 56} = 6 \text{ ft.} = \text{distance of } E \text{ to the right of wheel } 10$$

$$M_E = 205.9 \times 54 - \underset{(10\text{-}2)}{6110} + (\underset{(f\text{-}10)}{228} - \underset{(f\text{-}1)}{15}) \times 6 = 3730 \text{ kip ft.}$$

Using Table II:

E is 54 ft. from A or wheel 2

$$\underset{(2\text{-}10)}{54 - 48} = 6 \text{ ft.} = \text{distance of } E \text{ to the right of wheel } 10$$

$$M_E = 205.9 \times 54 - (\underset{(2\text{-}10)}{6108} + \underset{(2\text{-}10)}{213} \times 6) = 3730 \text{ kip ft.}$$

91. Figures 154 and 155[9] are of great assistance in determining the exact position of Cooper's loading which produces the maximum moment at any point in any span length from 10 to

[9] These diagrams were prepared by Prof. W. M. Scheurman of Vanderbilt University and were published in part 3, vol. 12, *Proceedings* of the American Railway Engineering Association.

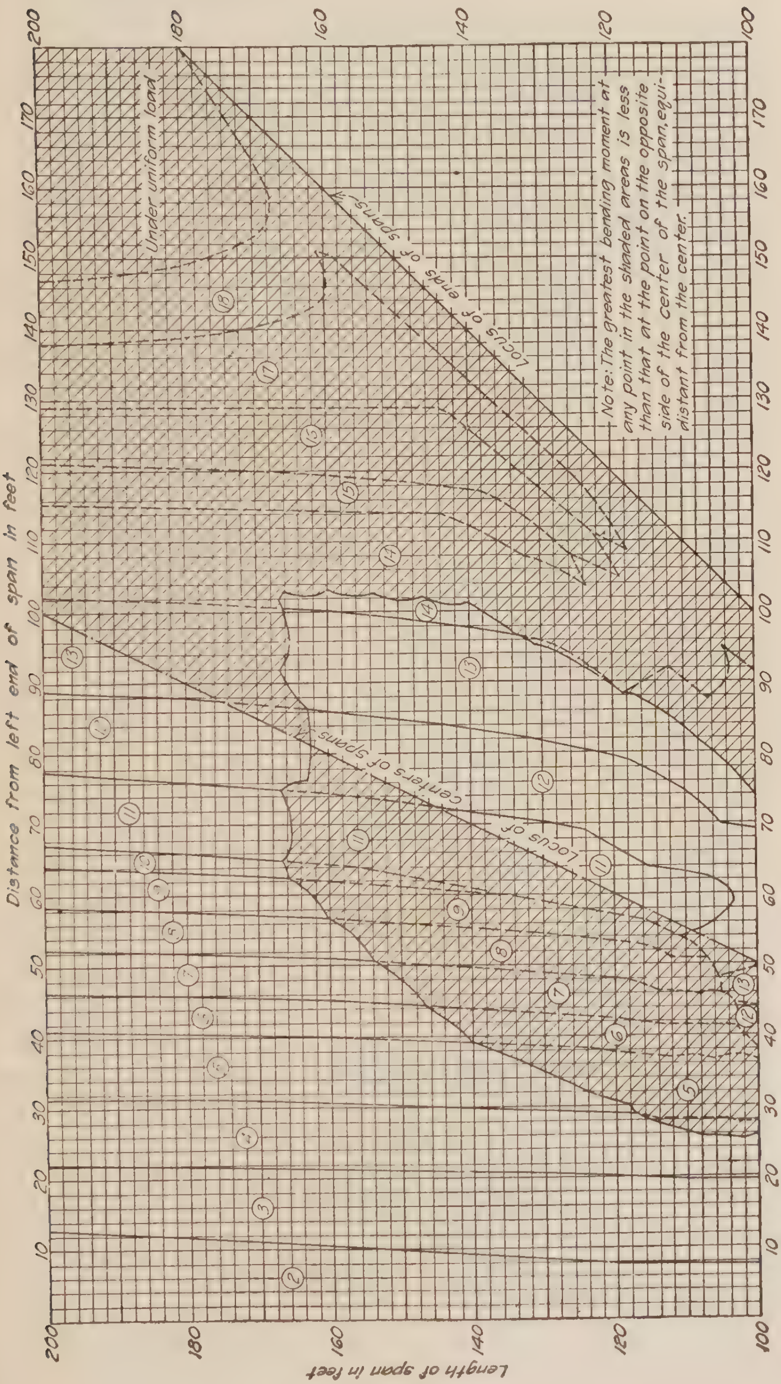

Distance from left end of span in feet
Length of span in feet
Under uniform load
Locus of ends of spans
Locus of centers of spans
Note: The greatest bending moment at any point in the shaded areas is less than that at the point on the opposite side of the center of the span, equidistant from the center.

FIG. 154.

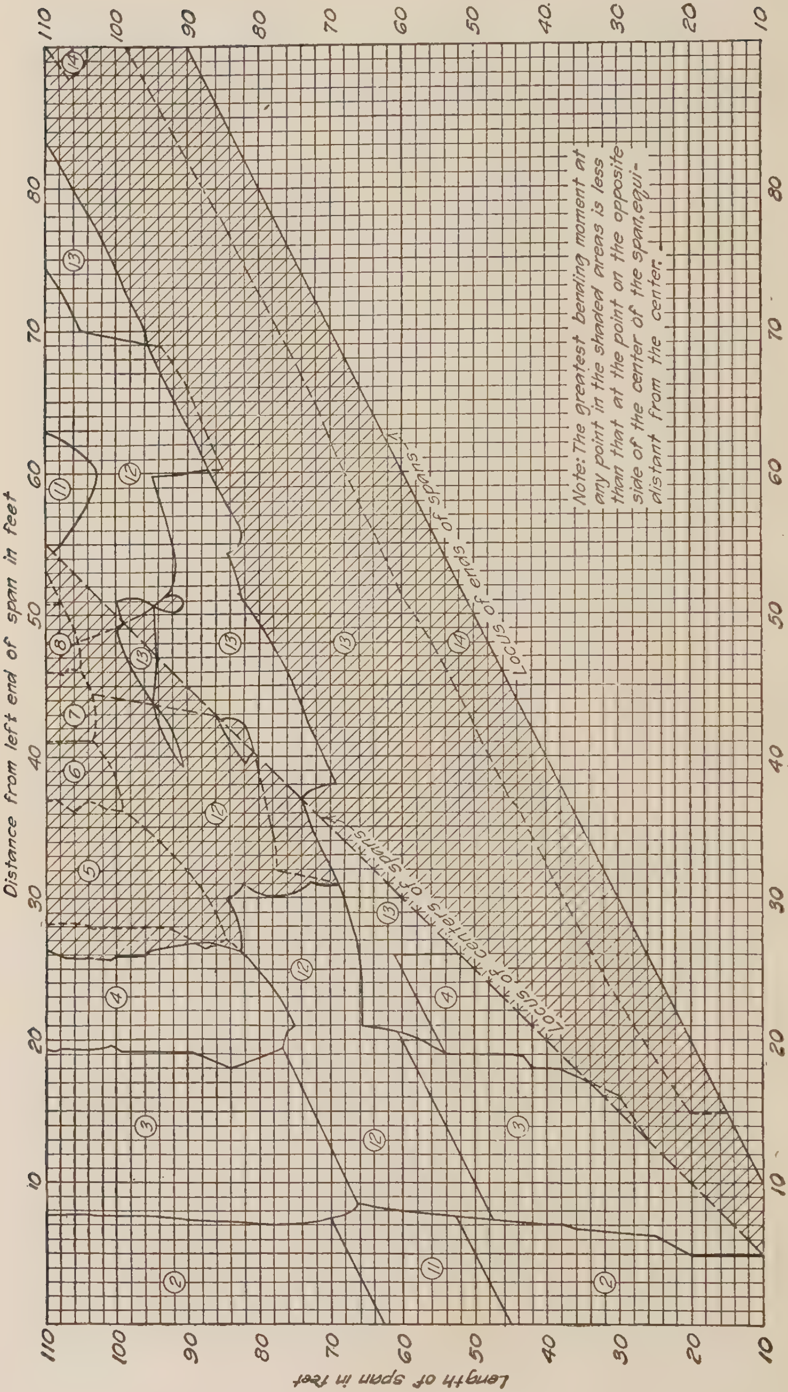

Distance from left end of span in feet
Length of span in feet
Note: The greatest bending moment at any point in the shaded areas is less than that at the point on the opposite side of the center of the span, equi-distant from the center.
Locus of ends of spans
Locus of centers of spans

FIG. 155.

200 ft. For example, in a span of 80 ft., the maximum moment at the center is obtained with wheel 13 at the center. The maximum moment 5 ft. from the center occurs on the right-hand portion of the span with wheel 13 at the section and not on the left-hand portion with wheel 12 at the section. For the shorter and longer spans, the larger moments will be found on the right-hand portion of the span.

CHAPTER VIII

CONCENTRATED MOVING LOADS ON TRUSSES

Trusses with Horizontal Chords and Single Web Systems

92. Since the effect of the loads is transferred by the floor system to the trusses at the panel points *only*, the methods of calculation used for the determination of shears, moments, reactions, and floor-beam reactions are similar to those used in the previous chapter for a girder with floor beams. The shear in any section determines the vertical component of the diagonal or vertical cut by the section and its stress is easily computed from this component. In determining chord stresses, the center of moments for each computation is in a panel point of the opposite chord. The moment at such a point, determined as in the previous chapter, divided by the depth of the truss gives the

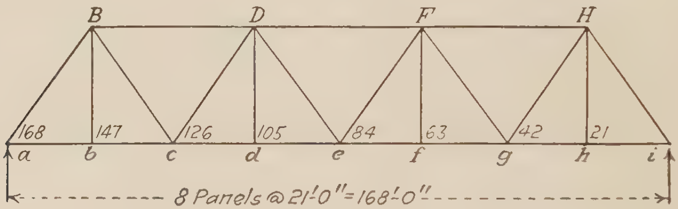


Fig. 156.

stress in the chord. The stress in a suspender is dependent only upon, and equal to, the floor-beam reaction at the panel point where it intersects the loaded chord. The total reaction of a truss is computed in exactly the same manner as if it were a simple beam.

In the solution of trusses, using the moment tables, the student will find it advisable to draw a diagram of the truss, whose stresses are to be computed, and mark thereon the distances from each panel point of the loaded chord to the right support. This is done in Fig. 156, 21 ft. being the distance from the panel point h to the right support, 42 ft. the distance from panel point g to the right support, etc.

The following example will serve to show the application of the moment tables to the determination of the stresses in a simple truss.

In the truss whose skeleton diagram is shown in Fig. 156 it is desired to compute for Cooper's *E*-60 loading:

a. The shear in the panel *bc* with wheel 3 at *c*.

b. The shear in the panel *de* with wheel 4 at *c*.

c. The bending moment in the section *Dd* with wheel 14 at *e*.

d. The floor-beam reaction at *b* with wheel 3 at *b*.

Using Tables I and IV:

a. $126 + \underset{(b\text{-}3)}{13} - \underset{(b\text{-}19)}{109} = 30$ ft. of uniform train load on the span

From Table IV:

$$M_i = \underset{(30\text{-}1)}{38{,}676} \text{ and } R_a = 230.2$$

$r_b = \underset{(3\text{-}1)}{345} \div 21 = 16.4 \qquad V_{bc} = 230.2 - 16.4 = 213.8$ kips.

b. $126 + \underset{(b\text{-}4)}{18} - 109 = 35$ ft. of uniform train load on the span

From Table IV:

$$M_i = \underset{(35\text{-}1)}{41{,}294} \text{ and } R_a = 245.8$$

$18 + 21 = 39$ and $18 + 42 = 60$, showing that wheels 8, 9, and 10 are on the panel *cd* since $39 > \underset{(b\text{-}7)}{37}$ and $60 > \underset{(b\text{-}10)}{56}$, with wheel 10, $60 - 56 = 4$ ft. to the left of *e*.

$$r_d = [\underset{(10\text{-}8)}{410} + (\underset{(f\text{-}10)}{228} - \underset{(f\text{-}7)}{174}) \times 4] \div 21 = 29.8$$

$$V_{de} = 245.8 - 174 - 29.8 = 42.0 \text{ kips.}$$

c. $84 + \underset{(b\text{-}14)}{79} - 109 = 54$ ft. of uniform train load on the span

From Table IV:

$$M_i = \underset{(50\text{-}1)}{49{,}596} + \underset{(50\text{-}1)}{576} \times 4 + \frac{3 \times 4^2}{2} = 51{,}924$$

$79 - 21 = 58$, showing, since $58 > \underset{(b\text{-}10)}{56}$ and $< \underset{(b\text{-}11)}{64}$ that wheels 1 to 10 are to the left of *d* with wheel 10, 2 ft. to the left of *d*.

$$M_d = \frac{51{,}924}{168} \times 63 - (\underset{(10\text{-}1)}{6950} + 2 \times \underset{(f\text{-}10)}{228}) = 12{,}070 \text{ kip ft.}$$

d. c is $13 + \underset{(b\text{-}3)}{21} - \underset{(b\text{-}6)}{32} = 2$ ft. to the right of wheel 6

$$M_c = \underset{(6\text{-}1)}{2460} + \underset{(f\text{-}6)}{154.5} \times 2 = 2770 \quad M_b = \underset{(3\text{-}1)}{345}$$

$$r_b = \frac{2770 - 2 \times 345}{21} = 99.0 \text{ kips.}$$

Using Tables II and IV:

a. $126 + \underset{(1\text{-}3)}{13} - \underset{(1\text{-}19)}{109} = 30$ ft. of uniform train load on the span

From Table IV:

$$M_i = \underset{(30\text{-}1)}{38{,}676} \text{ and } R_a = 230.2$$

$$r_c = \underset{(1\text{-}3)}{345} \div 21 = 16.4 \qquad V_{bc} = 230.2 - 16.4 = 213.8 \text{ kips.}$$

b. $126 + \underset{(1\text{-}4)}{18} - 109 = 35$ ft. of uniform train load on the span

From Table IV:

$$M_i = \underset{(35\text{-}1)}{41{,}294} \text{ and } R_a = 245.8$$

$18 + 21 = 39$ and $18 + 42 = 60$ showing that wheels 8, 9, and 10 are on the panel *de* since $39 > \underset{(1\text{-}7)}{37}$ and $60 > \underset{(1\text{-}10)}{56}$, with wheel 10, $60 - 56 = 4$ ft. to the left of *e*.

$$r_d = (\underset{(8\text{-}10)}{410} + \underset{(8\text{-}10)}{54} \times 4) \div 21 = 29.8$$

$$V_{de} = 245.8 - \underset{(1\text{-}7)}{174} - 29.8 = 42.0 \text{ kips.}$$

c. $84 + \underset{(1\text{-}14)}{79} - 109 = 54$ ft. of uniform train load on the span

From Table IV:

$$M_i = \underset{(50\text{-}1)}{49{,}596} + \underset{(50\text{-}1)}{576} \times 4 + \frac{3 \times 4^2}{2} = 51{,}924$$

$79 - 21 = 58$, showing, since $58 > \underset{(1\text{-}10)}{56}$ and $< \underset{(1\text{-}11)}{64}$ that wheels 1 to 10 are to the left of d, with wheel 10 2 ft. to the left of d.

$$M_d = \frac{51{,}924 \times 63}{168} - (\underset{(1\text{-}10)}{6948} + 2 \times \underset{(1\text{-}10)}{228}) = 12{,}068 \text{ kip ft.}$$

d. c is $13 + \underset{(1\text{-}3)}{21} - \underset{(1\text{-}6)}{32} = 2$ ft. to the right of wheel 6

$$M_c = \underset{(1\text{-}6)}{2460} + \underset{(1\text{-}6)}{154.5} \times 2 = 2769 \quad \underset{(1\text{-}3)}{M_b = 345}$$

$$r_b = \frac{2769 - 2 \times 345}{21} = 99.0 \text{ kips.}$$

93. The Pratt Truss without Counters. For the Pratt truss, three of the criteria developed in the previous chapter may be applied without change, to determine the position of the load producing the maximum stresses in the various members.

a. For the maximum stress in the suspenders, the loads must be so placed that the criterion for maximum floor-beam reaction is satisfied, that is

$$W_1 = 2P \qquad \text{(page 171)}$$

b. For the maximum stress in the diagonals and the verticals (other than the suspenders and the center vertical), the loads must be so placed that the criterion for the maximum shear between any two floor beams is satisfied, that is,

$$W = nP \qquad \text{(page 170)}$$

c. For the maximum stress in the chords, the loads must be so placed that the criterion for maximum moment is satisfied, that is,

$$\frac{m}{n} W = P \qquad \text{(page 170)}$$

In (*a*) the position is obtained by placing a wheel of the specified loading directly under (or over in a deck truss) the suspender, considering it first on one side and then on the other side of the panel point.

In (*b*) a wheel is placed at the panel point of the loaded chord just to the right of the section taken, and the two positions considered as in (*a*).

In (*c*) a wheel is placed directly at, or directly over or under, the center of moments, the two positions being considered as before.

94. Stresses in a Pratt Truss without Counters. The live-load stresses in the Pratt truss of Fig. 157, the complete data

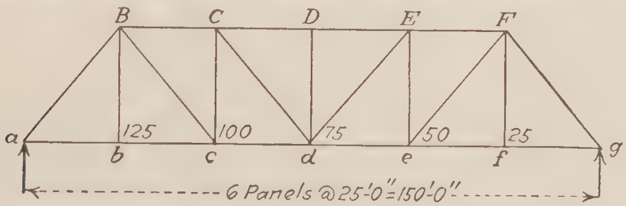


FIG. 157.

for which are given in Art. 64, will be computed (*a*) for the electric railway loads shown in Fig. 158, and (*b*) for Cooper's *E*-60 loading.

Stresses Due to Electric Railway Loads.

Bb, either wheel 4 or 5 at b

4 at b $\quad P = 12.5 - 25 \quad 2P = 25 - 50 \quad W_1 = 50$

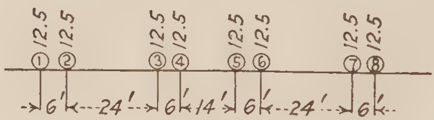
$$r_b = \frac{12.5(5 + 11 + 25 + 31) - 2 \times 12.5 \times 6}{25} = 30 \text{ kips.} = Bb$$

WEB STRESSES

Member	Wheel at right end of panel	P	nP	W	Moment at right support	Moment at right end of panel	Shear	Stress
aB	2	12.5–37.5	75–225	100	8800	75	55.7	−72.4
Bc	2	12.5–37.5	75–225	100	6300	75	39.0	+50.7
Cc	2	12.5–37.5	75–225	87.5	3863	75	22.8	−22.8
Cd	2	....	....	....	....	..	22.8	+29.6
dE	2	12.5–37.5	75–225	62.5–75	1975	75	10.2	
	1	....	....	50 −62.5	1600	0	10.7	−13.9
Ee	1	....	....	....	....	..	....	+10.7
eF	1	0 −12.5	0–75	25	550	0	3.7	− 3.7

CHORD STRESSES

Member	Wheel at section	W	$\frac{m}{n}W$	P	Moment at right support	Moment of wheels to left of section	M	Stress
$ab = bc$	2	100	16.7	12.5–37.5	8800	75	1392	+46.4
$cd = -BC$	3	100	33.3	25 −37.5	8700	675	2225	+74.2
CD	4	100	50	37.5–50	6800	700	2700	−90.0



Loads for One Rail
FIG. 158.

The stresses in dE, Ee, and eF are computed in order to obtain the stresses in the symmetrical members Cd, Cc, and Bc, when the cars are crossing the structure from left to right.

Stresses Due to Cooper's E-60 loading.

Bb Wheel 4 at b $P = 75 - 105$ $2P = 150 - 210$

$W_1 = 174 - 193.5$

$$r_b = \frac{4277 \dot{-} 2 \times 720}{25} = 113.5 \text{ kips.} = Bb$$

WEB STRESSES

Member	Wheel at right end of panel	P	nP	Length of uniform load	W	Moment at right support	Moment at right end of panel	Shear	Stress
aB	4	75–105	450–630	34	528	40,764	720	243.0	−315.9
Bc	4	75–105	450–630	9	453	28,502	720	161.2	
	3	45–75	270–450	4	438	26,274	345	161.4	+209.8
Cc	3	45–75	270–450	..	348–367.5	16,224	345	94.4	− 94.4
Cd	3	......	....	..	.....	......	...	94.4	+122.7
dE	2	15–45	90–270	..	228	7,404	120	44.6	− 58.0
Ee	2	......	....	..	.....	......	...	44.6	+ 44.6
eF	2	15–45	90–270	..	154.5	2,615	120	12.6	− 16.4

CHORD STRESSES

Member	Wheel at section	Length of uniform load	W	$\frac{m}{n}W$	P	Moment at right support	Moment of wheels to left of section	M	Stress
$ab = bc$	4	34	528	88	75 −105	40,764	720	6,074	+202.5
$cd = BC$	7	28	510	170	154.5–174	37,651	3,233	9,317	+310.4[1]
	8	34	528	176	174 −193.5	40,764	4,277	9,311	
CD	11	30	516	258	228 −2 8	38,676	8,772	10,566	
	12	35	531	265.5	258 −288	41,294	10,062	10,585	−352.8

[1] Considering the condition mentioned in the footnote of Art. 80, Wheel 14 at e satisfies the criterion, and the moment at section Ee is 9352, causing a stress in $de = -EF$ of 311.7 kips, which therefore would be the maximum stress in $cd = -BC$.

The special conditions encountered in the center panel of a Pratt truss with an uneven number of panels without counters are exactly similar to those occurring in the Parker truss (see Art. 99).

95. The Pratt Truss with Counters. From the preceding article it is seen that the web members near the center are subject to a reversal of stress during the passage of the load across the structure. This reversal of stress may be prevented, as has been shown in Art. 71, by the placing of counters in the panels where

the combined stresses due to dead and live loads would otherwise change sign.

Figure 159 shows a through Pratt truss with counters, the depth being 30 ft., and the secant of the angle between diagonal and vertical 1.30. When designed for Cooper's E-60 loading, counters are required in the three center panels in order to prevent reversal of stress. Upon assuming that a counter is also required in the panel bc, it is found that the dead-load tension in Bc is greater than any possible live-load compression, and that no counter is required.

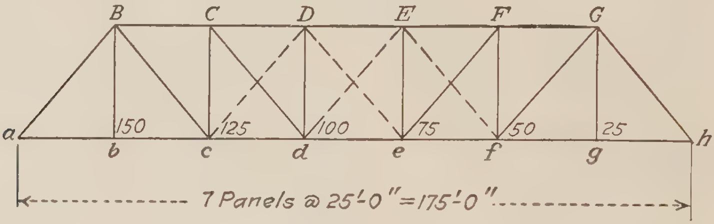


Fig. 159.

Stresses in Web Members. Since the moment tables are constructed for loads moving from right to left, the live load stresses will be computed for the web members shown in Fig. 159(a). The stress computed for Ef with the loads on the right is equal to the stress in the counter cD with the loads on the left. Similarly, the stress in Ee equals that in Dd, and the stress in De that in dE. Also, with Ef in tension there is no live-load stress in Ff, so that the stress in Cc need only be computed for the conditions shown in Fig. 159(a) with the load on the right.

Member	Wheel at right end of panel	P	nP	Length of uniform load	W	Moment at right support	Moment at right end of panel	Shear	Stress
aB	4	75–105	525–735	59	603	54,902	720	284.9	−370.4
Bc	4	75–105	525–735	34	528	40,764	720	204.1	
	3	45–75	315–525	29	513	38,162	345	204.3	+265.6
Cc	3	45–75	315–525	4	438	26,274	345	136.3	−136.3
Cd	3	......	....	..	....	......	...	136.3	+177.2
Dd	3	45–75	315–525	..	348–367.5	16,224	345	78.9	− 78.9
De	3	......	....	..	....	......	...	78.9	+102.6
Ee	2	15–45	105–315	..	228	7,404	120	37.5	− 37.5
Ef	2	......	....	..	....	......	...	37.5	+ 48.8
fG	2	15–45	105–315	..	154.5	2,615	120	10.1	− 13.1

Stresses in Chord Members. The stresses in the chord members near the ends of the span are calculated in a manner similar to those in a truss without counters, for the loading causing the greatest stresses in those members also produces positive shear in all but the center panel or panels. For the chord members near the center, it is necessary to determine which of the web members are stressed in order to compute the exact maximum stress in the chords.

The criterion for the maximum moment in the section *Dd* is satisfied with either wheels 11 or 12 at *d*. The maximum moment in section *Ee* may occur with either wheel 13 or 14 at *e*. The form of the truss, that is, whether *De*, or *dE*, or neither, is in tension, will have to be determined under these various loadings in order to ascertain which chord stresses may be obtained from the moments. The trusses under these various loadings assume

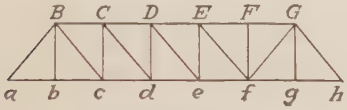


Fig. 159(*a*).

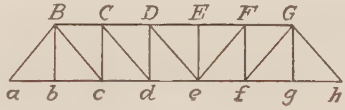


Fig. 159(*b*).

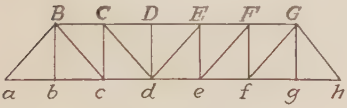


Fig. 159(*c*).

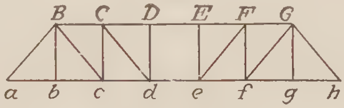


Fig. 159(*d*).

one of the forms shown in Fig. 159(*b*), 159(*c*), or 159(*d*), the sign of the shear in the panel *de* determining which form is assumed—a positive shear bringing into action the form shown in Fig. 159(*b*), a negative shear that shown in Fig. 159(*c*), and a zero shear that shown in Fig. 159(*d*). To obtain the stresses in the members *CD*, *DE*, *EF*, and *de*, the center of moments must be taken in Fig. 159(*b*) either at *e* or *D*, in Fig. 159(*c*) at *d* or *E*, while in Fig. 159(*d*) it may be taken at any point of *DE* or *de*. In the latter case, however, since the moment always varies uniformly between *d* and *e*, the maximum moment with zero shear in the panel *de* is the same no matter what point is selected as the center of moments.

In the accompanying table, since for the first position of the loads the shear in *de* is negative while for the fourth position it is positive, zero shear must occur, if at all, with some intermediate

position. There are only two such possible positions with a wheel placed at a panel point, namely, wheel 15 at e or wheel 10 at d. In some trusses, zero shear in the middle panel does not occur with the wheels so definitely placed, in which case the exact position can be obtained only after various trials.

Section	Wheel at section	Length of uniform load	Moment at right support	R_a	M of loads in de about e	r_d	V_{de}	Center of moment	M	Stress	Chord
Dd	11	55	52,514	300.1	2120	84.8	−12.7	*d*	13,734	457.8	*CDE*
								E	13,416	447.2	*de*
Dd	12	60	55,506	317.2	1937	77.5	−18.3	*d*	13,726	457.5	*CDE*
								E	13,269	442.3	*de*
Ee	13	40	43,986	251.3	720	28.8	+ 9.5	*e*	13,633	454.4	*DEF*
								D	13,394	446.5	*de*
Ee	14	45	46,754	267.2	1245	49.8	+ 4.4	*e*	13,624	454.1	*DEF*
								D	13,515	450.5	*de*
Dd	10	47	47,882	273.6	1515	60.6	0	*D, E, d* or *e*	13,573	452.4	*CDEF de*

From the above table it is seen that the maximum stress in $CD = DE$ occurs with wheel 11 at d and is 457.8 kips. For all seven panel trusses except those with short panels, the maximum stress in de occurs when the shear in the panel is zero. In the latter type a positive shear may occur with a wheel placed at d, in which case D may be taken as the center of moments. In stress calculations it is usually the practice to assume that the stress in de is equal to that in DE, since the former is only slightly less and never greater than the latter.

96. The Warren Truss without Verticals. In this type of truss the stresses in the web members are obtained by the application of the method of the preceding articles. In determining some of the chord stresses, a condition exists somewhat different from any yet encountered, so the method developed in Art. 86 cannot be applied.

Referring to Fig. 160, the moment center for the chord bc is the panel point C. Let P be the load to the left of the panel bc, Q the load in the panel bc, and W the total load on the span. Let g_1 be the distance from the center of gravity of the loads P to the center of moments, g_2 the distance from the center of gravity of the loads Q to the right end of the panel, p the panel length, c the distance of the center of moments from the left end of the

panel, k the distance of the center of moments from the left reaction, and g the distance of the center of gravity of the loads W from the right support. Then the moment at C is

$$M_C = \frac{Wg}{l}k - Pg_1 - \frac{Qg_2}{p}c$$

If the load advances a distance x, the increase in moment is

$$\frac{Wk}{l}x - Px - Q\frac{c}{p}x$$

whence from previous similar deductions, the maximum moment is obtained when

$$\frac{k}{l}W = P + \frac{c}{p}Q$$

The load Q changes in value whenever a wheel load passes either the point b or the point c, so that the position of the loads

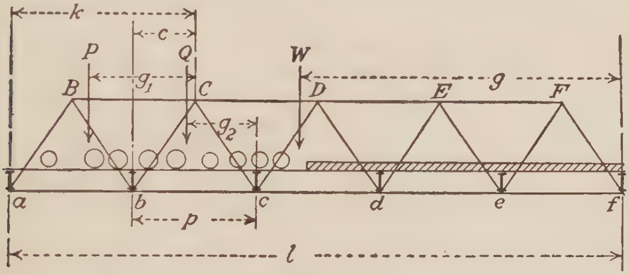


Fig. 160.

which causes the maximum moment at C and hence the maximum chord stress in bc occurs with a wheel load at either b or c. When a load passes c, Q may be the only quantity to change in value, while a load passing b causes both of the values P and Q to change.

For the usual case, $c = \frac{1}{2}p$ so that the criterion for maximum moment becomes

$$\frac{k}{l}W = P + \frac{1}{2}Q$$

and the expression for the moment

$$\frac{Wg}{l}k - Pg_1 - \frac{1}{2}Qg_2$$

The stresses in the opposite chord are obtained as in previous articles, as all the loads to the left of the center of moments have the same direct effect on the value of the moment (see Art. 86).

97. Calculation of Stresses. In the truss of Fig. 161 the maximum live-load stresses in the chord members due to Cooper's E-60 loading are obtained as follows:

ab (Fig. 161(a)) Center of moments at B $k = 12$ $\frac{k}{l} = \frac{1}{12}$

Wheel 4 at b $P = 0$ $W = 426 + (18 + 120 - 109)3 = 513$

$\frac{1}{12}W = 42.75$ $Q = 75 - 105$ $P + \frac{1}{2}Q = 37.5 - 52.5$

$M_g = 38{,}162$ $R_a = \frac{38{,}162}{144}$ $r_a = \frac{720^1}{24}$

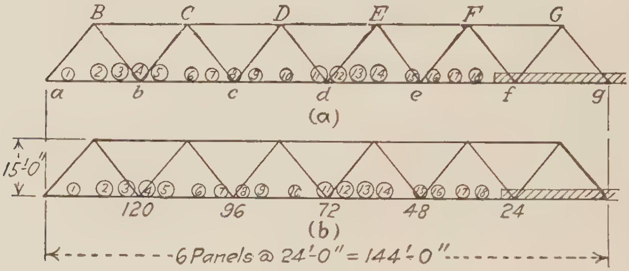


Fig. 161.

$$M_B = \frac{38{,}162}{144} \times 12 - \frac{720}{24} \times 12 = 2820 \qquad ab = 2820 \div 15 =$$

188.0 kips

bc (Fig. 161(a)) Center of moments at C $k = 36$ $\frac{k}{l} = \frac{1}{4}$

Wheel 4 at b $W = 513$ $\frac{1}{4}W = 128.25$

$P = 75 - 105$ $Q = 99 - 69$ $P + \frac{1}{2}Q = 124.5 - 139.5$

[1] An alternative method for the calculation of the moments at B, C, and D is evident by reference to Art. 77 and Fig. 143, *i.e.*, the moment at C, is the average of the moments at b and c, etc.

For example:

ab Wheel 4 at b $M_b = \frac{38{,}162 \times 24}{144} - 720 = 5640$

$M_a = 0$ $M_B = \frac{5640 + 0}{2} = 2820$

bc Wheel 4 at b $M_c = \frac{38{,}162}{144} \times 48 - (3233 + 5 \times 174) = 8618$

$M_C = \frac{5640 + 8618}{2} = 7129$

cd Wheel 11 at d $M_c = \frac{37{,}142 \times 48}{144} - (3233 + 174 \times 3) = 8626$

$M_d = \frac{37{,}142 \times 72}{144} - 8772 = 9799$ $M_D = \frac{8626 + 9799}{2} = 9212$

$M_g = 38{,}162$ $\quad$ M of wheels 1–4 about $C = 720 + 12 \times 105 = 1980$

$$r_b = \frac{518 + 69 \times 5}{24} = \frac{863}{24}$$

$$M_C = \frac{38{,}162}{144} \times 36 - 1980 - \frac{863}{24} \times 12 = 7129$$

$$bc = 7129 \div 15 = 475.3 \text{ kips}$$

Wheel 7 at c also satisfies the criterion, but the moment produced at C is smaller.[2]

cd (Fig. 161(b)) Center of moments at D $\quad k = 60 \quad \frac{k}{l} = \frac{5}{12}$

Wheel 11 at d $\quad W = 426 + (64 + 72 - 109)3 = 507$

$$\tfrac{5}{12}W = 211.3$$

$P = 174 \quad Q = 84 - 54 \quad P + \tfrac{1}{2}Q = 216 - 201$

$M_g = 37{,}142$ $\quad$ M of wheels 1–7 about $D = 3233 + 174 \times 15 = 5843$

$$r_c = \frac{842}{24}$$

$$M_D = \frac{37{,}142 \times 60}{144} - 5843 - \frac{842 \times 12}{24} = 9212$$

$$cd = 9212 \div 15 = 614.7 \text{ kips}$$

BC Center of moments at b $\quad \frac{m}{n} = \frac{1}{6}$

Wheel 4 at b $\quad W = 513 \quad \tfrac{1}{6}W = 85.5 \quad P = 75 - 105$

$$M_b = \frac{38{,}162 \times 24}{144} - 720 = 5640 \qquad BC = -376.0 \text{ kips}$$

CD Center of moments at c $\quad \frac{m}{n} = \frac{2}{6} = \frac{1}{3}$

Wheel 7 at c $\quad W = 426 + (37 + 96 - 109)3 = 498$

$$\tfrac{1}{3}W = 166 \qquad P = 154.5 - 174$$

$$M_c = \frac{35{,}634 \times 48}{144} - 3233 = 8644 \qquad CD = -576.3^{3} \text{ kips}$$

DE Center of moments at d $\quad \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$

Wheel 11 at d $\quad W = 507 \quad \tfrac{1}{2}W = 253.5 \quad P = 228 - 258$

$$M_d = \frac{37{,}142 \times 72}{144} - 8772 = 9799 \qquad DE = -653.3 \text{ kips}$$

[2] The wheels satisfying the criterion for the symmetrical chords ef and de produce smaller moments than those computed for bc and cd.

[3] Wheel 13 at e satisfies the criterion for the greatest bending moment in that section, which is 8723 kip ft. making the maximum stress in EF, and hence also in $CD = -581.5$.

Trusses with Inclined Chords and Single Web Systems

98. In a truss where the two chords are not parallel, the stresses in the web members due to a system of concentrated loads cannot be obtained directly by computing the shears, but the method of moments must be employed. The position of the loads for the maximum stresses having been obtained, the remainder of the computation is similar to that used for determining the stresses under uniform live load.

In the truss of Fig. 162, it is assumed that the maximum stress in any web member, such as *Bc*, will be obtained when the load extends from the right support to some point between *c* and *b*, but not to the left of *b*. This is true for practically all trusses, the exception occurring in trusses entirely out of the ordinary and rarely, if ever, encountered. The position of the loads which produces a maximum moment at the intersection of the two chords through which the section is passed, is the position which produces the maximum stress in the web member under consideration.

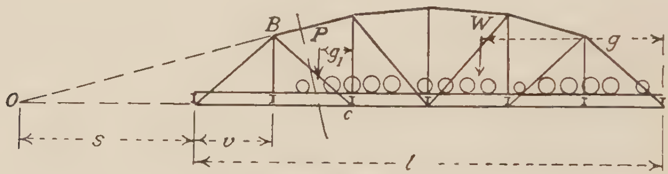


Fig. 162.

Let s be the distance from the center of moments to the left support, and v the distance from the left support to the left end of the panel through which the section cutting the web member is passed. Let W be the total load on the span, and g the distance of its center of gravity from the right support. Let P be the load in the panel through which the section is passed, and g_1 the distance of its center of gravity from the right end of the panel. Let p be the panel length and l the span, then

$$M_o = -\frac{Wg}{l}s + \frac{Pg_1}{p}(s + v)$$

If the loads are moved to the left a short distance x, the change in M_o is

$$-\frac{Wx}{l}s + \frac{Px}{p}(s + v)$$

and the rate of change per unit of length

$$-\frac{W}{l}s + \frac{P}{p}(s + v)$$

so that for maximum moment at O

$$W = \frac{Pl}{p}\frac{(s + v)}{s} \qquad (a)$$

In the usual case, with the panels equal and $l = np$, the criterion for the maximum stress in a web member due to any system of concentrated loads is

$$W = nP\frac{(s + v)}{s}$$

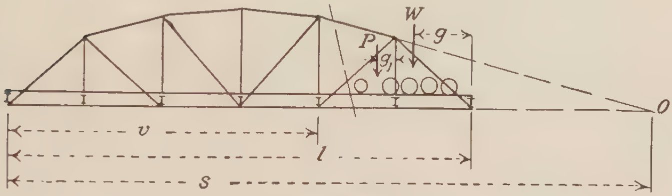


FIG. 163.

In computing the stresses on the right of the center, in order to obtain minimum stresses, or stresses in counters, the center of moments is on the right of the truss. In such a case (see Fig. 163)

$$M_o = \frac{Wg}{l}s - \frac{Pg_1}{p}(s - v)$$

and the criterion is

$$W = nP\frac{(s - v)}{s} \qquad (b)$$

In the equations (a) and (b) above, the multiplier of $\frac{nP}{s}$ is, in each case, the distance from the left end of the panel through which the section is passed to the center of moments. In equation (a) the multiplier of nP is always greater than unity, while in equation (b) it is always less than unity.

99. The Parker Truss without Counters. The stresses in the through Parker Truss of Fig. 164, due to Cooper's E-60 loading are calculated below. The dead load stresses for panel loads at 10 kips and 30 kips at the upper and lower panel points, respectively, were computed in Art. 65.

The stresses in aB and Bb are calculated in the same manner as the stresses in the corresponding members of the through Pratt truss of Fig. 157. For the stress in the diagonal Bc, wheel 3 at c satisfies the criterion of equation (a); for $s = 3$ panel lengths, $v =$ one panel length, P varies from 45 to 75 and $\frac{nP(s + v)}{s}$ from 420 to 700, while the total load on the span is $426 + 29 \times 3 = 513$. The moment about the right support is 38,162 and that about the panel point c is 345, then

$$M_o = \frac{38{,}162}{7 \times 25} \times 3 \times 25 - \frac{345}{25} \times 4 \times 25 = 14{,}975$$

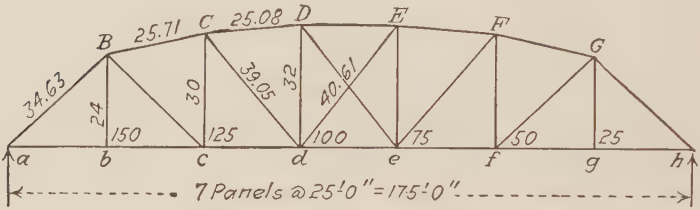


FIG. 164.

and the vertical component of Bc is

$$\frac{14{,}975}{125} = 119.8$$

and the stress in Bc

$$119.8 \times \frac{34.63}{24} = +172.9 \text{ kips.}$$

The stresses in Cc and Cd are obtained in a similar manner.

In the center panel the two diagonals De and dE are designed to resist either tension or compression. In such cases it is sufficiently accurate to assume that each member carries one-half of the shear in the panel, one diagonal being in tension while the other is in compression. Since the chords DE and de are parallel, the maximum live-load shear in the panel de has the same value as that given for the truss of Fig. 159. With the load advancing from the right and with wheel 3 at e, the live-load shear in the panel is 78.9 kips, and the stress in De is $\frac{78.9}{2} \times \frac{40.61}{32} = +50.1$ kips, and that in dE is -50.1 kips. With the load advancing from the opposite direction, the stresses are reversed.

Since these center diagonals are, under all conditions of loading, assumed to resist equally the shear in the center panel, they are both stressed when there is any portion of the live load on the bridge. Therefore, in determining the exact stress in the vertical *Dd*, the vertical component of the stress in one of these diagonals enters into the calculations. Passing a section through *CD*, *Dd*, *dE*, and *de* as shown in Fig. 165 and placing wheel 2 at *e*, the criterion for the maximum stress in *Dd* is satisfied, and $M_o = 24{,}979$. With the load so placed, the shear in the center panel is 78.0 kips and the vertical component of *dE* is 39.0 kips. The moment of this force about the center of moments *O* is $16 \times 25 \times 39 = 15{,}600$ and the maximum stress in

$$Dd = -\frac{24{,}979 - 15{,}600}{16 \times 25} = -23.4 \text{ kips.}$$

The maximum stress in *Ee* which furnishes the value for the minimum stress in the symmetrical member *Dd* is obtained with

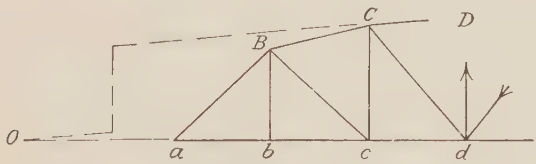


FIG. 165.

wheel 3 at *e*. $M_o = 40{,}489$, and the vertical component of *De* is $78.9 \div 2$, and the equation for the stress in *Ee* is

$$40{,}489 - \frac{78.9}{2} \times 16 \times 25 - Ee \times 16 \times 25 = 0$$

from which $Ee = +61.8$ kips.

The maximum stresses in *eF*, *Ff*, and *fG* are computed in a manner similar to that in which the stress in *Bc* was determined, and furnish the values for the minimum stresses in the corresponding symmetrical members.

The stresses in the lower chord members *ac* and *cd*, and the horizontal components of the stresses in the upper chord members *BC* and *CD* are calculated as if the chords were parallel and the principles of Art. 86 are directly applicable. In the center panel, since both diagonals act simultaneously and have equal

and opposite stresses, if the section is cut as shown in Fig. 166 and moments are taken about either J or j, the diagonal stresses will not affect the bending moment in the section. Since these centers of moments are not at the panel points but midway between two of them, the methods used for the upper chord of the Warren Truss of Fig. 160 are applicable to the determination of the stresses in DE and de. Three positions of the loading satisfy the criterion $\frac{k}{l}W = P + \frac{1}{2}Q$, namely wheel 10 at d, and either 14 or 15 at e. With the loading in the latter position, the length of uniform load on the span is 54 ft. $W = 588$ and

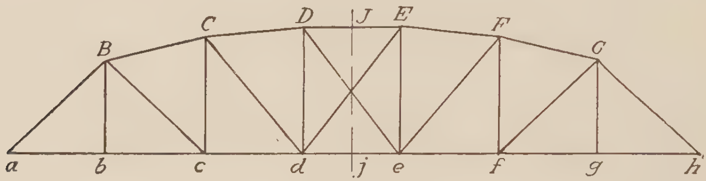


FIG. 166.

$\frac{1}{2}W = 294$, $P = 228$, $Q = 120 - 139.5$, $\frac{1}{2}Q = 60 - 69.7$ and $P + \frac{1}{2}Q = 288 - 297.7$.

From Table IV:

$$M_h = 49{,}596 + 576 \times 4 + 1.5 \times 4^2 = 51{,}924$$
$$M_d = \tfrac{3}{7} \times 51{,}924 - (6948 + 7 \times 228) = 13{,}709$$
$$M_e = \tfrac{4}{7} \times 51{,}924 - 16{,}224 = 13{,}447$$

from which

$$M_j = 13{,}578$$

and

$$DE = -de = \frac{13{,}578}{32} = -424.3 \text{ kips}$$

The moments for the other two positions are slightly smaller than 13,578.

The live-load stresses for all of the web members of the truss are tabulated on the following page:

This table furnishes sufficient data for the determination of the maximum and minimum live load stresses in the web members of the truss of Fig. 166. The values of these stresses in the

STRESSES DUE TO COOPER'S E-60 LOADING IN THE PARKER TRUSS OF FIG. 164

Member	Wheel at right end of panel	s (in panel lengths)	v (in panel lengths)	$\frac{s \pm v}{s}$	P	$\frac{nP(s \pm v)}{s}$	Length of uniform load	W	Moment at right end of truss (Wg)	Moment at right end of panel (Pg_1)	$M_o = Wg\left(\frac{s}{l}\right) - \frac{Pg_1(s \pm v)}{p}$	Vertical component $M_o \div$ lever arm	Stress
aB		See computations for shear in Ab of the truss of Fig. 159									$284.9 \times$	$\frac{34.63}{24} =$	−411.1
Bb		See computations for member Bb of the truss of Fig. 157											+113.5
Bc	3	3	1	$\frac{4}{3}$	45–75	420–700	29	513	38,162	345	14,975	119.8	+172.9
Cc	2	3	2	$\frac{5}{3}$	15–45	175–525	0	426	24,118	120	9,836	78.7	− 78.7
Cd	3	13	2	$\frac{15}{13}$	45–75	363–606	4	438	26,272	345	43,616	109.1	+143.0
Dd	2	13	3	$\frac{16}{13}$	15–45	129–388	0	348	14,484	120	24,979	23.4	− 23.4
De	3	..	..	....	45–75	315–525	0	348–367.5	16,224	345	......	39.5	+ 50.1
dE	..	..	..	....	.....	......	..	....	.....	...	....	.....	− 50.1
Ee	3	20	3	$\frac{17}{20}$	45–75	268–446	0	348–367.5	16,224	345	40,489	61.8	+ 61.8
eF	2	20	4	$\frac{16}{20}$	15–45	84–252	0	228	7,404	120	19,234	48.1	− 62.5
Ff	3	10	4	$\frac{6}{10}$	45–75	189–315	0	228	8,544	345	10,136	81.1	+ 81.1
fG	2	10	5	$\frac{5}{10}$	15–45	52–157	0	154.5	2,614	120	3,134		
	3	..	..	....	45–75	157–262	0	174	3,407	345	3,142	25.1	− 36.2

members on the left half of the truss are given in the following table.

MAXIMUM AND MINIMUM LIVE-LOAD STRESSES IN WEB MEMBERS

Member	Maximum	Minimum
aB	-411.1	0
Bb	$+113.5$	0
Bc	$+172.9$	-36.2
Cc	-78.7	$+81.1$
Cd	$+143.0$	-62.5
Dd	-23.4	$+61.8$
De	$+50.1$	-50.1

100. The Parker Truss with Counters. In Fig. 167 the diagonals are designed to resist tension only. Reference to Arts. 65 and 99 shows that the live load causes no reversal of stress in the members aB, Bb, and Bc, so that their maximum and minimum stresses have the same value as in a truss without

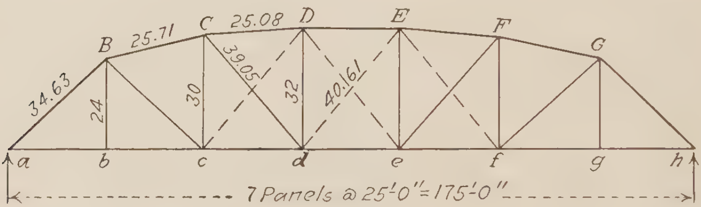


FIG. 167.

counters. The same is true of the chord members near the ends of the truss. The maximum stresses in the diagonal Cd and the vertical Cc are the same, and that in the diagonal De is twice as great as in the truss without counters.

The stress in the counter cD is obtained by calculating the stress in the symmetrical member Ef. With wheel 2 at f, and the section passed through EF, Ef, and ef, $s = 20$, $v = 4$, $\frac{nP(s-v)}{s} = 84 - 252$, and $W = 228$.

$$M_o = \frac{7404 \times 20 \times 25}{7 \times 25} - \frac{120 \times 16 \times 25}{25} = 19{,}234$$

and the live load stress in Ef =

$$\frac{40.61}{32}\left(\frac{19{,}234}{15 \times 25}\right) = +65.1 \text{ kips.}$$

The dead load stress in the counter as computed in Art. 73 is -33.8 kips. The maximum stress in the counter is, therefore, $+31.3$ kips.

As in the calculations for uniform live load of Art. 73, the maximum tension in the member Ff occurs, when there is no stress in either the diagonal eF or the counter Ef. The dead-load compression in the counter Ef is 33.8 kips, its vertical component being 26.7 kips. Therefore, the live load must be so placed as to cause a tensile stress in Ef of 33.8 kips. This position can only be found by trial. With the section passed through EF, Ef, and ef the equation for the vertical component of the stress in Ef may be written as follows:

$Ef_v(s - v + p) = R_a s - r_e(s - v) - r_d(s + p - v)$ . . . . etc.,

in which s and v are the distances as defined in Art. 98, R_a the left reaction, p the panel length, r_e and r_d the floor beam reactions at the respective panel points on the left of the section.

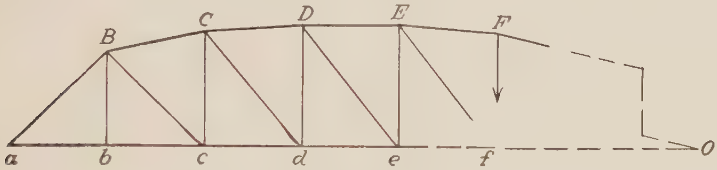


FIG. 168.

With wheel 5, 1.6 ft. to the left of the panel point f,

$$M_h = 11{,}502 + 318 \times 0.6 = 11{,}693$$

and

$$M_f = 1245 + 135 \times 1.6 = 1461.$$

There is no load on the left of the panel point e so that r_e is the only floor beam reaction to be considered. Then

$$Ef_v = \left[\frac{11{,}693 \times 20}{7} - 1461 \times 16\right] \div 15 \times 25 = +26.7$$

indicating a combined dead and live load stress in Ef of zero.

The live load stress in the vertical Ff with this loading is (Fig. 168)

$$\left[\frac{11{,}693 \times 10}{7} - 1461 \times 6\right] \div 5 \times 25 = +63.5 \text{ kips.}$$

The dead load stress is (Fig. 168)

$$\frac{120 \times 10 - 40(9 + 8 + 7 + 6) - 10 \times 5}{5} = -10.0 \text{ kips.}$$

and the maximum tensile stress in $Ff = Cc = +53.5$ kips.

Similarly, the maximum tension in the vertical Ee, occurs when neither of the diagonals in the center panel is stressed. Since there is no dead-load stress in either of these two diagonals (Fig. 169) the live-load shear in the center panel must be zero in order that this condition may exist. Wheel 10 at d (see Art. 95) causes zero shear in the center panel of this truss, and, with this loading, taking moments about O (Fig. 169) the live load tension in Ee is

$$\left[\frac{47{,}882 \times 20}{7} - (6948 + 228 \times 17 \times 25) - (1245 + 2 \times 135 \times 17)\right] \div 16 \times 25 = +35.4 \text{ kips.}$$

The dead load stress in Ee is $+5.0$ and the maximum tensile stress in $Ee = Dd = +40.4$ kips.

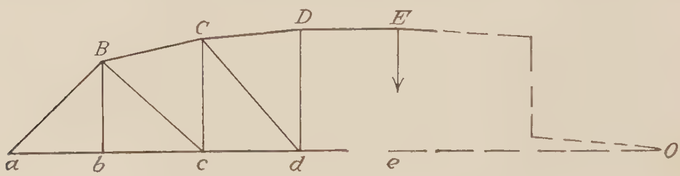


FIG. 169.

Since the span and the number of panels are the same as those of the through Pratt Truss of Art. 95, the moments computed there may be used in determining the chord stresses for this truss. Therefore, the maximum live load stress in de is $13{,}573 \div 32 = 424.2$ kips, that in DE, $13{,}734 \div 32 = 429.2$ kips, while the maximum stress in CD is $429.2 \times \frac{25.08}{25} = 430.6$ kips.

TRUSSES WITH SUBDIVIDED PANELS

101. The Baltimore Truss without Counters. Web Members. The stresses in many of the members of a Baltimore truss, due to a system of concentrated loads, may be determined by methods similar to those used for the Pratt truss. The stresses in the subverticals are equal to the floor-beam reactions, and those in the subdiagonals are obtained directly from them, as in the calculation of the dead-load stresses. The stresses in the main diagonals, in the panels where there are no subdiagonals, are not affected by the subdivision, and the method used for the Pratt truss is directly applicable.

a. Main Diagonals Where There Is also a Subdiagonal in the Panel. As in the calulation of stresses under uniform live load, where the subdiagonal is on the right of the double panel, the maximum stress is obtained when the load extends far enough to the left so that the subdiagonal is stressed. In Fig. 170, let P_1 and P_2 be the loads in the panels CD and DE, respectively, and g_1 and g_2 the distances of their respective centers of gravity from the panel points C and E. Let W be the total load on the span l, and g the distance of its center of gravity from the right support. Let p be the panel length, and n the number of panels in the truss.

Then the shear in section 1–1 is

$$\frac{Wg}{l} - P_1 - \frac{P_2g_2}{p}$$

and the vertical component of de is

$$\frac{Wg}{l} - P_1 - \frac{P_2g_2}{p} + \frac{1}{2}\left(\frac{P_1g_1 + P_2g_2}{p}\right) \tag{a}$$

which reduces to

$$\frac{Wg}{l} - P_1 - \frac{P_2g_2 - P_1g_1}{2p}$$

If the loads are moved to the left a distance x, the rate of change is

$$\frac{W}{l} - \frac{(P_2 - P_1)}{2p}$$

and since for a truss with equal panels $l = np$, the position of the loads producing a maximum stress in the diagonal must be such as to satisfy the criterion,

$$W = \frac{n(P_1 + P_2)}{2}$$

This is similar to the criterion for the position of the loads producing the maximum shear in a diagonal of a Pratt truss of equal span and of half the number of panels. In using this criterion, one of the concentrated loads must be placed at the panel point E, in order to have the quantity $P_1 + P_2$ variable as the load passes from just to the right of E to the left of E.

Equation (a) may be written

$$de_V = R_a - r_C - r_D + \frac{r_D}{2}$$

From Art. 87,

$$r_C = \frac{M'_D}{p} \text{ and } r_D = \frac{M'_E - 2M'_D}{p}$$

$$\therefore de_V = \frac{M'_M}{l} - \frac{M'_E}{2p}$$

where M' is the moment of all the loads to the left of the point designated by the subscript, or the *vertical component of the diagonal is the moment of all the loads on the span about the right support divided by the span length, minus the moment of all the loads in the double panel about the right end of the double panel divided by the double panel length.* In many trusses the position of the loads which causes the maximum stress will be such that

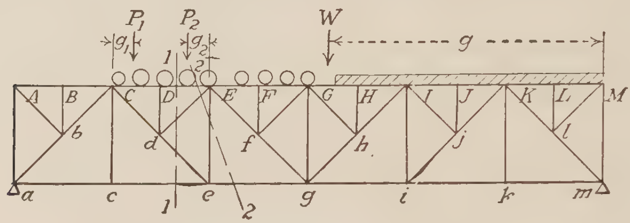


FIG. 170.

there will be no load on the panel CD. In this case P_1 vanishes from the criterion equation, while the equation for the calculation of the stress is not affected.

The methods used for the Pratt truss in determining the position of the loads for maximum stress are directly applicable where the subdiagonal is on the left of the double panel. By an analysis similar to that given above, the vertical component of the stress may again be shown to be

$$\frac{M'_M}{l} - \frac{M'_E}{2p}.$$

b. Verticals. Three cases are encountered in the calculation of the stresses in the verticals of a Baltimore Truss.

Case I. *Where the Vertical Is an Integral Part of the Main Web System.* In Fig. 170, Ee is a vertical of this type. The stress may be obtained from the shear in the section 2—2, hence the stress in Ee is a maximum under the same conditions that cause the maximum stress in de, being equal and opposite in sign to the vertical component of that member. In a truss of the type illustrated in Fig. 171, the criterion for the position of the

loads causing a maximum stress in the vertical Ee is somewhat different from that for Ee of Fig. 170. The same system of notation as was used for the deck truss will be applied. In Fig. 171 the shear in the section is

$$\frac{Wg}{l} - P_1 - P_2 - \frac{P_3 g_3}{p}$$

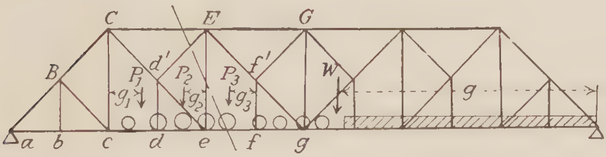


FIG. 171.

and the vertical component of Ee is

$$\frac{Wg}{l} - P_1 - P_2 - \frac{P_3 g_3}{p} + \tfrac{1}{2}\frac{(P_1 g_1 + P_2 g_2)}{p} \qquad (b)$$

and for the maximum stress the loads must be so placed that

$$W = n\left[\frac{(P_1 - P_2)}{2} + P_3\right]$$

In this case one of the concentrated loads must be placed at either e or f. For trusses of not more than 12 panels there will be no load on the panels cd and de, and hence the criterion reduces to $W = nP_3$ and the computation of the stress is similar to that for a Pratt truss. For trusses with a greater number of panels, the calculation of the stress is more involved and requires the solution of equation (b).

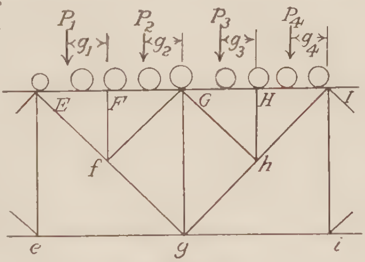


FIG. 172.

Case II. *Where the Vertical Is a Subvertical of the Main Web System, and the Web Members of the Adjacent Panels Are Symmetrical.* In Fig. 172, Gg is a vertical of this type. The stress in Gf is equal to one-half the floor-beam reactions at the points F and H, plus the full reaction at point G; or $Gg = \tfrac{1}{2}r_F + r_G + \tfrac{1}{2}r_H$.

Let the loads in the panels EF, FG, GH, and HI be P_1, P_2, P_3, and P_4, respectively, and g_1, g_2, g_3, and g_4, the respective distances

of their centers of gravity from the right ends of the respective panels. Then

$$r_F = \frac{P_1(p + g_1) + P_2g_2 - 2P_1g_1}{p}$$

$$r_G = \frac{P_2(p + g_2) + P_3g_3 - 2P_2g_2}{p}$$

$$r_H = \frac{P_3(p + g_3) + P_4g_4 - 2P_3g_3}{p}$$

Substituting these values in the previous expression for the stress in Gg and reducing

$$Gg = \frac{P_1(g_1 + 3p) + P_2(g_2 + 2p) + P_3(g_3 + p) + P_4g_4 - 2[P_1(g_1 + p) + P_2g_2]}{2p}$$

or

$$Gg = \frac{M'_I - 2M'_G}{2p}$$

which is the expression for the maximum floor-beam reaction at G due to the loads in the two adjacent panels EG and EI (see Art. 87), that is, the position is the same as if the members Ff, fG, Gh, and Hh were removed, and the panel length doubled.

Case III. *Where the Vertical Is a Sub-vertical of the Main Web System, and the Web Members of the Adjacent Panels Are Not Symmetrical.* Verticals of this type are of two classes: (*a*) *end verticals*, and (*b*) *intermediate verticals.*

In Fig. 170 the stress in the vertical Aa is equal to the floor-beam reaction at the point A plus one-half the reaction at the point B, or

$$Aa = r_A + \tfrac{1}{2}r_B$$

Let P_1 be the load in the panel AB and g_1 the distance of its center of gravity from B. Also let P_2 be the load in the panel BC and g_2 the distance of its center of gravity from C. Then

$$r_A = \frac{P_1g_1}{p}$$

and

$$r_B = \frac{P_1(p - g_1)}{p} + \frac{P_2g_2}{p}$$

and

$$Aa = \frac{P_1(p + g_1) + P_2g_2}{2p} = \frac{M'_C}{2p}$$

The maximum stress, therefore, in Aa occurs when the system of loads is so placed that the moment of the loads to the left of C

is a maximum about C. Some load placed at A will fulfill this condition.

In Fig. 171 the stress in the vertical Cc is equal to one-half of the floor-beam reaction at b plus the full floor-beam reaction at c, or

$$Cc = \tfrac{1}{2}r_b + r_c$$

From Art. 87

$$\tfrac{1}{2}r_b = \frac{M'_c - 2M'_b}{2p}$$

and

$$r_c = \frac{M_d - 2M'_c}{p}$$

and

$$Cc = \frac{2M'_d - 3M'_c - 2M'_b}{2p}$$

In all trusses but those with extremely short panels, the position of the load producing the maximum stress will be such that there will be no loads to the left of b, in which case $M'_b = 0$ and

$$Cc = \frac{2M'_d - 3M'_c}{2p}$$

In a span of length ad the bending moment at c is

$$\frac{M'_d}{3p} \times 2p - M'_c = 2M'_d - 3M'_c$$

As $2p$ is a constant, it follows that the maximum stress in an intermediate subvertical of the main web system, where the adjacent panels are not symmetrical, occurs when the loads are placed in the position to produce a maximum moment at the third point of a beam having a span equal to three panel lengths (see Arts. 80 and 81).

102. The Baltimore Truss without Counters. Chord Members. The methods used in obtaining the stresses in the chord members of the Pratt truss are directly applicable to the lower chord of a Baltimore truss with subties, or to the upper chord of such a truss with substruts. For example, the maximum stress in member ce, of Fig. 173, is computed by placing the loads so as to produce a maximum moment at C, the moment center, and this moment having been calculated, the stress in ce is obtained by dividing the moment by the depth of the truss.

In the opposite chord, two general conditions occur; (a) when the moment center is on the right of the section: and (b) when the moment center is on the left of the section.

a. Moment Center on the Right of the Section. In Fig. 173, the center of moments for the chord CE is at e, on the right of the section passed through CD, Cd, and ce. Using the same general system of notation as before, and taking moments about e of the external forces to the right of the section,

$$M = R_a k - P_1(g_1 + 2p) - \frac{P_2 g_2}{p} \times 2p$$
$$= \frac{Wg}{l}k - P_1 g_1 - 2P_1 p - 2P_2 g_2 \qquad (a)$$

If the loads are moved to the left a distance x, the rate of change in M is

$$\frac{Wk}{l} - P_1 - 2P_2$$

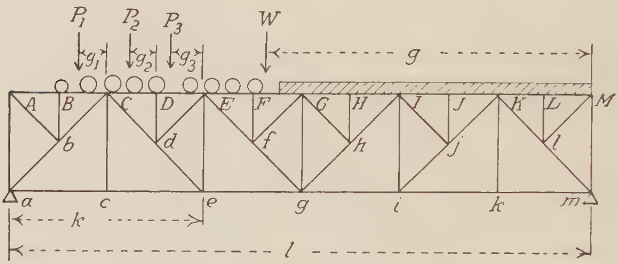


FIG. 173.

and the maximum moment at e, and hence the maximum stress in the chord CE, is obtained when the loads are so placed that

$$\frac{k}{l}W = P_1 + 2P_2$$

To satisfy this criterion, a load must be placed at panel point D.

In order to simplify equation (a), the equation for the bending moment at e or E, due to all the loads on the span may be written

$$M_E = R_a k - P_1(g_1 + 2p) - P_2(g_2 + p) - P_3 g_3$$

and the moment at D in a span of length CE caused by the loads P_2 and P_3 is

$$M_{D'} = \frac{[P_2(g_2 + p) + P_3 g_3]}{2p} \times p - P_2 g_2 = \frac{P_2 p - P_2 g_2 + P_3 g_3}{2}$$

but $M_E + 2M_{D'} = M$ as written in equation (a). Therefore, the stress in the chord member is

$$\frac{M_E + 2M_{D'}}{d}$$

in which M_E is the bending moment at the moment center caused by the loads on the span, M_D' the bending moment at the panel point next on the *left* caused by the loads on the two adjacent panels, and d the depth of the truss.

b. Moment Center on the Left of the Section. In Fig. 174, the center of moments for the chord eg is at E, on the left of the section passed through EG, $f'g$, and fg. The bending moment of the external forces on the left of the section is

$$M = R_1k - P_1g_1 + r_fp$$

From Art. 87,

$$r_f = \frac{P_2(g_2 + p) + P_3g_3 - 2P_2g_2}{p}$$

so that

$$M = \frac{Wg}{l}k - P_1g_1 - P_2g_2 + P_2p + P_3g_3 \qquad (b)$$

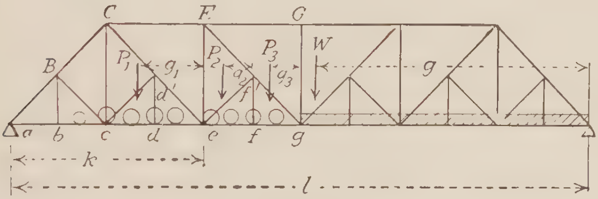


FIG. 174.

If the loads are moved to the left a distance x, the rate of change in M is

$$\frac{Wk}{l} - P_1 - P_2 + P_3$$

and the maximum moment at E, and hence the maximum stress in the chord eg, is obtained when the loads are so placed that

$$\frac{Wk}{l} = P_1 + P_2 - P_3$$

To satisfy this criterion, a load may be placed either at f or g.

Equation *(b)* may be simplified in the same manner as equation (*a*) previously, and the stress in the chord member is

$$\frac{M_e + 2M_{f'}}{d}$$

in which M_e is the bending moment at the moment center caused by the loads on the span, and $M_{f'}$ the bending moment at the panel point next on the *right* caused by the loads on the two adjacent panels.

103. Calculation of Stresses. The maximum and minimum live-load stresses in the deck Baltimore Truss of Fig. 175 are determined as follows:

Web Members

Aa It is evident that the maximum moment of the loads to the left of C about C (see page 208) occurs with wheel 2 at A. Its value in 6108, and the stress in

$$Aa = \frac{6108}{2 \times 24} = -127.3 \text{ kips.}$$

$Bb = Dd = Ef =$ maximum floor-beam reaction for a panel length of 24 ft. (see Art. 87)

$$Bb = +111.0 \text{ kips}$$

$$Ab = dE = fG = \frac{111.0}{2} \times 1.39 = +77.1 \text{ kips.}$$

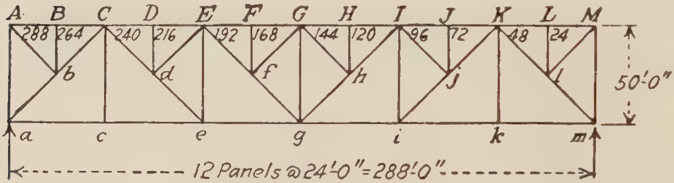


Fig. 175.

ab Wheel 4 at B $P = 75 - 105$ $nP = 900 - 1260$ $W = 945$

$$M'_M = 24{,}546 + 426 \times 173 + 173^2 \times 1.5 = 143{,}137$$

$$M_C = 3233 + 174 \times 5 = 4103$$

$$ab = \left[\frac{M'_M}{l} - \frac{M'}{2p}\right] \sec\,\theta = \left[\frac{143{,}137}{288} - \frac{4103}{48}\right] 1.39 = 572.0 \text{ kips.}$$

bC Wheel 3 at C $P = 45 - 75$ $nP = 540 - 900$ $W = 858$

$$M'_M = 113{,}586 + 4 \times 846 + 4^2 \times 1.5 = 116{,}994$$

$$M'_C = 345$$

$$bC = \left[\frac{M'_M - nM'_C}{l}\right] \sec\,\theta = \left[\frac{116{,}994 - 12 \times 345}{288}\right] 1.39 = -544.7 \text{ kips.}$$

Cd Wheel 3 at D $Cd = +488.2$ kips

de Wheel 5 at E $P_1 = 0$ $P_2 = 105 - 135$ $\dfrac{n(P_1 + P_2)}{2} = 630 - 810$ $W = 744$

$$M'_M = 85{,}814 + 741 \times 1 + 1^2 \times 1.5 = 86{,}556$$

$$M'_E = 1245$$

$$de = \left[\frac{M'_M}{l} - \frac{M'_E}{2p}\right] \sec\,\theta = \frac{86{,}556 - 6 \times 1245}{288} \times 1.39 = 381.7 \text{ kips.}$$

Ee Wheel 5 at E $Ee = -274.6$ kips.

Ef Wheel 3 at F $Ef = +284.1$ kips.

fg Wheel 4 at G $fg = +227.0$ kips.

Gg Wheel 8 at G satisfies the criterion $P = 2P'$ for a panel length of 48 ft. (see page 207)

$$M'_I = 16{,}224 + 367.5 \times 3 = 17{,}327$$
$$M'_G = 4277$$
$$Gg = \frac{M'_I - 2M'_G}{2p} = \frac{17{,}327 - 2 \times 4277}{48} = -182.7 \text{ kips.}$$

gh As in the computations for uniform live load when less than one-half of the truss is loaded, the maximum stress occurs when the subdiagonal is not stressed. Therefore, the maximum stress in gh (load from the right) is obtained under the same loading that produces the maximum stress in hI, which is wheel 2 at I. $M'_M = 22{,}416$ and $gh =$

$$\left[\frac{22{,}416 - 6 \times 120}{288}\right] 1.39 = -104.6 \text{ kips.}$$

hI Wheel 2 at I $hI = -101.2$ kips.

Ii Similarly the stresses in Ii and ij are obtained with the same position of the loads as that for jK.

Wheel 2 at K $Ii = +21.6$ kips.

$ij = -30.0$ kips.

$jK = -26.6$ kips.

Kk Wheel 1 at L $Kk = +6.7$ kips.

There is no live-load tension in either lm or Mm.

The maximum and minimum live-load stresses in the web members are summarized in the following table:

Member	Live-load, maximum	Live-load, minimum
Aa	−127.3	0
Bb	−111.0	0
Ab	+ 77.1	0
ab	−572.0	0
bC	−544.7	+ 6.7
Cd	+488.2	− 26.6
de	+381.7	− 30.0
Ee	−274.6	+ 21.6
Ef	+284.1	−101.2
fg	+227.0	−104.6
Gg	−182.7	0

Chord Stresses

AC The stress in *AC* is equal to the horizontal component of *Ab* and is a maximum when *Bb* is a maximum.

$$AC = \frac{111.0}{2} \times \frac{48}{50} = -53.2 \text{ kips.}$$

CE Wheel 11 at *D* $P_1 = 174$ $P_2 = 54 - 84$

$P_1 + 2P_2 = 282 - 342$

$k = 4$ $l = 12$ $W = 939$ $\frac{k}{l}W = 313$

$$M_E = \frac{141{,}253 \times 4}{12} - 16{,}224 = 30{,}860$$

$$M_{D'} = \frac{4118}{2} - 842 = 1217$$

$$CE = \frac{M_E + 2M_{D'}}{d} = \frac{30{,}860 + 2 \times 1217}{50} = -641.5 \text{ kips.}$$

EG Wheel 13 at *F* $EG = -730.7$ kips

$ac = ce$ Wheel 7 at *C* $ac = \left[\frac{155{,}676}{6} - 3433\right] \div 50 =$

$+454.3$ kips.

eg Wheel 13 at *E* $eg = +626.0$ kips.

104. The Baltimore Truss with Counters. In the truss of Fig. 175, the stresses due to dead loads prevent a reversal of stress occurring in any of the members of the four panels on either end of the truss. The diagonal *Eg*, however, receives a large compressive stress with the live load on the left. If it is desired to design the diagonals for tension only, another member *fe* must be placed in the truss. With the live load on the left half of the truss, this member, together with the sub-diagonal *fG*, acts as the main diagonal of the double panel, receiving a tensile stress, while *Ef* acts as the subdiagonal and also receives a tensile stress. Under these conditions there will be no stress in *fg*.

Considering the symmetrical portion of the truss on the right, in order to bring the load on from the right, the active members of the truss are shown in Fig. 176.

The maximum stress in the member $Gh(= fG)$ occurs with wheel 2 at *H*, the position for maximum stress and the stress itself being determined as for a Pratt truss. Its value is $+153.0$ kips.

The maximum stress in the member $hi(= ef)$ is computed by the method used for de. With wheel 3 at I, $P_1 = 0\,P_2 = 45 - 75$

$$n\frac{(P_1 + P_2)}{2} = 270 - 450 \text{ and } W = 426.$$

$$M'_M = 24{,}546,\ M'_I = 345 \text{ and}$$

$$hi = \left[\frac{24{,}546 - 6 \times 345}{288}\right] 1.39 = +108.5 \text{ kips}$$

In all of the other members of the truss, the stresses produced by the loading causing the counters to act, are of the same sign as their maximum stresses but smaller in amount. Hence such stresses are unimportant.

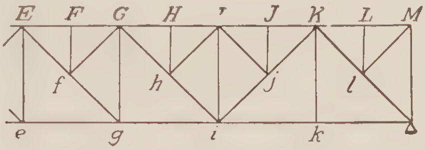


FIG. 176.

Therefore, in the truss with counters the only important differences from the truss without counters are: (1) The maximum live-load stress in fG is $+153.0$ kips instead of $+77.1$ kips; (2) the minimum live-load stress in fg is 0 instead of -104.6 kips; (3) the minimum live-load stress in Ef is 0 instead of -101.2 kips; and (4) the maximum stress in the counter ef is $+108.5$ kips.

105. Impact. In previous articles, the live load has been considered as if it were sliding smoothly over a horizontal surface, and stresses have been computed for the load at rest in some definite position. In reality the load is often moving at a high velocity, while the track or roadway surface has considerable irregularity even under the most favorable conditions. In structures where the ratio of design live load to dead load is large, the deflection of the bridge causes an additional dynamic action. On railroad bridges unbalanced locomotive drivers and the vibration of the machinery add to the general effect. All these conditions tend to increase the stresses produced by the live load. This increase in stress over that caused by the live load at rest is called the *impact stress* or simply the *impact.*

Many investigations with varying results have been made to determine the amount of this impact stress, and various methods

of calculating its value have been proposed from time to time. In many highway bridge specifications the impact stress is specified as a constant percentage of the live-load stress usually about 25 or 30 per cent. In other highway bridge specifications, the impact effect is determined by an empirical formula based upon actual stress measurements of some one or more investigations. The coefficient of impact as determined by these formulae is usually dependent upon the amount of the live load on the structure and sometimes upon the ratio of live-load to dead-load stresses.

The most widely used impact formula for railway bridges is that of the American Railway Engineering Association, which is

$$I = S\frac{300}{300 + \frac{L^2}{100}}$$

in which I = the impact stress

S = the maximum live-load stress

L = the length in feet of that portion of the span which is loaded to produce the maximum live-load stress in the member.

In applying this formula, when using a system of concentrated loads, it is customary to consider L as equal to the span length in determining the impact stresses in the chords and end posts, and equal to two panel lengths in determining the impact stress in a subvertical.

The impact stresses for the through Pratt truss of Fig. 157 are computed and tabulated below.

Member	Loaded length	Impact coefficient	Live-load stress	Impact stress
aB	150	0.571	−315.9	−180.2
Bc	113	0.702	+209.8	+147.2
Cc	88	0.794	− 94.4	− 75.0
Cd	88	0.794	+122.7	+ 97.5
dE	58	0.899	− 58.0	− 52.2
Ee	58	0.899	+ 44.6	+ 40.1
eF	33	0.965	− 16.4	− 15.8
Bb	50	0.924	+113.5	+104.8
ab = *bc*	150	0.571	+202.5	+115.6
cd = *BC*	150	0.571	±311.7	±177.9
CD	150	0.571	−352.8	−201.3

106. Equivalent Uniform Loads. An equivalent uniform load is one so chosen as to produce stresses that will differ as little as possible from those due to the specified concentrated load system. Three methods of obtaining a single equivalent uniform live load have occasionally been used, based, respectively, on the following conditions: first, that the shear in the end panel shall be the same as for the concentrated loads; second, that the bending moment shall be the same at the quarter point of the span; and third, that the bending moment at the center of the span shall be the same. The first method gives positive shears and bending moments in sections near the end of the span agreeing very closely with those calculated from the concentrated loads, but the moments near the center and the negative shears are too great. The second method gives positive shears and bending moments near the end of the span which are too small, while the negative shears and the moments near the center are very nearly the same as those obtained with the concentrated loads. The third method gives results too low except for the center portion of the span. Hence, it is evident that no single equivalent uniform load can be selected which will produce exact results for any span.

Exact results may be obtained for the moment at any section by computing the maximum moment due to the concentrated loads, equating this to the moment caused by a uniform load of w per foot, and solving for w. Each moment center will have a different value of w. Exact equivalent uniform live loads for shear may be obtained in a similar manner.

Equivalent loads obtained in this way are of no assistance in the calculation of the stresses in one span, as the concentrated load stresses are required as a basis for their determination.

For a loading frequently used, such as Cooper's, a chart constructed for a wide range of span lengths will be found very useful in determining stresses or in checking stresses calculated with the moment tables. For the simpler trusses, besides furnishing an alternative method for all stresses, it greatly shortens the computations for bridges on curves, while for the more complex structures it furnishes practically the only analytical method of attack that is warranted, when the amount of labor involved and the degree of accuracy obtainable are taken into consideration. In many of these latter cases exact results may be obtained, while in the others so close an approximation is possible that the error does not affect the design.

The most desirable form for a chart of equivalent uniform loads is one which is based on the lengths of the segments of the influence line triangle. Such a chart is directly applicable to all cases where the influence line of the function, or that part of the influence line corresponding with the loaded length, is triangular, and close approximations may be made for other cases. Figure 177[4] is a chart of equivalent uniform loads for Cooper's E-60 loading, based on the lengths of these segments, which may be used as an alternative to the moment tables in computing the stresses in any simple span.

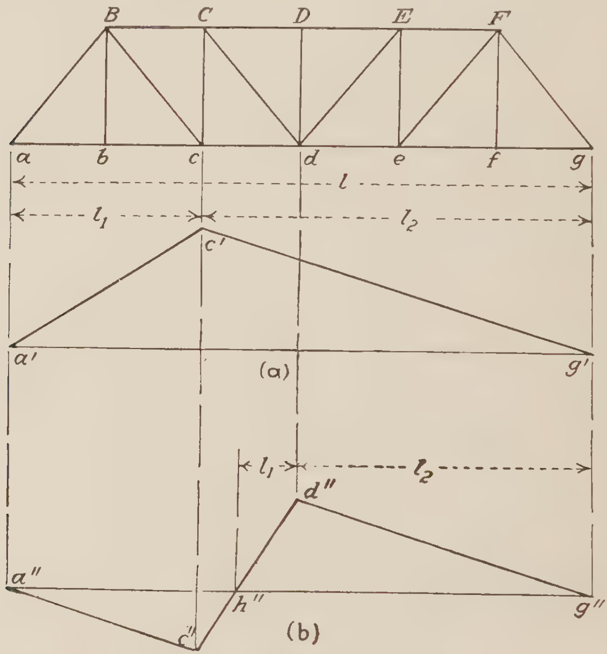
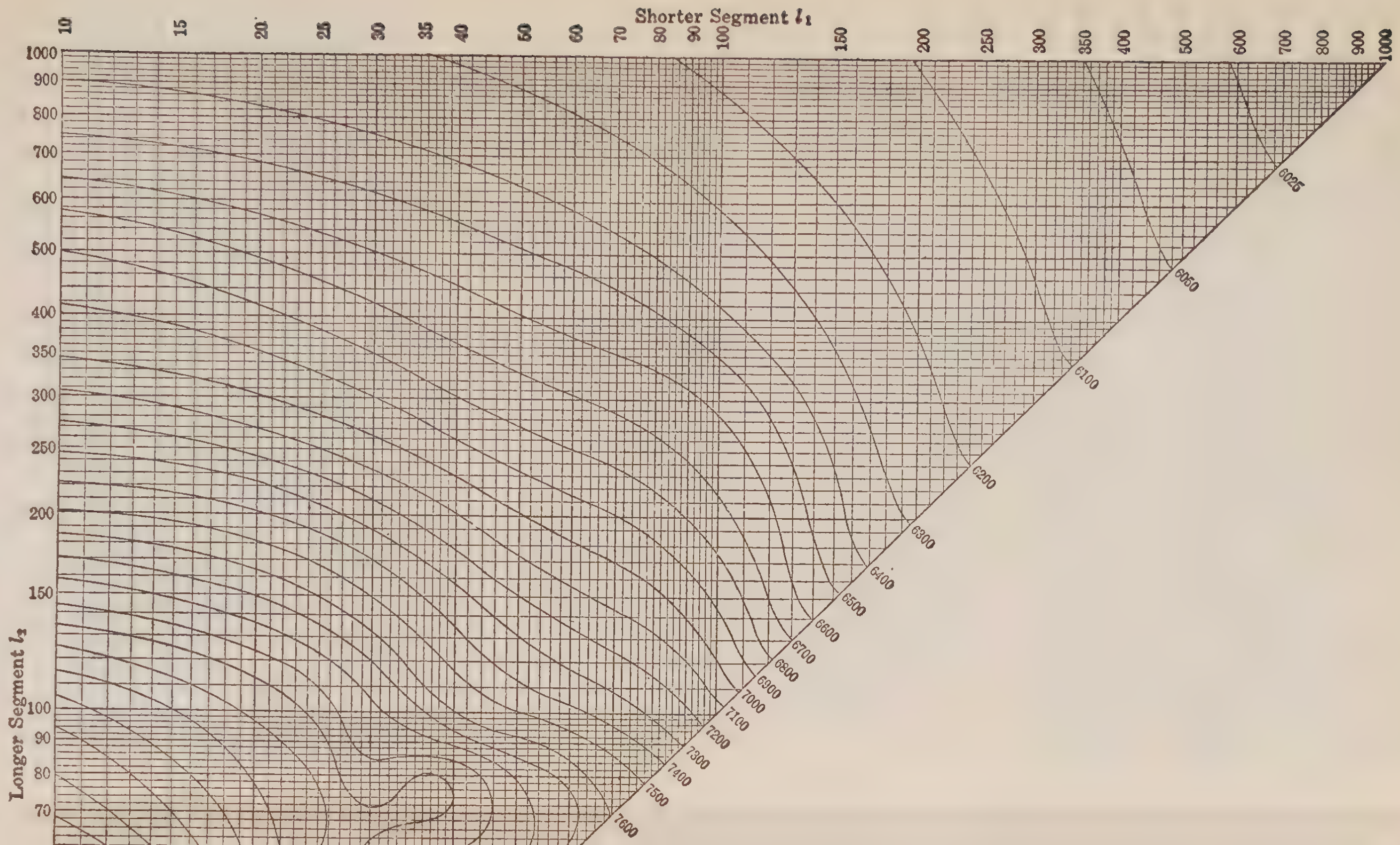


Fig. 178.

Its application to a simple truss with parallel chords is shown as follows: Fig. 178(a) is the influence line for moment at the point c in the span l; Fig. 178(b) is the influence line for shear in the panel cd of the same span. The portion $h''g''$ is loaded in order to obtain the maximum shear. The influence line for shear for the portion of the span $h''g''$ is similar to the influence line for moment at d in a span of length $h''g''$. From Art. 69,

[4] From *Locomotive Loadings for Railway Bridges*, by D. B. Steinman in *Transactions* A. S. C. E., Vol. 86, page 606.



Shorter Segment l_1
10
15
20
25
30
35
40
50
60
70
80
90
100
150
200
250
300
350
400
500
600
700
800
900
1000
Longer Segment l_2
1000
900
800
700
600
500
400
350
300
250
200
150
100
90
80
70
6025
6050
6100
6200
6300
6400
6500
6600
6700
6800
6900
7000
7100
7200
7300
7400
7500
7600

$x = \dfrac{p(n-m)}{n-1}$, or l_1 of Fig. 178(*b*) $= \dfrac{l_2}{n-1}$. For both moment and shear the exact equivalent uniform live load is dependent only upon the lengths of the shorter (l_1) and the longer (l_2) segments of the influence line triangle. In determining maximum floor-beam reactions l_1 and l_2 are the lengths of the adjacent panels.

In a truss with an inclined chord, the value of l_1 is more easily determined graphically. In Fig. 179 let it be required to determine the distance l_1 in order to obtain from Fig. 177 the equiva-

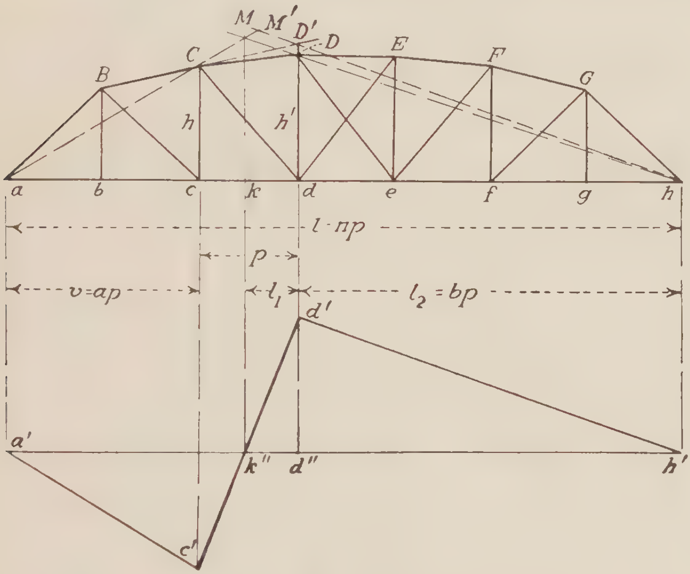


Fig. 179.

lent uniform load for the stress in the diagonal *Cd*. The lower portion of the figure is the influence line for the stress in the member *Cd*, obtained by computing the stress for a load of unity at *c* and again at *d*. The horizontal distance $k''d''$ is l_1. It may be determined graphically as follows: The lines *aC* and *hD* are drawn to their intersection at *M*, which is vertically over the point k''. The figure *aMh* represents to some scale the moment diagram for a load at the point *k*. Each ordinate of the diagram therefore represents to the same scale the moment at that particular point in the span of the truss caused by a load at the point *k*. At *c* the ordinate *Cc* or *h* represents the moment at the

joint C and the stress in cd is $M_C \div Cc$. Similarly at the joint d, the moment is represented by Dd or h' and the horizontal component of the stress in CD is $M_d \div Dd$. These two expressions are equal and opposite so that for this loading the diagonal Cd has no horizontal component and hence no stress.[5]

The analytical determination of l_1 is as follows: Referring to Fig. 179, by similar triangles

$$\frac{h'}{l_2} = \frac{Mk}{l_1 + l_2}$$

and

$$\frac{Mk}{p + v - l_1} = \frac{h}{v}$$

Reducing and solving for l_1

$$l_1 = \frac{(hp + hv - h'v)l_2}{hl_2 + h'v}$$

By placing $\frac{h'}{h} = r$ and substituting for v and l_2 in terms of the panel length p

$$l_1 = \left(\frac{n}{ar + b} - 1\right) l_2$$

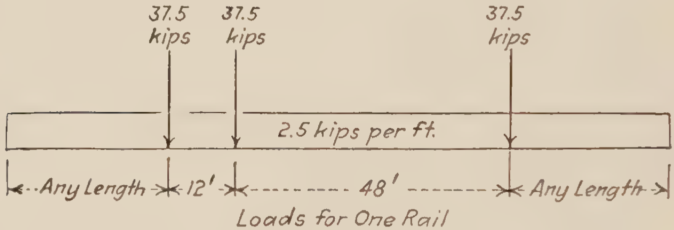


FIG. 180.

Another type of equivalent load consists of a uniform load usually equal to the train load and one or more excess concentrated loads. When more than one excess load is specified, the distance between them is fixed. Such a simplified loading can be obtained by a sufficient number of trials which will produce stresses very nearly the same throughout the span as the wheel-load concentrations. Once determined, it is comparatively simple in application and allows rapid stress calculation. The

[5] The length l_1 for the vertical Cc may be obtained graphically from the same figure. BC is produced to intersect Dd produced at D'. aC and hD' are produced to intersect at M' which is vertically above the point where the influence line for Cc crosses the axis $a'h'$.

simplified loading of this type, proposed by D. B. Steinman in the paper previously mentioned as an alternative for his M-50 loading, is shown in Fig. 180.

107. Equivalent Uniform Load on the Pratt Truss. In order to show the method of applying equivalent uniform loads, as obtained from Fig. 177, to simple trusses, the stresses in the

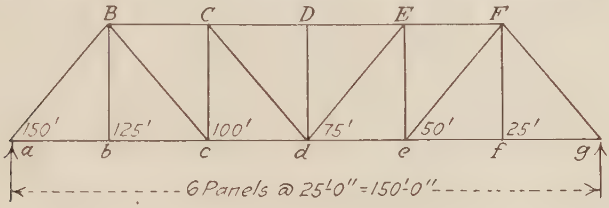


FIG. 181.

Pratt truss of Fig. 181 are computed below. The corresponding stresses, calculated with the aid of the moment tables, are given in Art. 94.

WEB MEMBERS

Member	l_1	l_2	Uniform load per foot per rail	Moment at right support	Moment at right end of panel	Shear	Stress
aB	25	125	3890	43,760	1,216	243.1	−316.0
Bc	20	100	4040	29,090	808	161.6	+210.1
Cc	15	75	4180	16,930	470	94.1	− 94.1
Cd	15	75	4180	......	.....	94.1	+122.3
dE	10	50	4500	8,100	225	45.0	− 58.5
Ec	10	50	4500	......	.....	45.0	+ 45.0
eF	5	25	5600	2,520	70	14.0	− 18.2
Bb	25	25	4560	$\frac{4.560 \times 50^2 - 2 \times 4.560 \times 25^2}{2 \times 25} = +114.0$ kips			

CHORD MEMBERS

Member	l_1	l_2	Uniform load per foot per rail	Moment at right support	Moment of loads to left of section	Bending moment	Stress
$ab = bc$	25	125	3890	43,760	1,216	6,077	+202.6
$cd = BC$	50	100	3750	42,190	4,688	9,379	±312.6
CD	75	75	3770	42,410	10,603	10,602	−353.4

A comparison of the stresses computed above with those obtained from the moment tables shows a very close agreement throughout. Only one stress, the compressive stress of the diagonal in the second panel, shows a variation of more than 1 per cent, while the majority of the stresses show variations considerably less than that amount. These slight errors are caused, partly by the impossibility of making a perfect chart, and partly because of the difficulty in selecting the exact value from the chart. It is interesting to note that the stress in the chord $cd = BC$ is more nearly in agreement with the value obtained on the right-hand portion of the truss with the moment tables than with the corresponding apparent maximum stress obtained in the usual manner in the left-hand portion.

CHAPTER IX

LATERAL FORCES ON BRIDGE TRUSSES

108. In addition to the effect of the live and dead loads, a bridge structure is subjected to lateral forces due to wind pressure and the vibration caused by the impact of the moving loads. For railroad bridges on curves, the centrifugal force developed, together with the usual elevation of the outer rails, causes additional stresses. These stresses are resisted by the lateral trusses (see Fig. 79) placed between the chords of the main vertical trusses, and the portal and sway bracings, placed between each pair of end posts and verticals, respectively. When the chords of the main trusses are horizontal, the lateral trusses are horizontal trusses, the chords of which are the chords of the main trusses. The sway bracing lies in a vertical plane and the portal bracing in the plane of the end posts.

109. Forms of Lateral Trusses. The type of truss used for the lateral trusses depends upon the size and type of the main structure. In short spans where the diagonal length is not great, a bracing of the Warren type is often used. This same type, too, is usually found in deck-plate girder bridges, where often only one set of laterals (the upper) is used, and the lower flange is stiffened by the transverse bracing or cross-frames. The usual type of lateral truss for through bridges is illustrated in Fig. 182. The upper lateral system is shown in Fig. 182(a) and the lower lateral system in Fig. 182(c). The lateral forces acting upon the upper lateral truss are carried by that truss to the points BB' and FF' and taken thence through the portal bracing and the end posts shown in Fig. 182(d) to the abutments of the bridge at aa' and gg'. In a through truss, this compels a portion of the end posts to act as beams, as the portal bracing cannot extend to their lower extremities. Loads on the lower lateral systems are carried directly to the abutments at aa' and gg'.

The diagonals are usually made rigid, *i.e.*, capable of resisting both compressive and tensile stresses, and the lateral shear in each panel is assumed to be distributed equally to the two diag-

onals. If the unsupported diagonal length is long, it is often better to consider the diagonals as resisting tensile stresses only.

In the upper lateral system of a through bridge, the upper member of the sway bracing (CC' of Fig. 182(e)) acts as the lateral strut, while in the lower lateral system the floor beams perform this function.

In the smaller and lighter trusses, the sway bracing is often omitted, the lateral strut being the only transverse member between the intermediate panel points.

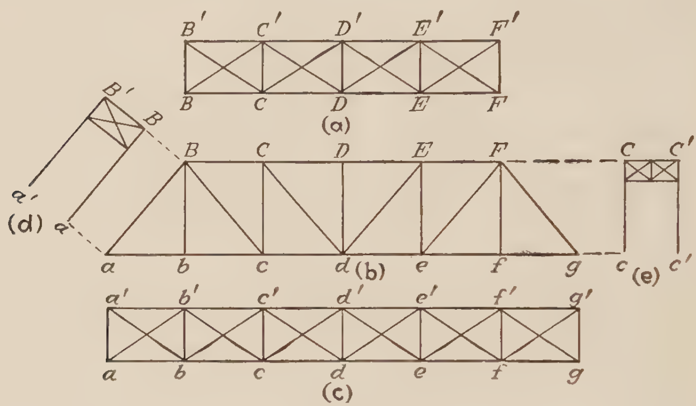


FIG. 182.

110. Lateral Forces. The wind is regarded as blowing horizontally at right angles to the structure, and exerting a pressure of about 50 lb. per square foot on the unloaded bridge, the exposed area being taken as the exposed surface of the trusses and floor system, as seen in elevation. When the bridge is loaded, this pressure is reduced to 30 lb. per square foot on the loaded structure and its load. The pressure acting upon the live load is always considered as a moving load, but specifications differ as to whether the pressure on the structure itself is to be considered as a moving or a static load.

The pressure upon the upper half of the truss is assumed to be transferred to the upper laterals, and that upon the lower half to the lower laterals. In each case the load is transferred at the panel points. The pressure upon the live load is assumed transferred to the lateral system of the loaded chord.

On account of the vibration caused by the impact of the moving load, it is usual to specify lateral forces considerably

greater than would be obtained by the use of the unit pressures mentioned above in order to secure the desired rigidity. The American Railway Engineering Association specifies a moving load of 30 lb. per square foot on one and one-half times the vertical projection of the structure on a plane parallel with its axis, which is in no case to be less than 200 lb. per linear foot at the loaded chord and 150 lb. per linear foot at the unloaded chord. The wind force on the train is specified as a moving load of 300 lb. per linear foot on one track, applied 8 ft. above the base of the rail. In addition, in order to provide for the effect of the sway of the engines and train, this specification requires that all spans shall be designed for a moving load equal to 5 per cent of the specified live load on one track, but not more than 400 lb. per linear foot, applied at the base of the rail.

111. Stresses in Lateral Trusses. With the load per foot known, the panel loads are computed and the stresses determined by the methods of Chapter V. The loads may be assumed as all applied on the windward side, or may be assumed applied equally on the two sides. In the former case the stresses in the lateral struts are one-half panel load greater than if the latter assumption were made, but this is of no practical consequence. Where the diagonals are considered as tension members only, counter stresses need not be computed, as the reversal of the direction of the wind gives greater stresses in the members concerned than any partial loading from the opposite direction. Where a rigid system of diagonals is used, the two diagonals of a panel may be assumed to be equally stressed. The chord stresses of the lateral trusses must be combined with the stresses in the chords of the main trusses due to dead and live loads.

Since the stresses in the lateral system of the loaded chord are caused both by the effect of the wind on the truss and the wind on the live load, and the latter cannot occur except when the live load covers the structure, the actual determination of stresses is the more easily obtained by using the *method of coefficients.*

In this method all panel loads are assumed equal to unity and so placed on the truss as to produce the maximum stresses in the respective members. The coefficients of the web members are numerically equal to their respective shears. The actual stresses in the lateral struts are obtained by multiplying the coefficient by the computed panel load, and in the diagonals by the panel

load times sec θ where θ is the angle between the diagonal and the lateral strut. The coefficient of each chord member equals the sum of the coefficients[1] of all the diagonals on its left when a section is passed cutting the chord member and those diagonals. The chord stress is obtained by multiplying the coefficient by the panel load times tan θ. The coefficients for the lateral systems of a six-panel through Pratt truss are shown in Fig. 183, those on the left being for a tension system, and those on the right for a rigid system. The signs of the coefficients indicate the signs of the stresses with the wind in the direction shown. With the direction of the wind reversed, the other diagonals (not shown) of Fig. 183(*a*) have stresses equal to those shown on the diagram, and the chord stresses of each panel are interchanged. In Fig. 183(*b*) a reversal of the direction of the wind causes a change of sign of stress in every member of the system. In all cases one-half of the load is considered applied on the windward and one-half on the leeward truss.

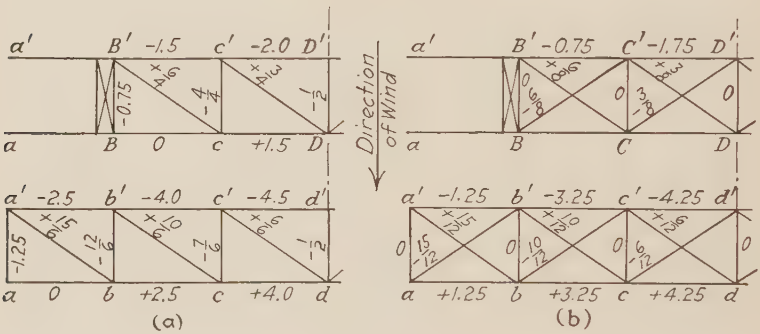


FIG. 183.

In bridges whose trusses have inclined chords, the lateral system of the chords which are inclined lies in several planes, and the exact determination of all the wind stresses is rather difficult. The stresses in the lateral members, however, may be determined by considering the truss flattened out into one plane. The panel lengths will vary, but the panel loads will be equal and may be determined from the horizontal panel length. The resulting chord stresses are not the exact stresses, but the error is not great enough to be of any importance.

[1] These coefficients are not the maximum coefficients, but those obtained with live loads over the whole structure.

As an example of the determination of the stresses in the lateral system, the stresses will be computed for a rigid lateral system for the six-panel through Pratt truss bridge, whose dead- and live-load stresses were determined in Chapters V and VIII. The span is 150 ft., the depth of trusses 30 ft., and the distance between trusses 17 ft. 6 in. Using the specifications of the American Railway Engineering Association, given in Art. 110, with Cooper's E-60 loading, the panel load on the upper lateral system is $25 \times 150 = 3750$ lb. $= 3.75$ kips. For the lower lateral system the

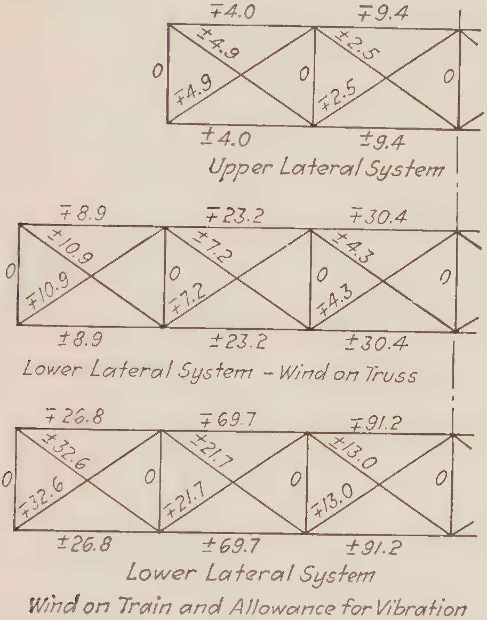


FIG. 184.

panel load due to the wind on the structure is $25 \times 200 = 5000$ lb. $= 5.0$ kips. Sec $\theta = 1.74$ and tan $\theta = 1.43$. Using the coefficients of Fig. 183 the stresses are computed and are shown on the diagrams of Fig. 184.

112. Stresses in Main Trusses Due to Lateral Forces. Since some of the lateral forces are applied considerably above the horizontal plane of the end supports of the bridge, these forces tend to overturn the structure.

The lateral forces of the upper lateral system are carried to the portal struts, and the horizontal loads at these points pro-

duce an overturning moment about the horizontal plane of the supports. In Fig. 185, P represents the horizontal load brought to each portal strut by the upper lateral bracing, h the

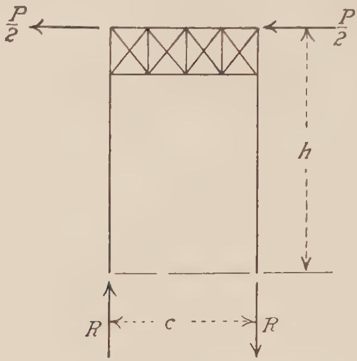


Fig. 185.

depth of the truss, and c the distance between trusses. The overturning moment produced at each end of the structure is Ph, which is balanced by a reaction couple Rc. The value

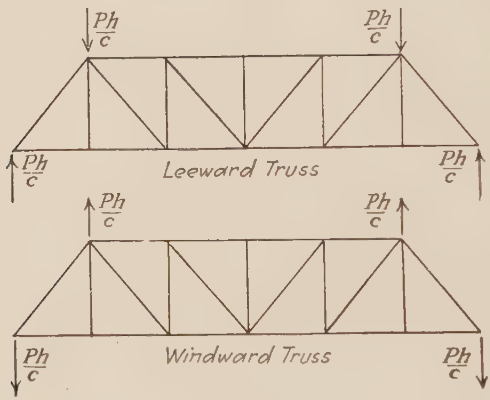


Fig. 186.

of the reaction R is then $\frac{Ph}{c}$, and the same effect is produced on the main trusses as is caused by the wind on the truss if loads equal to $\frac{Ph}{c}$ are applied to the main trusses at the panel points B and F as shown in Fig. 186. These loads produce stresses in

the end posts and in the chord members, but the web members are not stressed.

The wind force on the train is considered to be applied some distance above the base of the rail, usually 7 or 8 ft., and the base of the rail in a through truss is usually several feet above the plane of the end supports. Hence another overturning moment is produced by these forces, which may be treated in a similar manner to that caused by the wind on the upper lateral system. There is a difference, however, as far as the web members of the main trusses are concerned. Since the wind on the train produces an effect corresponding to the position of the train load on the bridge, it is necessary to determine equivalent vertical panel

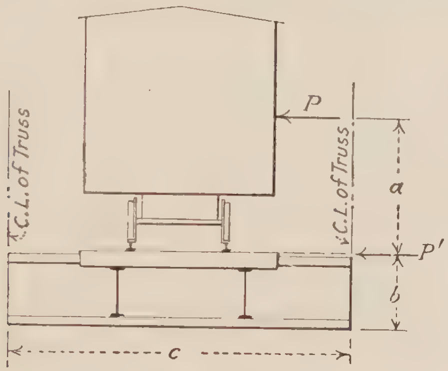


FIG. 187.

loads rather than equivalent reactions as in the previous case. From Fig. 187 it can be readily seen that the equivalent vertical panel load is $\frac{P(a + b)}{c}$, where P is the horizontal panel load due to wind on the train. The stresses produced are then computed by the methods of Chapter VI.

The effect of the lateral force specified to compensate for the sway of the engines and train is determined in a similar manner, the equivalent vertical panel load being $\frac{P'b}{c}$, where P' is the horizontal panel load.

The two equivalent vertical panel loads $P\frac{(a + b)}{c}$ and $\frac{P'b}{c}$ may be added together and treated as one panel load, as the two lateral forces from which they are obtained act simultaneously.

Applying the above principles to the six-panel through Pratt Truss bridge, the stresses for the lateral systems of which were computed in Art. 111, the equivalent vertical loads and reactions producing the same effect as the wind on the upper lateral system are

$$\frac{3750 \times 5 \times 30}{2 \times 17.5} = 16{,}100 \text{ lb.} = 16.1 \text{ kips}^2.$$

and the stresses produced in the leeward truss are:

$$\begin{array}{lll} \text{Upper chords} & 16.1 \times 0.83 & = -13.4 \text{ kips}^3. \\ \text{End posts} & 16.1 \times 1.30 & = -20.9 \text{ kips.} \\ \text{Lower chords} & & = +13.4 \text{ kips.} \end{array}$$

The stresses in the corresponding members of the windward truss have opposite signs.

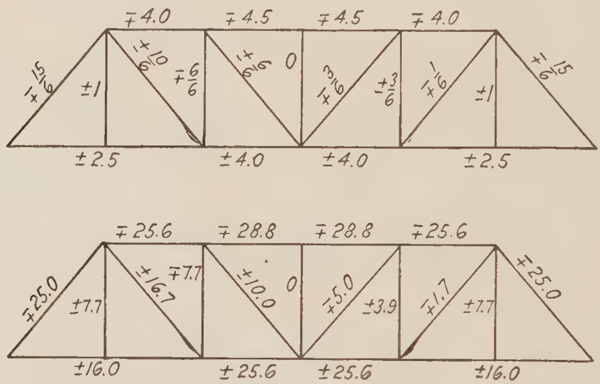


FIG. 188.

From the specifications of Art. 110, the horizontal force on the train per panel is $300 \times 25 = 7500$ and that at the base of rail $0.05 \times 6000 \times 25 = 7500$. The equivalent vertical panel load producing the same effect as these horizontal forces due to the wind on the train and the lateral force specified to allow for the sway of engines and the train is (considering the base of rail 5 ft. above the plane of the end supports):

$$\frac{7500 \times (8 + 5)}{17.5} + \frac{7500 \times 5}{17.5} = 7700 \text{ lb.} = 7.7 \text{ kips.}$$

[2] Considering a full horizontal panel load at BB' and FF' to allow for the wind on the end posts and portal bracing.

[3] See Art. 64.

Treating this panel load as a moving load, the coefficients of the stresses produced are marked on the upper diagram of Fig. 188 and the stresses are given on the lower diagram. The upper sign preceding the values of the coefficients and stresses is for the leeward, and the lower for the windward truss.

113. Stresses Due to Tractive Forces. When a train with its brakes set crosses a bridge, a horizontal force is exerted upon the track through the friction of the braked wheels. The amount of this force is usually taken as 20 per cent of the vertical live load. The American Railway Engineering Association specifies "a longitudinal force of 20 per cent of the live load . . . applied 6 ft. above the top of the rail." The chord members receive a tensile stress when the train comes on to the bridge from the anchored end, and a compressive stress when it approaches from the expansion end. This stress increases uniformly from the free to the fixed end, receiving an increment at each panel point. The effect of this force on the structure is materially reduced by the continuity of the track over the bridge and by the continuity of the span itself, these conditions tending to transfer a part of the force to the roadbed beyond the bridge. The specifications referred to above allow a reduction of 50 per cent in the specified force for the usual conditions.

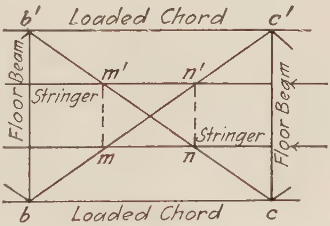


Fig. 189.

With the lateral diagonals (Fig. 189) connected to the stringers at m', m, n', and n, the tractive force is resisted mainly by the bending resistance of the floor beams. The addition of members $m'm$, and $n'n$ forms a truss of the laterals and stringers which resists the tractive force by direct stresses. These stresses are small and unimportant.

In through trusses no stresses of any importance are produced in any of the main truss members except the lower chords, and these are often neglected in design. In a deck bridge supported in the plane of the lower chord, the overturning effect of the traction force, lengthwise of the bridge, is sufficient to produce a change in the vertical reactions, and hence stresses throughout the structure. The vertical distance, however, from the point of application of the tractive force to the plane of the bridge supports, is rarely greater than one-fifth of the span, which

produces a maximum additional reaction of not more than 4 per cent of the reaction for full vertical live load.

114. Bridges on Curves. The spacing of trusses for deck bridges on curves must be increased over that required for a straight track by an amount equal to the midordinate of the curve which is equal to $R - \sqrt{R^2 - \left(\frac{l}{2}\right)^2}$, in which R is the radius of the curve and l the span of the bridge. In through bridges, an additional increase in the spacing must be made to allow for the tilting of the cars due to the superelevation of the outer rail, and for the swing of the portion of the cars overhanging the trucks.

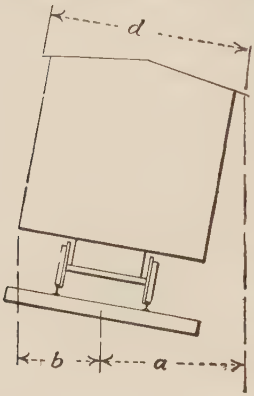


Fig. 190.

The distance $a + b$ (Fig. 190) may be calculated from the car dimensions and from the superelevation of the outer rail. Assume that the clear width between trusses in the horizontal plane of the tops of the cars is the same at the end posts as at the center of the span, and let l_1 equal the distance between the intersections of this plane with the end posts at opposite ends of the span, c_1 the midordinate for the distance l_1, and c_2 the midordinate for the distance equal to the length of a car. Then the

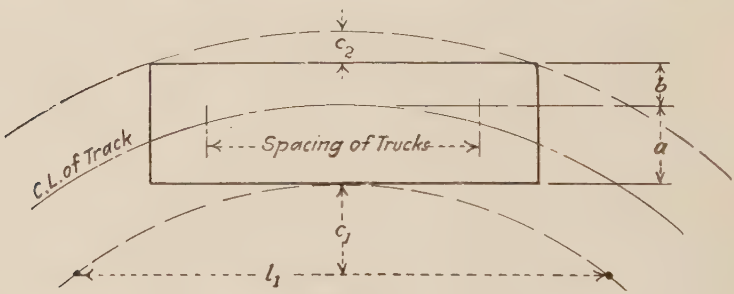


Fig. 191.

total increase in the spacing of trusses required for a through bridge on a curve is $a + b + c_1 + c_2 - d$ (see Fig. 191).

The live load in moving over such a bridge produces additional stresses in its members due (a) to the centrifugal force developed,

and (*b*) to the eccentricity of the load on account of the curvature of the track and the superelevation of the outer rail.

The track is sometimes placed on the structure so that the axis of the bridge bisects the midordinate. In order, however, more nearly to equalize the stresses in the trusses, it is the usual practice to place the track somewhat nearer to the inner truss than this position would locate it.

115. Stresses Due to Centrifugal Force. The amount of centrifugal force developed by a load P moving in a curve of radius R is

$$F = \frac{v^2}{gR} P$$

in which g = the acceleration of gravity, and v = the velocity of the load. Expressing both v and g in miles per hour, and R in terms of the degree of curvature D,

$$F = \frac{v^2}{\dfrac{32.2 \times \overline{60}^4}{5280} \times \dfrac{5730}{D \times 5280}} P$$
$$= 0.0000117 v^2 D P$$

The centrifugal force F may be obtained from the above equation in terms of the live load P.

The American Railway Engineering Association specifies centrifugal force as follows:

On curves, the centrifugal force (assumed to act 6 ft. above the rail) shall be taken equal to a percentage of the live load including impact according to the following table:

Degree of curve	0 deg.	0 deg.	1 deg.	2 deg.	3 deg.	4 deg.	5 deg.	6 deg.	7 deg.	8 deg.	9 deg.	10 deg.	11 deg.	12 deg.
	20′	40′												
Percentage	2½	5	7½	10	10	10	10	10	10	10	10	10	10	10
Speed in miles per hour	80	80	80	65	53	46	41	38	35	33	31	29	28	27

These values are somewhat less than those determined by the equation developed above.

The effect of the centrifugal force is similar to any other lateral force acting on the moving load. It stresses the lateral system of the loaded chords, and it causes an overturning effect, which produces stresses in the main trusses.

The stresses produced in the lateral truss between the loaded chords of the bridge are computed as for a vertical truss for a series of loads each equal[4] to K times (or a specified percentage of) the vertical loads for *both* rails. If the vertical loads are a system of concentrated loads, the horizontal force is a similar system of concentrated loads, while if the vertical load is equivalent uniform load, the horizontal load will likewise be a uniform load.

In the main trusses, the equivalent vertical loads are determined as in Art. 112. Thus the equivalent vertical loads or panel loads are $\frac{F(a + b)}{c}$, in which a is the distance above the base of the rail to the point of application of the centrifugal force, b the distance from the base of the rail to the plane of the end supports of the bridge, and c the distance center to center of trusses. Hence, the equivalent vertical loads are $\frac{K(a + b)}{c}$ times the vertical live loads, and the stresses in the outer truss may be obtained by multiplying the live load plus impact stresses by this ratio. The stresses in the inner truss have the same numerical values but are of opposite sign.

Additional stresses are also developed in the stringers and floor beams. Since a pair of stringers and their lateral system constitute a simple bridge of one panel length, their stresses may be determined in a manner similar to that used for the main trusses and bracing. The portion of the floor beams adjacent to the outer truss has stresses equal to $\frac{K(a + b)}{c}$ times those caused by the vertical loading, due to the overturning effect of the centrifugal force.

116. Stresses Due to Eccentricity. The most convenient method of calculating the stresses in a bridge truss carrying a curved track is to determine the equivalent live load for a straight track and then compute the corrections to the panel loads due to the eccentricity. In determining the correct value of the eccentricity of the center of gravity of the load, the superelevation of the outer rail as well as the degree of curvature must be considered. Also in determining the eccentricity of the load on a floor beam it is better to consider, not the eccentricity of the panel point in question but the average eccentricity for a half-panel length on each side of this joint.

[4] $K = 0.0000117v^2D$.

In Fig. 192, let P represent the total load on any beam and e the average eccentricity for that load. The panel load at b' will be $\frac{P}{2} + P\frac{e}{c}$ and that at b, $\frac{P}{2} - P\frac{e}{c}$. It is only necessary to consider the portions $P\frac{e}{c}$ and $-P\frac{e}{c}$ as these are the amounts by which the loads differ from those produced by a central load. A plus sign indicates a downward load, but, as toward the ends of the bridge e is usually minus, both downward and upward loads may be expected on either truss.

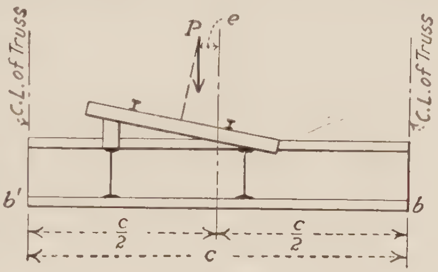


FIG. 192.

After determining the equivalent uniform load, the corresponding panel loads, and the corrections to the panel loads on account of the eccentricity, the same joints are loaded for the respective maximum stresses regardless of whether the individual eccentric loads act downward or upward.

The arrangement of the stringers in a bridge carrying a curved track depends on the degree of the curve. Where the midordinate is small, the spacing of the stringers is slightly increased, and

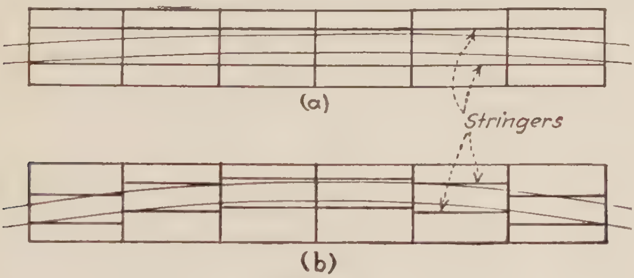


FIG. 193.

they are arranged as shown in Fig. 193(a). When the degree of curvature is large, however, the stringers are usually offset as shown in Fig. 193(b). In calculating the effect of the eccentricity on the stresses in the stringers, the average eccentricity over the length of the stringer is considered. The increase in stress in the outer stringer is then equal to $\frac{2e'}{s'}$ times the stress

computed for a central load, in which e' is the average eccentricity as defined above, and s', the spacing of the stringers.

The variation in the value of the floor-beam reactions is the same as that for the panel loads; that for the outer truss being $+r\frac{e}{c}$ and that for the inner truss $-r\frac{e}{c}$. The eccentricity to be used is the average for the two adjacent panels. In computing the bending moments in the floor beams, the actual spacing and arrangement of the stringers must be considered.

117. Stresses Due to Centrifugal Force and Eccentricity. As an example of the calculations required for bridges on curves, the stresses in the six-panel through Pratt truss of Fig. 194(a) will be computed below. The bridge is located on a 7-deg. curve and the spacing center to center of trusses is 19 ft.-6 in. The stresses for a bridge consisting of two similar trusses spaced 17 ft.-6 in. center to center and carrying a straight track were computed in Art. 94, using Cooper's E-60 loading, and in Art. 107 for an equivalent uniform live load. Figure 194(b) represents the lower laterals and the line of the center of gravity of the train. The eccentricities at the several panel points are marked on the diagram.

Using the specifications of the American Railway Engineering Association, the stresses in the stiff lateral system due to the centrifugal force are obtained as follows: The maximum live load plus impact shears in the three panels on the left half of the main truss are 351.6, 274.6, and 169.4. The shears in the corresponding panels of the lateral system are, therefore, 70.3, 54.9, and 33.9, respectively, and the stresses in the various diagonals as given in Fig. 194(c) are obtained by multiplying these shears by one-half the secant of the angle between the diagonal and the floor beam.

In computing the chord stresses, the effect of the stiff diagonals must be considered. With a tension system of diagonals, the members ab', bc', etc. are in action, and the stress in cd is the largest chord stress. For the stiff system, a reference to Figs. 183(a) and 183(b) shows that the stress in both $a'b'$ and ab is the average of the stresses occurring with a tension system of diagonals. The same relation holds in the other panels. The sum of the live load and impact stresses in $ab = bc$ of the main truss is 318.1 kips. The bending moment at B is $318.1 \times 30 = 9543$ kip ft. Therefore, the bending moment at b' in the lateral

truss is $9543 \times 0.10 \times 2 = 1908$ kip ft., and the stress in $ab = bc$ (tensile diagonals being considered) is 97.9 kips. Similarly, $bc = c'd' =$

$$\frac{489.6 \times 30}{19.5} \times 0.10 \times 2 = 150.7 \text{ kips and } cd = \frac{554.1 \times 30}{19.5}$$

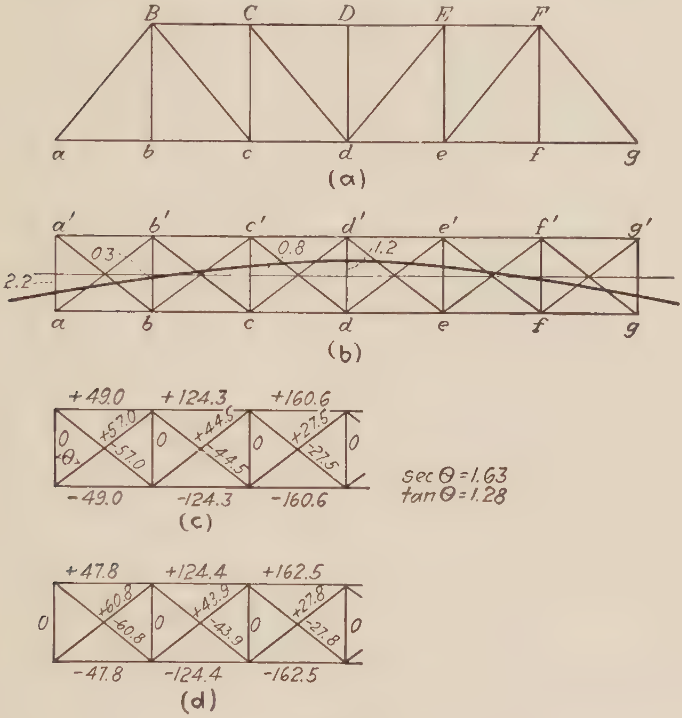


FIG. 194.

$\times 0.10 \times 2 = 170.5$ kips. From these stresses the actual stresses for the stiff system are readily obtained as follows:

First panel.	$(0 + 97.9) \div 2 = 49.0$
Second panel.	$(97.9 + 150.7) \div 2 = 124.3$
Third panel.	$(150.7 + 170.5) \div 2 = 160.6$

These stresses may be obtained just as satisfactorily by selecting an equivalent uniform load, computing the horizontal panel load, and making use of the coefficients of Art. 111 together with the impact coefficients of Art. 105. For this truss an equivalent uniform load of 3800 lb. per foot of rail was used to determine the stresses given on the diagram of Fig. 194(d).

The stresses due to the overturning effect of the centrifugal force are obtained by multiplying the live-load impact stresses by the ratio $\frac{2K(a + b)}{c}$ (Fig. 187) which in this case is

$$\frac{2 \times 0.10(6 + 5)}{19.5} = 0.113.$$

In calculating the stresses due to eccentricity a uniform load of 3800 lb. per foot per rail is taken. This is about the average equivalent load for the chords and end posts and the errors in the stresses in the other members will not be of any importance. For a central load this gives a panel load of 95 kips for one truss, or 190 kips for both trusses. The average eccentricity at b is -0.4 ft., at c, 0.5 ft., and at d, 1.1 ft. The eccentric panel loads on the outer truss are then at b'

$$150 \times \frac{-0.4}{19.5} = -3.1, \text{ at } c', \; 150 \times \frac{0.7}{19.5} = +5.4$$

and at d, $150 \times \frac{1.1}{19.5} = +8.5$. The loads on the right are symmetrical. With these panel loads, the stresses in the outer truss are calculated as follows:[5]

Web Stresses			Chord Stresses			
Member	Shear	Stress	Member	Shear in diagonal	Horizontal component	Stress
aB	10.4	−13.5	ab,bc	10.4	8.6	+ 8.6
Bc	15.5	+20.2	BC,cd	15.2	12.6	∓21.2
Cc	9.9	− 9.9	CD	6.8	5.6	−26.8
Cd	9.9	+12.9				
dE	2.5	− 3.3				
Ee	2.5	+ 2.5				
eF	−1.0	+ 1.3				
Average eccentricity panels ab and bc = −0.5						
$Bb = \frac{2 \times 218.3 \times 0.5}{19.5} = -11.2$						

[5] As the specification used requires impact to be considered, the shears obtained from the above panel loads are multiplied by one plus the impact coefficient as used in Art. 105. For example, the maximum live-load shear in the panel Bc is $(5.4 \times 6 + 8.5 \times 3 - 3.1 \times 1) \div 6 = 9.1$. The live load plus impact shear is $9.1 \times 1.702 = 15.5$.

The complete tabulation of the stresses in the outer truss for live load, impact, centrifugal force, and eccentricity is given in the accompanying table.

The stresses in the inner truss due to centrifugal force and eccentricity are all of the opposite sign from those given above for the outer truss. With zero velocity there would be only stresses due to eccentricity to be considered, but as the impact stresses are much greater than those due to the overturning effect, the maximum stresses in the inner truss usually occur under normal conditions.

In the truss just analyzed, the stresses in the outer truss are all greater than those for the corresponding members of the inner truss. This is the usual condition, except where the eccentricity at the first panel point is very large, in which case the member Bb may have a larger stress in the inner truss.

Member	BC	CD	ab	bc	cd	aB	Bb	Bc	Cc	Cd	dE	Ee	eF
Live load + Impact	−489.6	−554.1	+318.1	+318.1	+489.6	−496.1	+218.3	+357.0	−169.4	+220.2	−110.2	+84.7	−32.2
From laterals	......	......	+ 49.0	+124.3	+160.6								
From overturning effect	− 55.4	− 62.6	+ 36.0	+ 36.0	+ 55.4	− 56.1	+ 24.7	+ 40.4	− 19.1	+ 24.9	− 12.5	+ 9.6	− 3.6
From eccentricity	− 21.2	− 26.8	+ 8.6	+ 8.6	+ 21.2	− 13.5	− 11.2	+ 20.2	− 9.9	+ 12.9	− 3.3	+ 2.5	+ 1.3
Total	−566.2	−643.5	+411.7	+487.0	+726.8	−565.7	+231.8	+417.6	−198.4	+258.0	−126.0	+96.8	−34.5

118. Portal Bracing. The chief function of the portal bracing is to transfer the reactions of the upper lateral system to the supports of the bridge. In addition, it stiffens the end posts against vibration. In order to keep the flexure in the end posts as small as possible, the portal framing should extend as low as the headroom will permit.

Where the headroom required is nearly equal to the depth of the trusses, the portal bracing may consist merely of a portal strut and two brackets or knee braces, as illustrated in Fig. 195(*a*), or of a shallow plate girder as shown in Fig. 195(*b*). Where more depth can be used, portal framing of the types shown in Fig. 195(*c*) or Fig. 195(*d*) may be used. The former is a simple but very effective type of framing. In bracing of the latter type the number of pairs of diagonals may vary from one

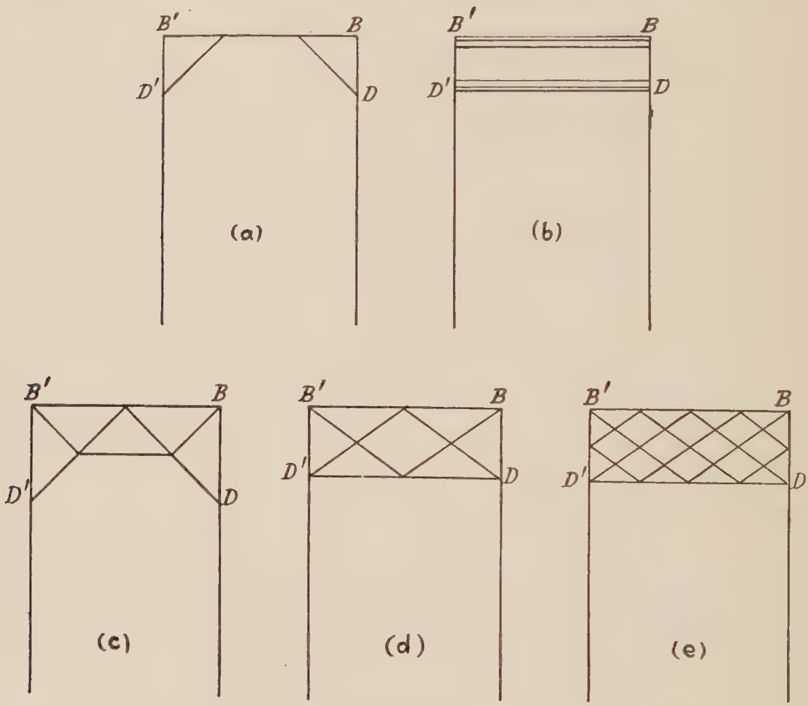


FIG. 195.

to eight depending upon the depth available. Another type often used is illustrated in Fig. 195(*e*). Here the lattice work performs the same function as the web of the plate girder portal of Fig. 195(*b*).

The bases of the end posts are partially fixed by virtue of the direct stress due to the vertical load. This prevents their tipping on the pins of the end supports. The exact location of the point of inflection in the end posts cannot be determined until the design has been made and the distance center to center of pin

bearings of each end post is known. In Fig. 196, let S represent the dead-load stress in the end post, c' the distance between the centers of the pin bearings at the end post, and H' the horizontal reaction on the pin caused by the horizontal wind-load forces. Taking moments about any point o, the distance x_0 of the point of inflection i above the pin bearing is $\frac{Sc'}{2H'}$.

In determining the stresses in the portal bracing, the first assumption may be that the bases are hinged and this later corrected, when the position of the point of inflection is known; or from previous designs a close approximation of the distance c' may be made and the stresses computed accordingly. The first method will be used in the analyses to follow.

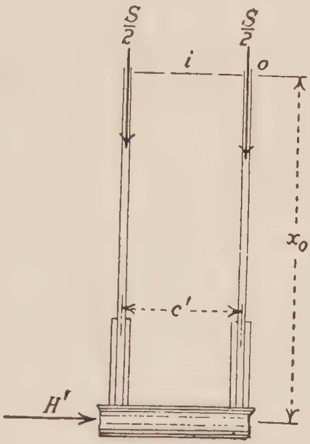


FIG. 196.

119. The Portal with Knee Braces. In Fig. 197, let P represent the total force, including the end panel loads, brought to the end of the upper lateral truss. Assuming that the horizontal resistance to this force P is provided equally by each of the end supports, $H = H' = \frac{P}{2}$. Taking moments about a and a', $V' = \frac{Ph}{c}$ and $V = -\frac{Ph}{c}$, respectively. Assuming the joint B to be hinged, the moment at this point is zero. Therefore, with the section s-s' taken through B, summation of moments about B, gives $CD_H \times d - Hh = 0$, from which $CD_H = \frac{Hh}{d}$ and

$$\text{the tension in } CD = \frac{Hh}{d} \cdot \frac{f}{e} = \frac{Phf}{2de} \qquad (a)$$

In $C'D'$ an equal and opposite compressive stress will exist.

The bending moments in the end posts have their maximum values at D and D', respectively, and are

$$H(h - d) = \frac{P(h - d)}{2} \qquad (b)$$

Below D and D' the shear in each post is $H = \frac{P}{2}$, while above, it is

$$\frac{Ph}{2d} - H = \frac{P}{2}\left(\frac{h}{d} - 1\right)$$

In the leeward post the compression below D' is $V' = \frac{Ph}{c}$, while

$$\text{the tension in } B'D' = \frac{Ph}{2e} - \frac{Ph}{c} = \frac{Ph}{2ec}(c - 2e) \qquad (c)$$

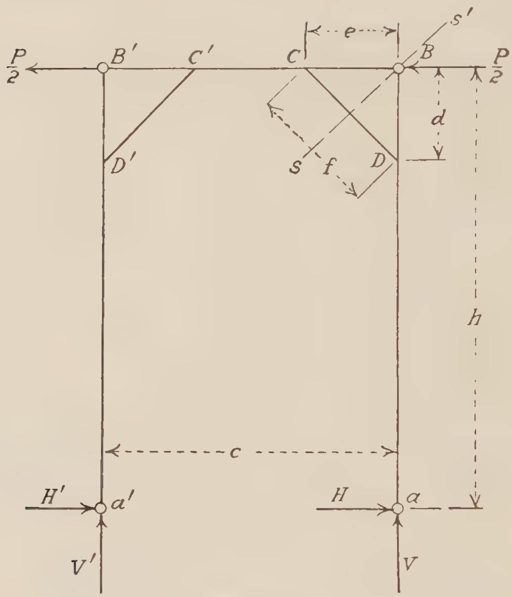


FIG. 197.

Similarly, in the windward post the tension below D' is $\frac{Ph}{c}$ and

$$\text{the compression in } BD = \frac{Ph}{2ec}(c - 2e) \qquad (d)$$

The stresses in the portal strut BB' are as follows: For the portion CC', the direct stress is $\frac{P}{2}$, the shear is V, while the moment at C is

$$V'(c - e) - H'h = Ph\left(\frac{1}{2} - \frac{e}{c}\right) \qquad (e)$$

and that at C' is $-Ph\left(\frac{1}{2} - \frac{e}{c}\right)$. With the center of moments at

D, $\Sigma M = 0$ of the forces to the right of section s-s' gives

$$\text{the compression in } BC = \frac{H(h-d)}{d} + \frac{P}{2} = \frac{Ph}{2d} \qquad (f)$$

and similarly the tension in $B'C'$ is $\frac{Ph}{2d}$. The shear in BC is the difference between the vertical component of CD and $V = \frac{Ph}{2e} - \frac{Ph}{c} = \frac{Ph}{2ec}(c - 2e)$ and in $B'C'$ it is $\frac{Ph}{2ec}(2e - c)$.

With the point of inflection determined as a definite distance x_0, above a, the above solution is modified in that the forces H and V may now be considered applied at the point i. The same method of analysis is applicable, the distance $h - x_0$ being substituted for h in all of the foregoing equations. It can readily be seen that under these conditions the stresses in the portals are greatly reduced.

The stresses in the end posts determined above must be added to those obtained for dead load, live load, etc., in obtaining the maximum stresses. The maximum stress occurs in the leeward post at D', where, on the inner side of the post, the stresses due to dead load, live load, wind load, and bending moment are all compressive.

120. The Plate Girder Portal. In this type of portal, as illustrated in Fig. 198(a), the values of H and V are determined as in Art. 119. The bending moments are maximum at D and D' and of the same value, as are the direct stresses and shears in the end posts below D and D'.

In the girder itself, the stresses in the flanges and the shear in the web are determined as follows: Consider the section shown in Fig. 198(b). Taking moments in the line of action of F_1, at any distance x from the leeward post, and assuming all of the moment taken by the flanges,

$$V'x - H'h - F_2 d = 0$$

from which the stress in the lower flange of the girder F_2 is

$$\frac{P\left(\frac{hx}{c} - \frac{h}{2}\right)}{d}$$

For $x = 0$, $F_2 = -\frac{Ph}{2d}$; for $x = \frac{c}{2}$, $F_2 = 0$; and for $x = c$, $F_2 = \frac{Ph}{2d}$.

Similarly, for the upper flange

$$V'x - H'(h - d) - F_1 d - \frac{Pd}{2} = 0$$

from which $F_1 = P\left(\frac{hx}{c} - \frac{h}{2}\right)$; and for $x = 0, F_1 = +\frac{Ph}{2d}$; for $x = \frac{c}{2}$, $F_1 = 0$; and for $x = c, F_1 = -\frac{Ph}{2d}$.

Thus the girder has a point of inflection at its center when both of the flange stresses are zero, while at the leeward end the upper flange is in tension, and the lower flange in compression, with

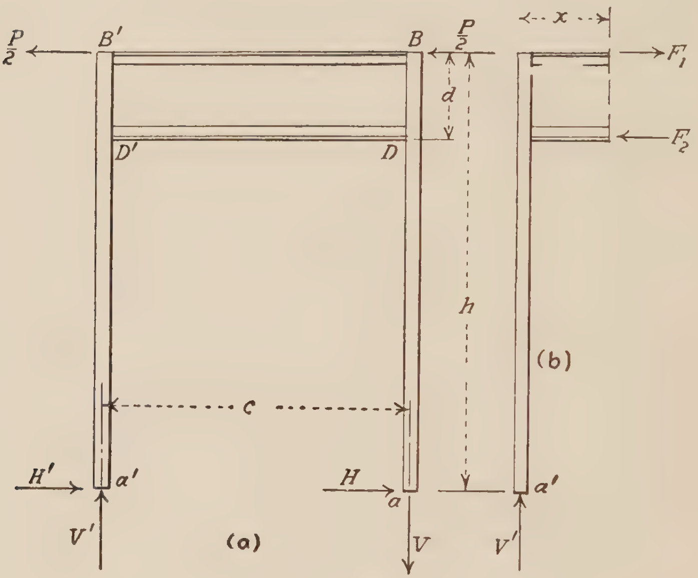


FIG. 198.

conditions reversed at the windward end. Since there are no external loads applied to the girder between B and B' the shear at all points is $V' = \frac{Ph}{c}$.

121. The Portal with Diagonal Bracing. The number of panels in a portal with diagonal bracing, and the character of the stresses which the diagonals are designed to resist, partially control the method of analysis to be used. In the type as represented in Fig. 199, with the diagonals designed for tension only, and the wind force acting in the direction shown, the member

BD' is not acting. With the section s-s' taken as shown, moments about B' give $DD' \times d + H'h = 0$, from which

$$DD' = -\frac{Ph}{2d};$$

moments about D give $B'B \times d + \frac{Pd}{2} + H(h - d) = 0$, from which

$$BB' = -\frac{Ph}{2d}$$

and $\Sigma V = 0$ give $V' - B'D \times \frac{d}{f} = 0$, from which $B'D = \frac{Phf}{cd}$. The stresses in the posts below D and D' are the same as in the previous cases.

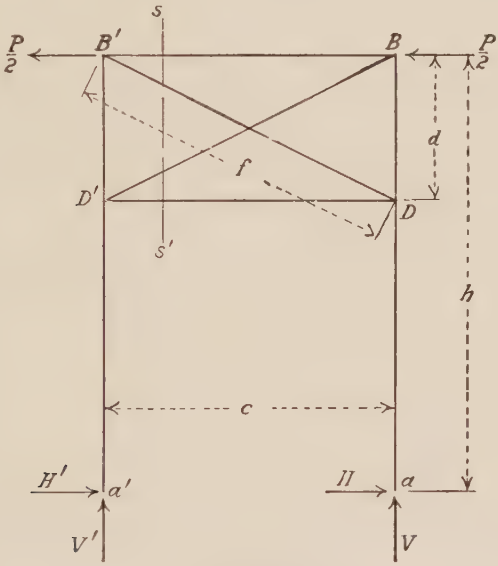


Fig. 199.

If the diagonals are designed to resist either tension or compression and are considered to act simultaneously, their stresses are $\pm \frac{Phf}{2cd}$, while the members BB' and DD' are not stressed.

With a portal of the type shown in Fig. 195(d) the stress in each of the diagonals is $\pm \frac{Phf}{2cd}$, while the members BB' and DD' must be designed for a stress of $\pm \frac{Ph}{4d}$.

The lattice portal, as illustrated in Fig. 195(*e*), may be analyzed for the stresses in the diagonals as in the previous case, each diagonal having a stress of $\pm \frac{Phf}{ncd}$, in which n is the number of diagonals cut by any vertical section. The stresses in the members BB' and DD' are determined as in the plate girder portal, but are in all cases somewhat less than for that type, as, no matter how numerous the diagonals, the distance x is never zero as the section must be taken through some intersection of the diagonals.

122. Stresses in Portal Bracing. For the through Pratt truss of Fig. 97 it is desired to compute the stresses in a portal bracing

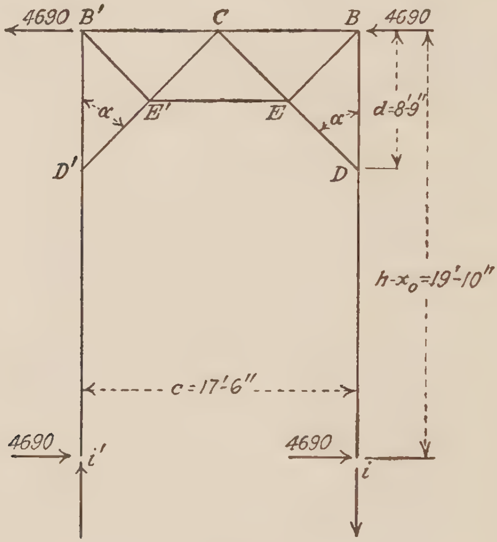


FIG. 200.

of the type shown in Fig. 200. The theoretical length of the end post is 39 ft.-½ in., and the distance between trusses is 17 ft.-6 in. The dead-load stress in the end post is 117,700 lb. and the distance between its centers of pin bearings is assumed as 20 in. For this design it is assumed that the connection details are so arranged that the center line of the member BB' lies in the plane of the center lines of the upper chords. The angles α are each 45 deg. Therefore, $d = \frac{c}{2} = 8$ ft.-9 in.

The distance of the point of inflection above the end pins is $\frac{117{,}700 \times 20}{2 \times 4690} = 230$ in., and the distance $h - x_0$ to be used in computing the stresses in the portal bracing is $39 \times 12 - 230 = 238$ in.

Since the bending moment at the center of the portal strut C is zero $\left(\text{for in equation } (e)\ \frac{e}{c} = \frac{1}{2}\right)$ there is no stress in the member EE', and, therefore, no stresses in the members $B'E'$ and BE. These members are used where the clearance requirements will permit, for the sake of appearance, and to stiffen the members $D'C$ and DC. The portal may then be analyzed by the method outlined in Art. 119.

$$V = V' = \frac{9380 \times 238}{17.5 \times 12} = 10{,}600$$

which is the value of compression in $D'i'$ and the tension in Di. Above D' and D, the end posts are not stressed by the action of the portal (since $c = 2e$ in equations (c) and (d) of Art. 119). The bending moments in the end posts are from equation (b), is

$$\frac{9380(238 - 105)}{2} = 624{,}000 \text{ in.-lb.}$$

From equation (f) the compression in BC and the tension in $B'C$ is

$$\frac{9380 \times 238}{2 \times 105} = 10{,}600$$

and from equation (a), which for $d = e$ becomes $CD = \frac{\sqrt{2}Ph}{2d}$,

$$\text{the stress in } CD = CD' = \frac{\sqrt{2} \times 9380 \times 238}{2 \times 105} = \pm 14{,}900$$

The shears in the various members are not computed, as usually they are not important, but it must be realized that sufficient sectional area must be provided to resist them.

123. Maximum and Minimum Stresses. The stresses for which the several members of any truss are designed are the summations of all of the simultaneous stresses caused by the dead-load, live-load, impact, and lateral forces. For bridges on curves, the stresses due to centrifugal force and eccentricity must also be included.

In those members in which there is no reversal of stress under any possible combination of the stress-producing forces, only the

maximum stress has any significance so far as the design is concerned. In those members in which a reversal of stress does occur, however, the minimum stress is oftentimes fully as important as the maximum stress, for such members must be designed to resist both kinds of stress.

It is usual in specifications for design to allow certain increases in the allowable values of the unit stresses when the full effect of the lateral forces is considered. It therefore becomes necessary to have two sets of maximum and minimum stresses, the first including only dead-load, live-load, and impact forces (and centrifugal force for bridges on curves), and the second including in addition the full effect of the lateral forces. In either case, the dead load must be included for both maximum and minimum stresses, for the weight of the bridge is a constant force. Live-load stresses and impact stresses are inseparable, if one is included in determining a maximum or minimum stress, the other must be included also.[6] Furthermore, if the lateral force of the wind on the live load, or that force which allows for the sway of engines and train is included, the effect of the live load and impact must also be included.

The final maximum and minimum stresses for the members of the through Pratt Truss for which the dead-load stresses were computed in Art. 64, the live-load stresses in Art. 94, the impact stresses in Art. 105, and the stresses due to the lateral forces in Arts. 111 and 112, are given on the next two pages for each of the conditions mentioned above. No maximum or minimum stresses are given for *dE*, *eF*, and *Ee*, as they have the same values as those given for the corresponding symmetrical members in the left half of the truss. The minimum stresses in *Bc*, *Cd*, and *Cc*, are obtained from the values given for the symmetrical members on the right half of the truss, as in all the computations of the various stresses the load advanced from the right.

[6] Unless the load at rest upon the structure makes possible a reversal in stress which would not otherwise occur. It is extremely unlikely that any such stress would be of much importance.

INCLUDING LATERAL STRESSES

Stresses due to:	End post	Upper chord		Lower chord		Diagonals				Verticals			
	aB	*BC*	*CD*	*bc*	*cd*	*Bc*	*Cd*	*dE*	*eF*	*Bb*	*Cc*	*Dd*	*Ee*
Dead load........	−117.7	−120.2	−130.2	+ 75.1	+120.2	+ 70.6	+ 23.5	+23.5	+70.6	+ 26.2	− 28.1	−10.0	−28.1
Live load.........	−315.9	−311.7	−352.8	+202.5	+311.7	+209.8	+122.7	−58.2	−16.4	+113.5	− 94.4	0	+44.6
Impact....	−180.2	−177.9	−201.3	+115.6	+117.9	+147.2	+ 97.5	−52.2	−15.8	+104.8	− 75.0	0	+40.1
Overturning effect of wind:													
On truss[7].....	∓ 20.9	∓ 13.4	∓ 13.4	± 13.4	± 13.4								
On live load..	∓ 25.0	∓ 25.6	∓ 28.8	± 16.0	± 25.6	± 16.7	± 10.0	∓ 5.0	∓ 1.7	± 7.7	∓ 7.7	0	± 3.9
Effect of wind:													
On upper laterals........	....	± 4.0	± 9.4										
On lower laterals........	....	....	....	± 23.2	± 30.4								
On live load...	....	....	....	± 69.7	± 91.2								
Maximum........	−659.7	−644.8	−717.1	+515.5	+770.4	+444.3	+253.7	......	......	+252.2	−205.2	−10.0	
Minimum.........	− 96.8	−110.8	−126.2	+ 38.5	+ 76.4	+ 36.7	− 91.9	......	......	+ 26.2	+ 60.5	−10.0	

[7] The upper sign preceding the values of the stresses given for the effect of the lateral forces is the sign of the stress in the leeward truss. The lower sign indicates the character of the stress in the windward truss.

Not Including Lateral Stresses

Stresses due to:	End post	Upper chords		Lower chords		Diagonals				Verticals			
	aB	BC	CD	bc	cd	Bc	Cd	dE	eF	Bb	Cc	Dd	Ee
Dead load........	−177.7	−120.2	−130.2	+ 75.1	+120.2	+ 70.6	+ 23.5	+23.5	+70.6	+ 26.2	− 28.1	−10.0	−28.1
Live load.........	−315.9	−311.7	−352.8	+202.5	+311.7	+209.8	+122.7	−58.2	−16.4	+113.5	− 94.4	0	+44.6
Impact...........	−180.2	−177.9	−201.3	+115.6	+177.9	+147.2	+ 97.5	−52.2	−15.8	+104.8	− 75.0	0	+40.1
Maximum........	−613.8	−609.8	−684.3	+393.2	+609.8	+427.6	+243.7	......	......	+244.5	−197.5	−10.0	
Minimum.........	−117.7	−120.2	−130.2	+ 75.1	+120.2	+ 38.4	− 86.9	......	......	+ 26.2	+ 56.6	−10.0	

CHAPTER X

INFLUENCE LINES AND DISPLACEMENT DIAGRAMS FOR TRUSSES

Influence Lines

124. The principle of the influence line is explained in Chapter VII, while in Art. 106 the lengths of the segments of the influence line triangles are used in the determination of equivalent uniform live loads. In this chapter it is proposed to show the method of constructing influence lines for typical simple truss members and their use in actual stress calculations.

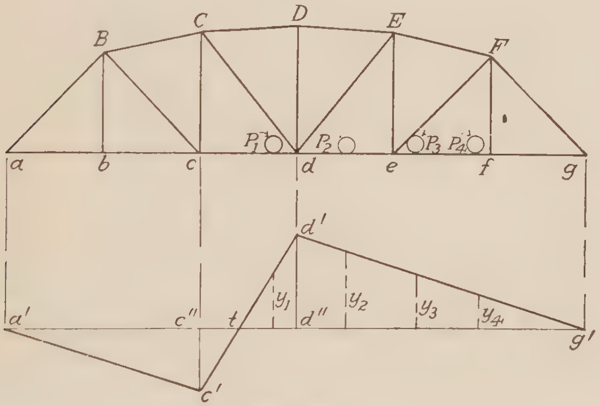


Fig. 201.

In Fig. 201, $a'c'd'g'$ is the influence line for the stress in the member Cd. The ordinate $c'c''$ is laid off equal to the stress in Cd when a load of unity is placed at the point c. It is measured downward from c'', since the stress in Cd caused by this single load at d is compression. Similarly, the ordinate $d'd''$ represents the tension in Cd caused by a unit load at d. As the load moves to the left from c, the stress in Cd decreases uniformly to zero when the load reaches a. A similar variation occurs as the load

moves from d to g. In Art. 77 it was shown that the influence line between panel points is a straight line. Therefore, the construction of the influence line is completed by drawing the straight lines $a'c'$, $c'd'$, and $d'g'$.

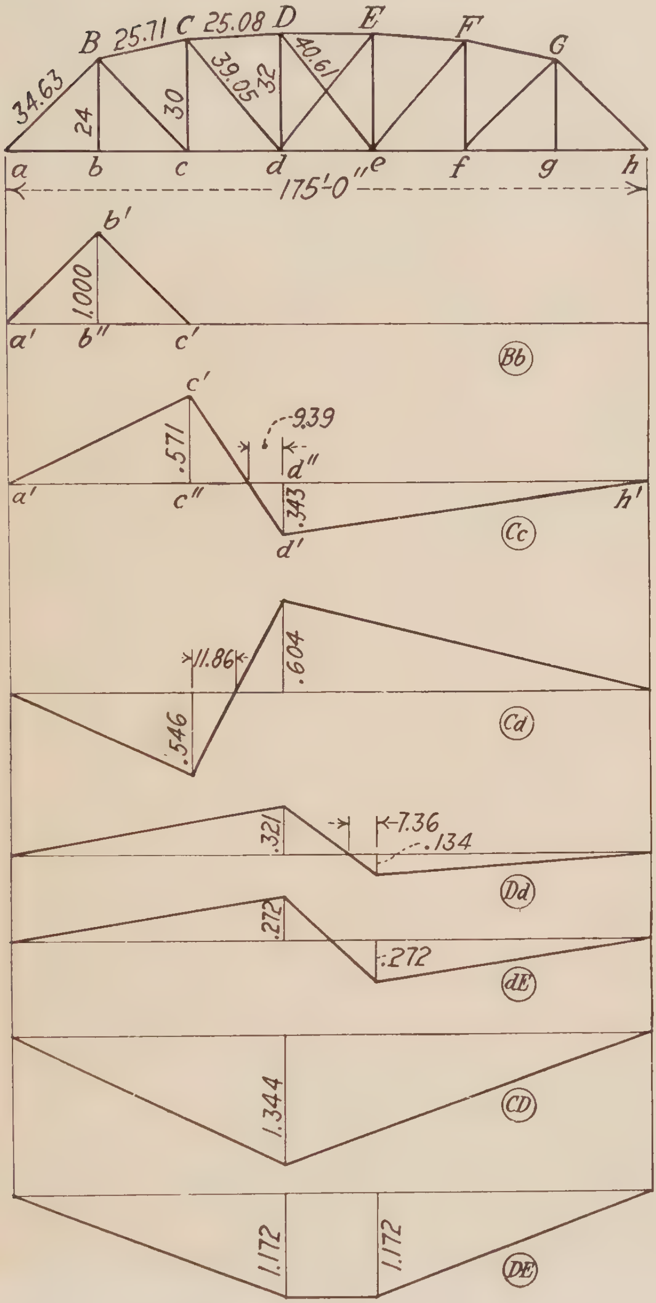
For any system of concentrated loads, the summation of the products of the loads times the ordinates of the influence line directly under the respective loads gives the value of the stress in the member for the loads in the given position. In the case shown, the stress in Cd is $P_1y_1 + P_2y_2 + P_3y_3 + P_4y_4$. For a loading such as Cooper's, the large number of products to be obtained makes computation of stresses by this method very laborious and susceptible to error, and for simple trusses, the analytical methods are much to be preferred. If, however, an equivalent uniform load is selected, the work is greatly reduced, and, for the trusses with subdivided panels and web members with different inclinations, the influence line offers an alternate method of computing and checking stresses.

The stress in Cd, for any position of the loads, is obtained by making a summation of the products of P times y as stated before. If the concentrated loads are increased to an infinite number of unit loads the ΣPy becomes ΣP times $\frac{d'd''}{2}$, and if P is given in units of weight w, per unit of length l, the maximum stress in Cd is

$$w\left[nl \times \frac{d'd''}{2}\right]$$

in which n is the number of unit lengths l in the distance tg'. In the above expression, $\frac{nl \times d'd''}{2}$ is the area of the influence line triangle $td'g'$. Since it is customary to express uniform load in pounds per foot, it follows that the stress in any member may be obtained from the product of the uniform load per foot times the area of its influence line triangle, the base of which is measured in feet.

125. The Parker Truss without Counters. Figure 202 shows the influence lines for the stresses in some of the typical members of a seven-panel through Parker truss. The stress in the suspender Bb depends only upon the load applied at b. Its influence line is, therefore, constructed by laying off the ordinate $b'b''$ equal to unity, and drawing the straight lines $a'b'$ and $b'c'$.



B 25.71 C 25.08 D
E
F
G
34.63
24
30
39.05
32
40.61
a
b
c
d
e
f
g
h
175'-0"
b'
1.000
a'
b"
c'
Bb
c'
.571
9.39
d"
a'
c"
.343
h'
d'
Cc
11.86
.604
.546
Cd
.321
7.36
.134
Dd
.272
.272
dE
1.344
CD
1.172
1.172
DE

Fig. 202.

The stress in Cc, with unit load at c is computed by the methods of Art. 65. With this position of the unit load $R_a = 5/7$ and $Cc = \dfrac{1 \times 5 - 5/7 \times 3}{5} = +0.571$. With the unit load at d, $Cc = \dfrac{-4/7 \times 3}{5} = -0.343$. These two computations give the value of the ordinates $c'c''$ and $d'd''$, respectively, and the influence line may now be drawn as shown.

The necessary computations for the influence line for the stress in Cd are as follows:

Unit load at c: $R_a = \dfrac{5}{7}$ $\quad Cd = \dfrac{39.05}{30}\left(\dfrac{5/7 \times 13 - 1 \times 15}{16}\right) = -0.465$

Unit Load at d: $R_a = \dfrac{4}{7}$ $\quad Cd = \dfrac{39.05}{30}\left(\dfrac{4/7 \times 13}{16}\right) = +0.604$

In determining the values of the ordinates of the influence line for Dd, the section passed through Dd also cuts the diagonal dE, so that its vertical component must be known before Dd can be computed.

Unit load at d: $R_a = 4/7$ Shear in center panel $= -3/7$

$$dE_V = +\frac{3}{14} \qquad Dd = \frac{1 \times 16 - 4/7 \times 13 - 3/14 \times 16}{16} = +0.321$$

Unit load at e: $R_a = 3/7$ Shear in center panel $= +3/7$

$$dE_V = -\frac{3}{14} \qquad Dd = \frac{3/14 \times 16 - 3/7 \times 13}{16} = -0.134$$

The stress in CD with unit load at the center of moments d, is $\dfrac{-\dfrac{25.08}{25}\left(\dfrac{4}{7} \times 75\right)}{32} = -1.344$, and the stress in DE with unit load at either d or e is 1.172, the center of moments for this computation being taken midway between d and e as in Art. 99.

The influence lines as constructed in Fig. 202 may be used to determine the stresses due to any system of loading. For the more simple systems of concentrated loads, an inspection of the influence line will show the position of the loads which produces either the maximum or minimum stresses. For uniform loading the stresses are determined by computing the areas of the influence line triangles and multiplying these respective areas by the value of the uniform load per foot. In the following table, equivalent uniform loads are used as taken from Fig. 177.

Member	Maximum: Maximum ordinate	Maximum: Base of triangle	Maximum: Area	Maximum: Uniform load per foot of bridge	Maximum: Stress in kips	Minimum: Maximum ordinate	Minimum: Base of triangle	Minimum: Area	Minimum: Uniform load per foot of bridge	Minimum: Stress in kips
Bb	1.000	50	25.00	9100	+113.8					
Cc	0.343	109.39	18.76	8360	− 78.4	0.571	65.61	18.73	8650	+81.0
Cd	0.604	114.13	34.47	8230	+141.8	0.465	60.87	14.15	8980	−63.6
Dd	0.321	92.64	14.87	8290	+ 61.6	0.134	82.36	5.52	8640	−23.2
dE	0.272	87.50	11.90	8450	+ 50.3	0.272	87.50	11.90	8450	−50.3
CD	1.344	175	117.6	7330	−431.0					
DE	1.172	......	117.2	7330	−429.5					

126. The Parker Truss with Counters. The influence lines for the members not in or adjacent to the panels containing counters are constructed in the same manner as those in the previous article. The influence lines for the chords *CD* and *DE* as drawn in Fig. 202 may be used for the determination of the stresses in *CD* and *DE* of Fig. 203. The maximum stress in the diagonal *Cd* of Fig. 203 may also be obtained from the influence line of Fig. 202, and since its minimum stress is zero, no further construction for *Cd* is necessary.

In order to determine the maximum tensile stress in the vertical adjacent to a panel containing a counter, it is first necessary to ascertain the exact position of the live load which causes no stress in either the counter or the main diagonal. As was shown in Art. 73, under these conditions the stress in the vertical is equal to the difference between the vertical components of the two upper chords meeting the vertical at its upper extremity. In order to place the live load in this position, the dead-load stress in either the counter or the main diagonal must be known. The following constructions and computations consider a total dead panel load of 40 kips, which was used in computing the dead-load stresses for this truss in Art. 65.

The influence line for the counter *cD* is constructed as shown in Fig. 203, the values of the ordinates for loads at *c* and *d* being computed as in the previous article. These ordinates are not equal and opposite to those computed for the diagonal *Cd*, on account of the different inclinations and lengths of the two members. The portion of the influence line to the right of *t*, is of no value so far as determining the maximum live-load

stress in cD, but it may be used for determining the dead-load stress, and also in locating the position of the load which produces zero stress in both cD and Cd. With a dead panel load of 40 kips at each panel point (assuming Cd not acting) the dead-load stress in $cD = 40 \ (0.242 + 0.484 - 0.628 - 0.471 - 0.314 - 0.157)$

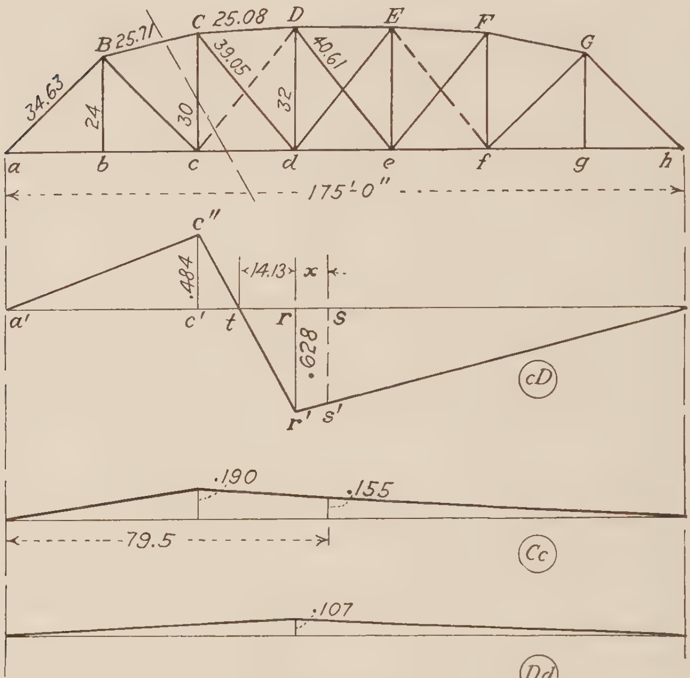


Fig. 203.

$= -33.8$ kips. Using an equivalent uniform live load as in Art. 125, the maximum live-load stress in cD is

$$\frac{0.484 \times 60.87}{2} \times \frac{8.98}{2} = +66.2 \text{ kips}$$

and the maximum stress in the counter cD is $+32.4$ kips.

The influence line for the vertical Cc for the truss without counters may be used to determine the maximum compressive stress in Cc, but since with a counter in the panel there is no position of the live load which causes compression in Cd, it is necessary to construct a separate influence line for the tension

in Cc when Cd is not acting. With unit load at c, the section being passed as shown in the figure,

$$\text{the tension in } Cc = \frac{-\frac{5}{7} \times 3 + 1 \times 5 - \frac{0.484 \times 32}{40.61} \times 5}{5} = 0.190$$

As this tension is equal to the difference in the vertical components of BC and CD, and therefore, a function of the moment, it follows that the tension in Cc caused by a load of unity decreases uniformly as the load moves in either direction from c, and the influence line will have the same form as an influence line for moment. In order for the stress in the diagonal cD to be zero, the live load must extend to the right from t, a distance such that the area $tss'r'$ times $\frac{8.98}{2}$ equals 32.4 kips. The area of the triangle trr' times $\frac{8.98}{2}$ equals 19.9 kips. The area of the trapezoid $rss'r'$ times $\frac{8.98}{2}$ must, therefore, equal 12.5 kips. The distance x may be obtained from a simultaneous equation or by trial. For the case in point it is found to be 4.5 ft. Therefore, the maximum tension in Cc is obtained with the live load extending from the left support for a distance of $75 + 4.5 = 79.5$ ft. and the live-load tension is $\left(\frac{0.190 \times 50}{2} + \frac{0.190 + 0.155}{2} \times 29.5\right) \frac{7.34}{2}$[1] $= 36.1$ kips. The apparent dead-load stress is $40 \times 0.19 \times 3.5 = 26.7$ kips, but since 10 kips of the dead panel load is applied at the upper panel point C, the actual dead-load stress is $26.7 - 10 = 16.7$ kips. The maximum tension in Cc is, therefore, 52.8 kips.

The stress in the member Dd is affected by the action of the diagonal in each of the adjacent panels. With a unit load at e the diagonals Cd and De are stressed, and the compression in Dd is 0.348. With unit load at d, the diagonals Cd and dE are in tension, and tension also exists in the member Dd in an amount equal to

$$\frac{1 \times 16 - \frac{4}{7} \times 13 - \frac{3}{7} \times 16}{16} = 0.107$$

With unit load at c, the diagonals cD and dE are stressed, and

[1] $l_1 = 50$ ft.,
$l_2 = 125$ ft.

Dd is again in compression. With full load on the structure, however, *Dd* is in tension. The influence line for tension in *Dd* is shown in the figure, and the maximum live-load tension is

$$\frac{0.107 \times 175}{2} \times \frac{7.33}{2} = 34.3 \text{ kips.}$$

In taking the dead-load tension from this influence line, account must be taken of the subdivision of the dead-load panel loads as in the case of *Cc*. The value of the dead load tension is 5.0 kips, so that the maximum tensile stress in *Dd* is 39.3 kips. If it is desired to determine the maximum compressive stress in *Dd* by this method, the load should be considered to cover the right half of the structure. With unit load at e the stress in *Dd* is -0.348 and the live-load compression is

$$\frac{0.348 \times 87.5}{2} \times \frac{8.45}{2} = 64.3 \text{ kips.}$$

The influence line constructed in Fig. 202 for dE may be used for the truss with counters. The ordinate under d will have twice the former value and only the left half of the influence line is applicable, since the minimum stress is zero.

127. The Baltimore Truss without Counters. In Fig. 204 influence lines for typical members of a deck Baltimore truss are shown. The truss has a span of 288 ft.-0 in. and a depth of 50 ft.-0 in. Dead- and live-load stresses have been computed for all of the members of this truss in Arts. 66, 74, and 103.

Ef is the only web member in the panel *EF*, and hence the stress varies directly as the shear in the panel. For a load between A and E, or between G and M, there is no stress in *Ff* or *fG*, and the stress in *Ef* is the same as in the diagonal *Eg* of the double panel *EG*. The lines $m'g'$ and $a'e'$ may now be drawn either by computing the ordinates $g'g''$ and $e'e''$ or by laying off $a'a''$ equal to sec θ and drawing $m'g'$ and $a'e'$ parallel to $a''m'$. Since F is on the same side of the section as G, the stress in *Ef* will follow the line of variation of $m'g'$ as far as f'. The line $e'f'$ completes the influence line. If n be the total number of panels, and r the number of panels from F to M, from the construction $tf'' = \frac{r}{n-1} \times e''f'' = \frac{f''m'}{n-1}$ or $tf'' \times n = tm'$, which graphically illustrates the criterion $W = nP$ as developed in Art. 85.

The stress in the member *Ee* is not affected if the subsidiary truss *CfE* (Fig. 205) is substituted for the framing as shown in

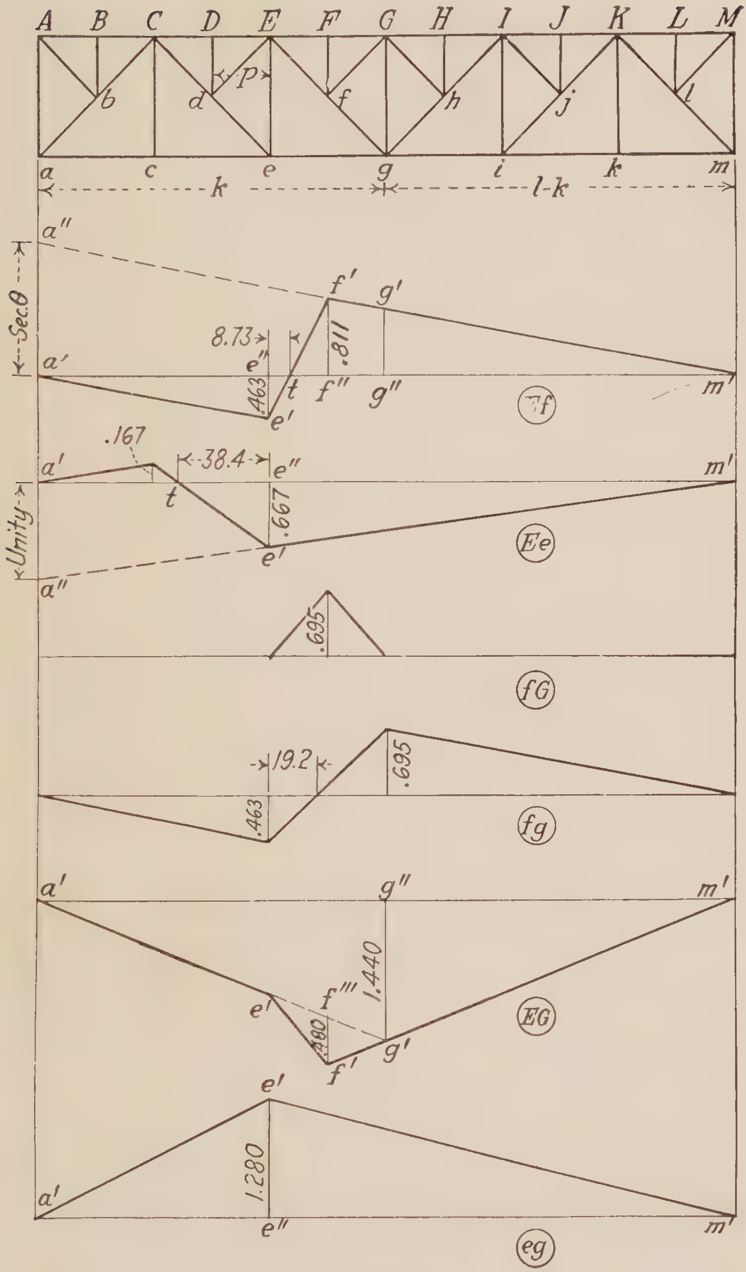

A B C D E F G H I J K L M
b d f h j l
p
a c e g i k m
k
l-k
a''
Sec θ
f' g'
8.73
.811
a' e''
.463
t f'' g''
e' m'
Ff
.167
a' t 38.4 e'' m'
Unity
.667
Ee
e'
a''
.695
fG
19.2
.695
.463
fg
a' g'' m'
1.440
e' f'''
.880
g' f'
EG
e'
1.280
a'
e'' m'
eg

FIG. 204.

Fig. 204. Any load at D is carried one-half to C and one-half to E as before, and the stress in Ee is the same for both systems of framing. The influence line may then be drawn by calculating the stress with unit load at C and again at E, or by laying off $a'a''$ equal to unity, and proceeding as in the construction of the influence line for Ef. Here again, since $\frac{te'' \times n}{2} = tm'$, a graphical illustration of the criterion $W = \frac{n(P_1 + P_2)}{2}$ of Art. 101 is presented.

The stress in fG depends entirely on the load applied at F, its values for unit load at F being $\frac{1}{2} \times \sec \theta$.

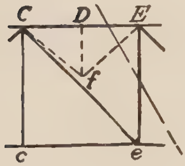


FIG. 205.

The influence line for fg is constructed in a manner similar to that used for the vertical Ee, the stress in fg being unaffected by the introduction of the subsidiary truss EfG.

With no load at F, the stress in the chord EG is computed as if the members Ef and fG did not exist, for neither of them is stressed. The ordinate $g'g''$ is computed for unit load at G, directly over g, the center of moments, and the lines $a'g'$ and $g'm'$ drawn. Since F is on the same side of the section passed through EF, Ef, and eg, as the center of moments g, the stress in EG will follow the line of variation $m'g'$ as far as f'. The line $f'e'$ completes the influence line. With unit load at G, $g'g''$ (representing the ordinate of a moment influence line) $= \left(\frac{l-k}{l}\right) k$ and from similar triangles $f'f''' = \frac{p}{k}\left[\frac{(l-k)k}{l}\right] + \frac{p}{l-k}\left[\frac{(l-k)k}{l}\right] = p$. In a beam of length $EG = 2p$, with unit load at F, the ordinate of the moment influence line is $\frac{p}{2}$, so that the triangle $e'f'g$ represents twice the moment in a beam of length $EG = 2p$, and, as in Art. 102, $EG = \frac{M_E + 2M_{D'}}{d}$.

The influence line for the lower chord eg is constructed by computing the value of the ordinate $e'e''$ with unit load at E and drawing the two lines $a'e'$ and $e'm'$.

The values of the ordinates necessary for the calculation of the stresses are marked on the diagrams. Equivalent uniform live loads are used as in Art. 125.

Member	Maximum					Minimum				
	Maximum ordinate	Base of triangle	Area	Uniform load per foot of bridge	Stress in kips	Maximum ordinate	Base of triangle	Area	Uniform load per foot of bridge	Stress in kips
Ef	0.811	183.27	74.32	7650	+284.3	0.463	104.73	24.31	8400	−102.1
Ee	0.667	230.40	76.80	7160	−274.9	0.167	57.60	4.80	9100	+ 21.8
fG	0.695	48.00	16.68	9220	+ 76.9					
fg	0.695	172.80	60.05	7560	+227.0	0.463	115.20	26.67	8120	−108.3
EG	1.440	288.00								
	0.480	48.00	218.88	6670	−730.0					
eg	1.280	288.00	184.32	6780	+624.8					

128. The Baltimore Truss with Counters. In a truss with tension diagonals as shown in Fig. 206, when the load is so placed

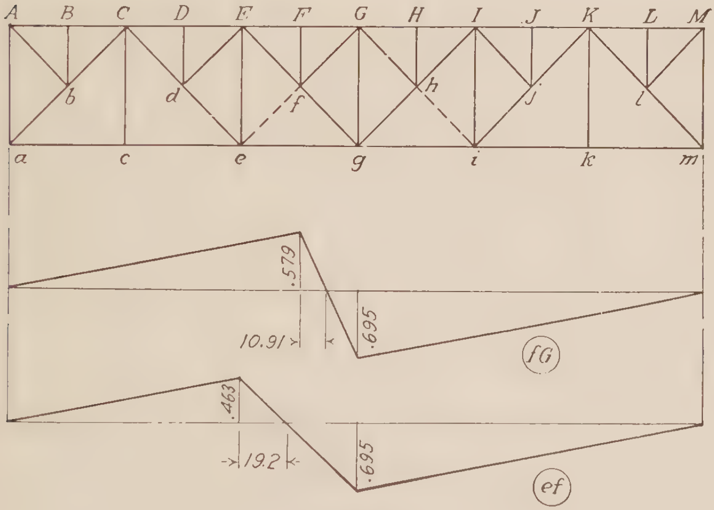
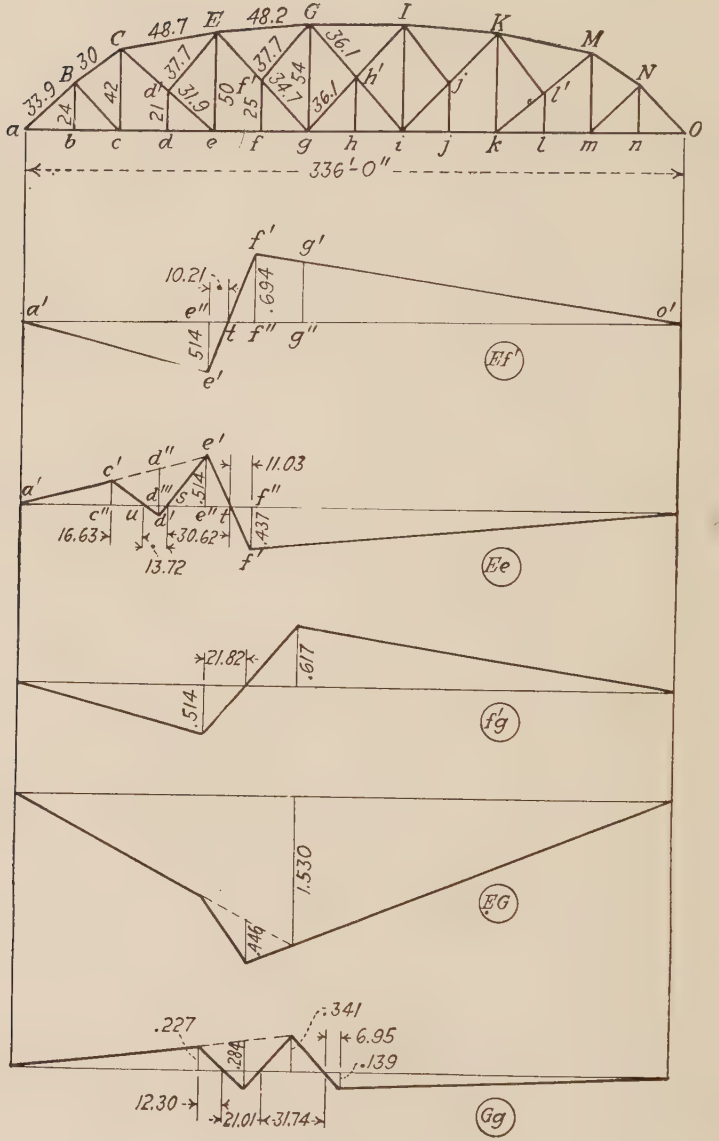


Fig. 206.

as to tend to cause compression in *Ef*, that member ceases to act as the main diagonal, and becomes the subdiagonal of the double panel, while the counter *ef*, together with the subdiagonal *fG* becomes the main diagonal. Under these conditions *fg* is not stressed.

The maximum stress in the diagonal *fG* is greater than in the truss without counters. Its influence line is similar to that for



a
B
C
E
G
I
K
M
N
O
33.9
30
48.7
48.2
24
42
21
37.7
31.9
50
25
37.7
34.7
54
36.1
36.1
d'
f'
h'
j'
l'
b c d e f g h i j k l m n
336'-0"
f'
g'
10.21
.694
a'
e''
o'
t
f''
g''
.514
e'
Ef'
e'
d''
c'
11.03
a'
d'''
s
.514
f''
c''
u
d'
e''
t
.437
16.63
30.62
13.72
f'
Ee
21.82
.617
.514
f'g
1.530
.446
EG
.341
.227
6.95
.284
.139
12.30
21.01
31.74
Gg

FIG. 207.

Ef in the previous article, and the maximum live-load stress is

$$\frac{0.579 \times 130.91}{2} \times \frac{8.13}{2} = +154.6 \text{ kips.}$$

Comparing the influence line for *ef* with that of *fg* of Fig. 204, it is seen that the left-hand portions are similar and the areas of the two triangles are equal. Therefore, the maximum live-load stress in the counter *ef* is +108.3 kips.

129. The Pettit Truss without Counters. In the Pettit truss, the combination of an inclined chord and subdivided panel requires the application of the methods previously used for the Parker truss, together with those used for the Baltimore truss.

In Fig. 207 the influence line for the diagonal Ef' is constructed by determining the values of the ordinates $e'e''$ and $g'g''$ with unit load at e and g, respectively, in the same manner that the corresponding ordinates were determined for the influence line for the diagonal Cd of the Parker truss of Fig. 202. The lines $a'e'$ and $o'g'$ may now be drawn, and for the same reason given in Art. 126 in regard to the construction of the influence line for the diagonal Ef of Fig. 204, $o'g'$ is produced to f' and the influence line computed by drawing the line $e'f'$.

The apparent influence line for the vertical Ee is $a'e'f'o'$, constructed by computing the ordinates $e'e''$ and $f'f''$ for unit load at e and f, respectively, as in the previous cases. Considering the subsidiary truss of Fig. 208, however, a load of unity applied at d will produce two equal reactions at C and E, each equal to ½ unit. The reaction at E is equivalent to ½ unit of compression in the member Ee, so that in constructing the influence line $d'd'' = ½$ is laid off downward from the apparent influence line $a'e'$, and the lines $c'd'$ and $d'e'$ drawn to complete the construction. The difference between this construction and that described in connection with Fig. 205 should be noted. In the Baltimore truss used in Art. 127, a load at E causes compression in Ee and a load at C causes tension, while in the Pettit truss under consideration, a load at either c or e causes tension in Ee.

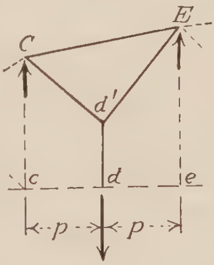


FIG. 208.

The influence lines for $f'g$ and EG are constructed in a manner similar to that used for the corresponding members in the

Baltimore truss, the critical ordinates being determined as for the Parker truss. The line for $f'g$ is exactly the same as that for Ef' except from e to g. The influence line for Gg is constructed, under the assumption that the diagonals Gh' and gh divide the shear in the panel gh. With this distribution of the shear, the dead-load stress in Gg is -8.3 kips instead of -24.4 kips as computed in Art. 67. The construction in the panel eg is similar to that for Ee.

The necessary computations for the construction of the influence lines for the members Ef' and Ee are as follows:

Ef' Unit load at f $\quad f'f'' = \dfrac{34.7}{25}\,\dfrac{(9/14 \times 21)}{27} = +0.694$

Unit load at e $\quad e'e'' = -\dfrac{34.7}{25}\,\dfrac{(1 \times 25 - 10/14 \times 21)}{27} = -0.514$

$$e''t = \frac{0.514}{0.514 + 0.694} \times 24 = 10.21$$

Ee Unit load at f $\quad f'f'' = -\dfrac{9/14 \times 8.5}{12.5} = -0.437$

Unit load at e $\quad e'e'' = \dfrac{1 \times 12.5 - 10/14 \times 8.5}{12.5} = +0.514$

$$tf'' = \frac{0.437}{0.437 + 0.541} \times 24 = 11.03$$

$$d'd''' = 3/4 \times 0.514 - 0.500 = -0.114$$

$$st = \frac{0.514}{0.514 + 0.114} \times 24 + 12.97 = 30.62$$

$$c'c'' = 1/2 \times 0.514 = 0.257$$

$$c''u = \frac{0.257}{0.257 + 0.114} \times 24 = 16.63$$

The computations for $f'g$, EG, and Gg are similar.

The maximum and minimum live-load stresses as determined with the aid of the influence lines are tabulated on page 265.

Member	Maximum					Minimum				
	Maximum ordinate	Base of triangle	Area	Uniform load per foot of bridge	Stress in kips	Maximum ordinate	Base of triangle	Area	Uniform load per foot of bridge	Stress in kips
Ef'	0.694	229.79	79.74	7400	+295.0	0.514	106.21	27.30	8370	−114.1
Ee	0.437	227.03	49.61	7420	−184.2	0.514 −0.114 0.257	30.62 13.72 64.63	15.39	8280	+ 63.7
f'g	0.617	218.18	67.31	7350	+247.4	0.514	117.82	30.28	8060	−122.0
EG	1.530 0.446	336 48	267.84	6470	−866.5					
Gg	0.139	174.95	12.16	7800	− 47.4	0.341 −0.216 0.227	31.74 21.01 108.30	15.43	7800	+ 60.2

130. The Pettit Truss with Counters. With tension diagonals, the truss of Fig. 207 is modified by the insertion of the counter *ef'* as shown in Fig. 209. This member, together with *f'G*, acts as the main diagonal of the double panel *eg*, when there is sufficient load to the left of the panel to cause compression in the diagonal *Eg*, if the counter were not placed in the truss. Under such conditions, *Ef'* becomes a subdiagonal, while *f'g* is not stressed. In the center double panel the distribution of the shear is different from that in the truss without counters. With the load on the right, *Gi* acts as the main diagonal and *h'I* as the subdiagonal, while *gh'* is not stressed. With the load on the left, *gI* is the main diagonal, *Gh'* the subdiagonal, and *h'i* is not stressed.

The influence lines for the members *ef'* and *f'G* are constructed by the methods previously described. Since, however, *f'G* produced does not pass through *e*, the influence lines for the two members are not similar outside of the panel *eg*, as in the previous cases.

The maximum live-load tension in *Ee* (if any) occurs when the stress in *Ef'* is zero, as in the analysis of the Parker truss. With the counter *ef'* in action, if there is any load at *f*, *Ef'* is stressed in tension as a subdiagonal. This reduces the possible tension in *Ee*, as does the effect of the subdiagonal *d'E* in the panel on the left. From an inspection of the influence line, it is evident that there will be no live-load tension in *Ee*, since with load extending from *a* to *c* only, the counter is not in action. In the truss under consideration, impact must be included in order to

have the counter in action with the load extending from a to e (see Art. 67 for dead-load stresses).

The maximum tension in Gg does not occur with full load as in the Parker truss, on account of the effect of the subdiagonals, but with the load extending from a to slightly beyond e.

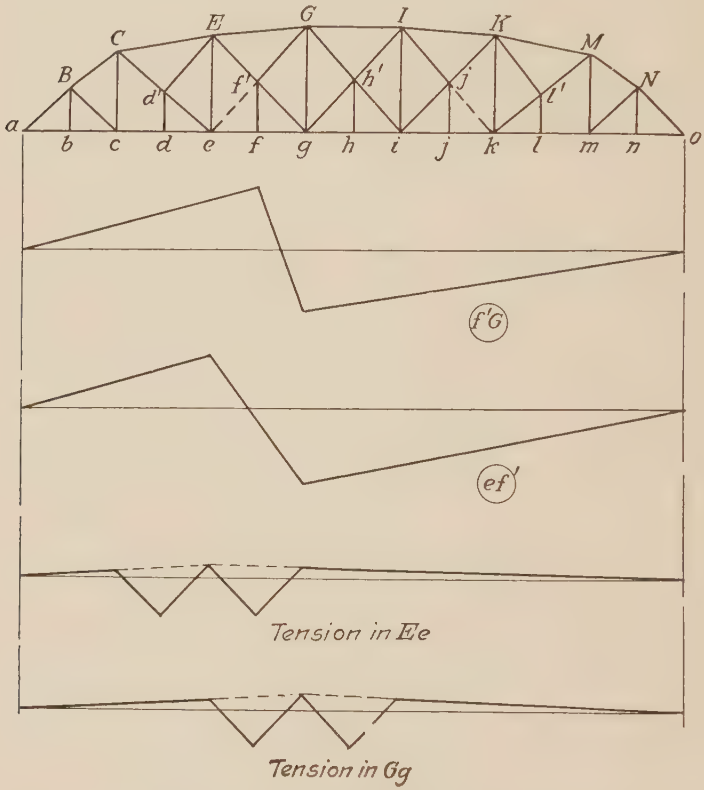


FIG. 209.

Figure 209 shows the influence lines discussed above. The actual stresses are determined as in the previous article.

DISPLACEMENT DIAGRAMS

131. The Displacement of a Joint. If any member of a framed structure be subjected to a stress or a change in temperature, its length is changed. When the change is caused by stress, its amount is

$$\lambda = \frac{pl}{AE}$$

in which λ is the change in length, p the total stress in the member, l its length, A its cross-section, and E the modulus of elasticity of the material. When the change in length is caused by a change in temperature, its amount is

$$\lambda_t = \omega t l$$

in which ω is the coefficient of linear expansion per degree of temperature change, and t the number of degrees of change.

In Fig. 210, consider the points a and c fixed as to horizontal or vertical movement, and the frame abc hinged at its three joints. The effect of a load applied at b, as shown, is to shorten the member ab and lengthen the member bc due to the respective compressive and tensile stresses produced in them. Let their lengths under this load be aa' and cc', respectively. With a as a center and aa' as a radius, let an arc be described. The point b must lie somewhere on this arc. Similarly, an arc is described with c as a center and cc' as a radius. The intersection of the two arcs b' is the position of the point b; and the frame abc, under the action of the load shown, assumes the shape $ab'c$.

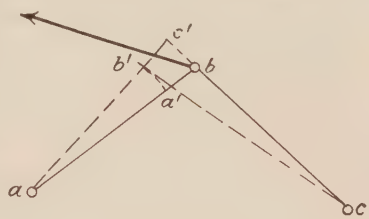


FIG. 210.

The deformations, as shown in the figure, are very much exaggerated, for if they were laid off to the same scale as the lengths they would not be visible. Since they are so small in comparison with the lengths of the members, the tangents to the arc may be substituted for the arc themselves without appreciable error. Therefore, the determination of the position of the point b' may be made by erecting perpendiculars to ab and bc at a' and c', respectively, their interesection being the position of the point b'.

132. The Displacement Diagram. In the previous discussion two points of the frame abc were considered fixed. If, however, this frame were considered merely as a portion of a larger frame, the stresses in which cause a shifting in the positions of the points a and c, the final position of b would also be affected. In Fig. 211, the magnitude and direction of the displacements of the points a and c, respectively, are represented by the lines aa' and cc'. Considering first the position of b as affected by the change in position of a, it is seen that b takes the position b_1; but the

shifting in the position of c to c' considered alone would cause b to assume the position b_2. The deformation in the members ab and bc causes a further shifting of b. Proceeding as in the previous article, the change in length of $ab = a'b_1$ is laid off from b_1 *toward* a', since the change in length is a shortening; its magnitude is represented by the heavy line. Similarly, from b_2 the elongation of $cb = c'b_2$ is laid off *away from* c'. At the extremities of these deformations perpendiculars are erected, their intersection locating the point b'. The final displacements of the three points a, b, and c are bb_1, bb', and bb_2, respectively.

The displacements of the points a and c, the deformations of ab and bc, together with the perpendiculars erected at the extremities of the deformations form a closed polygon. Therefore, that portion of the diagram may be constructed separately in

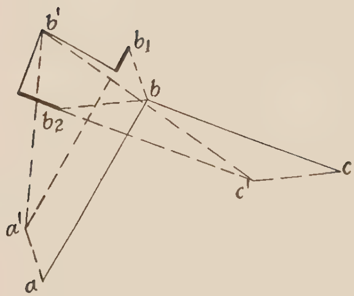


FIG. 211.

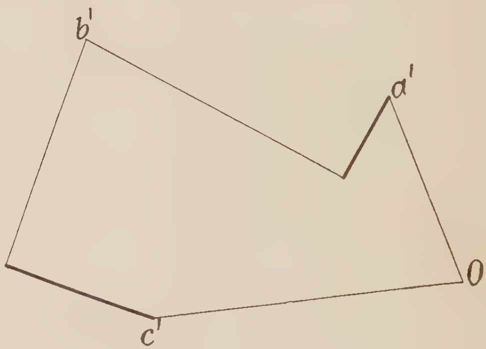


FIG. 212.

order to determine the final displacements. This is desirable on account of the exceedingly small values of the deformations compared with the lengths of the members. Figure 212 is the necessary diagram drawn to a larger scale. Such a diagram is called a displacement diagram.

133. The Displacement Diagram for a Truss. The changes in length of the several members of the truss of Fig. 213(a) due to a single load applied at d are marked on the diagram. The support a is fixed while the other support c is free to move in the inclined direction indicated.

The displacement diagram may be constructed by considering any joint fixed in position, and one of the members making the

joint may be considered fixed in direction. In the following construction, a and the direction of ad are regarded as fixed.

Beginning at a' in Fig. 213(b), the elongation of the member ad is laid off in the direction of a toward d, since in the final position

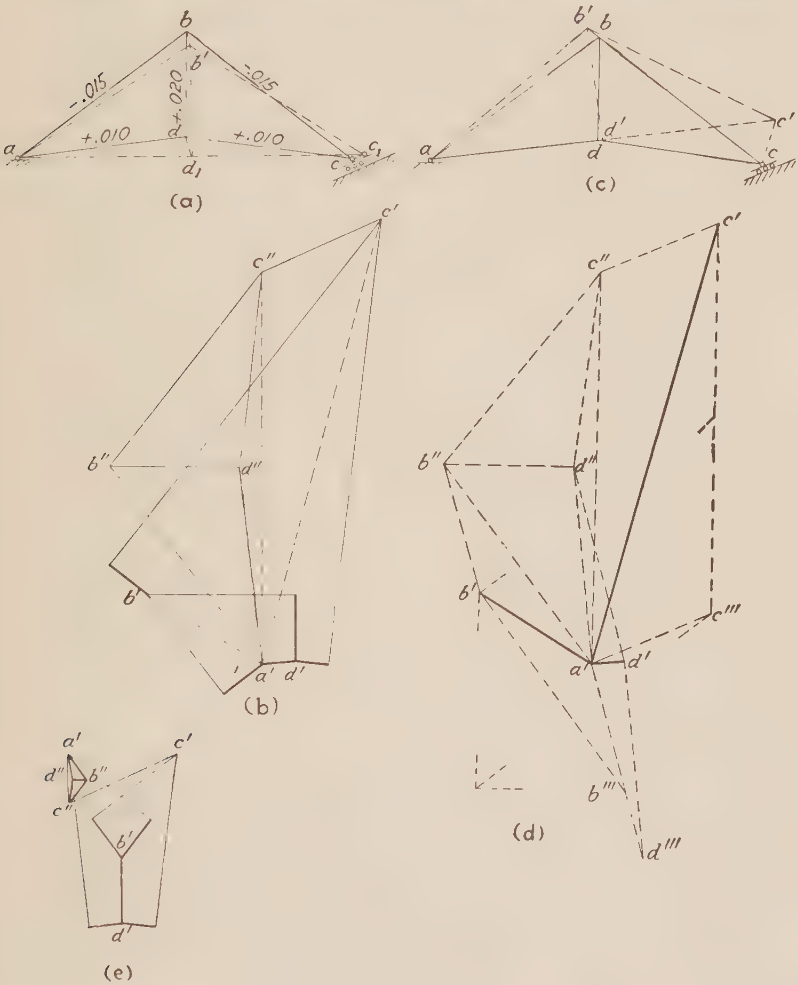


Fig. 213.

of the deformed truss d is pulled away from a. With d' thus determined, the displacement of b is found by regarding a and d in the triangle abd as fixed. The elongation of bd is laid off in the direction of d toward b from d', and the shortening of ab

in the direction of b toward a from a'. The intersection of the perpendiculars, erected at their extremities, is the point b'. In the same manner, c' is located by laying off the shortening of bc from b' and the elongation of dc from d' and erecting the perpendiculars. The lines $a'b'$ and $a'c'$ are the displacements of the points b and c, respectively.

The deformation of the truss, under the assumed conditions of a and the direction of ad being fixed, are shown in Fig. 213(c) where the displacements obtained in Fig. 213(b) are laid off to a smaller scale and the corresponding panel points joined by broken lines. The deformation is greatly exaggerated in order to show the general effect.

The original conditions, however, require that c shall move only on the inclined plane of the support, and therefore, the whole truss must be revolved about a as a center until c' falls into a line drawn through c parallel to the constrained line of motion. As the arc thus described by c' is very small compared with the radius ac', and as the direction of ac' is practically the same as that of ac, a perpendicular from c' to ac may be substituted for the arc.

In Fig. 213(d), in which the displacements $a'b'$, $a'c'$, and $a'd'$ are shown without the construction lines of Fig. 213(b), the corresponding path of rotation of c is represented by $c'c'''$, which is drawn perpendicular to ac of Fig. 213(a) to an intersection with a line drawn through a' parallel to the constrained line of motion of c. The actual displacement of c is $a'c'''$. The displacement of b caused by the rotation of the truss about a is $b'b'''$, which is drawn perpendicular to ab of Fig. 213(a) and whose length bears the same relation to $c'c'''$ as their respective radii of rotation bear to one another. That is $b'b''':c'c''' = ab:ac$. The length of $b'b'''$ may be determined by similar triangles as follows: Through the midpoint of $c'c'''$ draw a line through b' and from c''' another parallel line to its intersection with one drawn from b' perpendicular to ac of Fig. 213(a). From this intersecton a line perpendicular to bd of Fig. 213(a) locates b''' at its intersection with the line drawn from b' perpendicular to ab of Fig. 213(a). The construction is indicated on the diagram. The displacement of d caused by the rotation of the truss is determined in a similar manner. The resultant displacements are then represented in amount and direction by $a'b'''$, $a'c'''$, and $a'd'''$. Since a is fixed, there is no displacement of that joint, and a' and a''' coincide.

In Fig. 213(a) the final position of the deformed truss is shown in broken lines, the resultant displacements $a'b'''$, $a'c'''$, and $a'd'''$ being laid off to a smaller scale from the points b, c, and d, respectively. As in Fig. 213(c) the actual displacements are greatly exaggerated.

For the purpose of simplifying the construction, the parallelograms of Fig. 213(d) are completed; *i.e.*, $b'b''$ is drawn parallel to $a'b'''$, $a'b''$ parallel to $b'b'''$, $c'c''$ parallel to $a'c'''$, $a'c''$ parallel to $c'c'''$, $d'd''$ parallel to $a'd'''$, and $a'd''$ parallel to $d'd'''$. Since $b''b' = a'b'''$, $c''c' = a'c'''$, and $d''d' = a'd'''$, $b''b'$, $c''c'$, and $d''d'$, represent the respective final displacements of the points b, c, and d. Also, since $a'b''$, $a'c''$, and $a'd''$ are, respectively, perpendicular to ab, ac, and ad of Fig. 213(a), if the points a', b'', c'', and d'' are joined, they form a truss similar to the original truss of Fig. 213(a). It follows, therefore, that the actual displacements may be obtained from the diagram of Fig. 213(b) as follows: From c' draw $c'c''$ parallel to the constrained line of motion of the point c to an intersection with a perpendicular from a' to the radius of rotation of c, *i.e.*, to ac of Fig. 213(a). On $a'c''$ as a base construct a truss diagram similar to the original diagram. The required displacements are then given by the directions and distances of b', c', and d' from b'', c'', and d'', respectively. The necessary construction is shown in Fig. 213(b) in full lines.

Since the displacement diagram may be drawn, assuming any joint fixed and the direction of any one of the members making that joint also fixed, it is advisable to choose the joint and the direction which will make the diagram as compact as possible. This allows the use of a larger scale and reduces the probability of error. Such a result may be secured by selecting a member which suffers the minimum change in direction under the applied load. In a simple truss one of the chords in the center panel or the middle vertical should be chosen. In the truss whose displacement diagram has been constructed as described above, if the direction of the middle vertical and either one of its extremities are considered fixed, the resultant diagram is as shown in Fig. 213(e). If the constrained line of motion were horizontal, the truss diagram $a'b''c'd''$ would be reduced to a point, since a' and c'' would coincide.

134. The Deflection of a Bridge Truss. When live load covers the entire span of a simple truss bridge, the elongation and shortening of the various members of the trusses are due to the

stresses caused by the live and dead load, and the several panel points are displaced an appreciable amount from their theoretical positions with no stresses in the members. If this displacement were not considered in the design of the trusses, there would be, in any but the shortest spans, a noticeable sag at the midpoint of the span. In order to prevent any point of the lower chord falling below a line joining the two supports, the tension members are shortened by an amount equal to or greater than their elongation as computed with full live load on the bridge. Similarly,

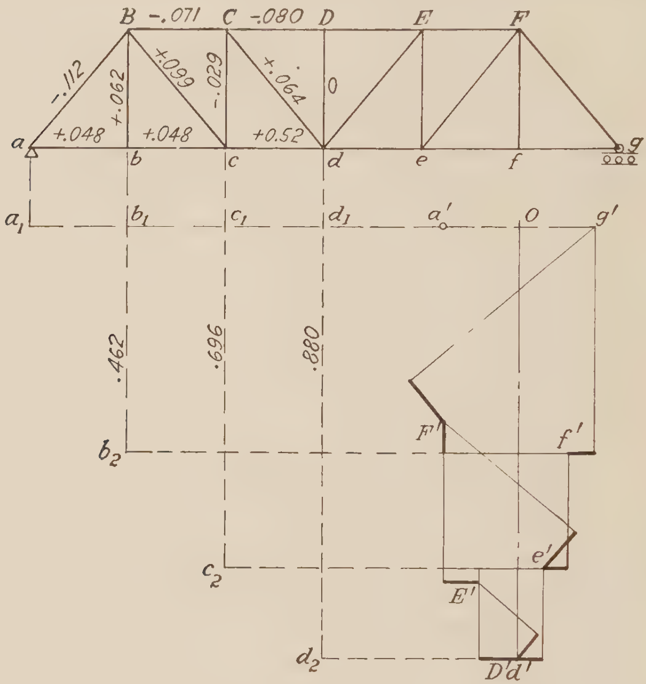


FIG. 214.

the compression members are lengthened a corresponding amount. Since full live load does not cause the maximum stresses in most of the web members of a simple truss, it is not necessarily the maximum stresses that are used in determining the various elongations and shortenings.

With the sections and lengths of the members known, together with their stresses under full live load, the change in length in each member may be computed by the equation given in Art.

131; and the displacement diagram constructed. Generally, the vertical components of the displacements of the lower chord panel points are all that are required. These components are the deflections.

Figure 214 shows the necessary construction to determine the deflections of the lower chord panel points of a six-panel through Pratt truss when the live load covers the entire span. The left support a is fixed while the right support g rests on a roller bearing and is free to move horizontally. The changes in the lengths of the several members are given on the truss diagram.

In constructing the displacement diagram, the direction of Dd is considered fixed. Since the stress in Dd is due to dead load only, and its amount inconsiderable, the shortening in Dd may be neglected; therefore, D' and d' coincide. In a symmetrical truss sustaining symmetrical load if the center member is chosen as the fixed member for the construction of the displacement diagram, the diagram itself is symmetrical about this member as was shown in Fig. 213(e). Therefore, for the truss under consideration it is necessary to construct only one-half of the displacement diagram in order to determine the deflection of the lower chord panel points. The construction for the left half is shown in Fig. 214.

The displacements are measured from a' ($a'O = Og'$) since g'' and a' coincide (see Art. 133) and a' is also $a''b''b''C''$, etc. The deflections or vertical components of the displacements for the panel points b, c, and d are easily determined graphically by drawing horizontal lines from a', f', c', and d' and scaling the distances b_1b_2, c_1c_2, and d_1d_2. Their values are given on the figure. The deflections of the symmetrical panel points on the right side of the truss are of the same amount.

INDEX

D

E

F

G

H

I

K

L

M

TABLE I

a		8′	5′	5′	5′	9′	5′	6′	5′	8′	8′	5′	5′	5′	9′	5′	6′	5′	5′
b		8′	13′	18′	23′	32′	37′	43′	48′	56′	64′	69′	74′	79′	88′	93′	99′	104′	109′
c	109′	101′	96′	91′	86′	77′	72′	66′	61′	53′	45′	40′	35′	30′	21′	16′	10′	5′	
d	15	30	30	30	30	19.5	19.5	19.5	19.5	15	30	30	30	30	19.5	19.5	19.5	19.5	
e	∠1	2	3	4	5	6	7	8	9	∠10	11	12	13	14	15	16	17	18	Uniform Load 3000 lb. per lin. ft.
f	15	45	75	105	135	154.5	174	193.5	213	228	258	288	318	348	367.5	387	406.5	426	
g	426	411	381	351	321	291	271.5	252	232.5	213	198	168	138	108	78	58.5	39	19.5	
h	0	120	345	720	1245	2460	3230	4280	5240	6950	8770	10060	11500	13090	16220	18060	20380	22420	24550
19	24550	22910	19880	17000	14270	11690	10190	8790	7500	6310	5510	4160	2970	1910	1010	605	290	97.5	
18	22420	20860	17980	15250	12670	10240	8830	7530	6340	5240	4520	3320	2280	1370	625	310	97.5		
17	20380	18900	16170	13590	11160	8880	7570	6360	5270	4280	3630	2580	1680	930	330	117			
16	18060	16670	14120	11720	9470	7370	6180	5090	4110	3230	2680	1810	1090	520	97.5				
15	16220	14900	12500	10260	8150	6200	5110	4120	3240	2460	1980	1260	690	270					
14	13090	11910	9780	7800	5970	4290	3370	2550	1850	1250	900	450	150						
13	11500	10390	8410	6580	4900	3370	2550	1830	1230	720	450	150							
12	10060	9030	7200	5520	3990	2510	1890	1260	755	345	150								
11	8770	7810	6130	4600	3220	1990	1370	840	430	120									
10	6950	6110	4670	3380	2240	1250	780	410	156										
9	5240	4520	3320	2270	1370	625	310	97.5											
8	4280	3630	2580	1680	930	330	117												
7	3230	2680	1810	1090	520	97.5													
6	2460	1980	1260	690	270														
5	1250	900	450	150															
4	720	450	150																
3	345	150																	
2	120																		

With a length of Uniform Load $= x$
$M = 24550 + 426x + 1.5x^2$
or not including wheel 1
$M = 22920 + 411x + 1.5x^2$

MOMENT TABLE
COOPER'S E-60 LOADING
TWO 213 TON ENGINES + 6000 LB. PER FOOT.
MOMENTS IN THOUSAND FOOT POUNDS FOR ONE RAIL.
LOADS IN THOUSANDS OF POUNDS FOR ONE RAIL.

TABLE II

MOMENT TABLE FOR COOPER'S E-60 LOADING.

WHEEL NO.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	UNIFORM LOAD 3000 LB. PER FT.
LOADS	15	30	30	30	30	19.5	19.5	19.5	19.5	15	30	30	30	30	19.5	19.5	19.5	19.5	
SPACING		8	5	5	5	9	5	6	5	8	8	5	5	5	9	5	6	5	5
DISTANCES	0	8	13	18	23	32	37	43	48	56	64	69	74	79	88	93	99	104	109
LOADS (1)	15	45	75	105	135	154.5	174	193.5	213	228	258	288	318	348	367.5	387	406.5	426	426
MOMENTS	0	120	345	720	1245	2460	3233	4277	5244	6948	8772	10062	11502	13092	16224	18062	20384	22416	24546
DISTANCES		0	5	10	15	24	29	35	40	48	56	61	66	71	80	85	91	96	101
LOADS (2)		30	60	90	120	139.5	159	178.5	198	213	243	273	303	333	352.5	372	391.5	411	411
MOMENTS		0	150	450	900	1980	2678	3632	4524	6108	7812	9027	10392	11907	14904	16667	18899	20856	22911
DISTANCES			0	5	10	19	24	30	35	43	51	56	61	66	75	80	86	91	96
LOADS (3)			30	60	90	109.5	129	148.5	168	183	213	243	273	303	322.5	342	361.5	381	381
MOMENTS			0	150	450	1260	1808	2582	3324	4668	6132	7197	8412	9777	12504	14117	16169	17976	19881
DISTANCES				0	5	14	19	25	30	38	46	51	56	61	70	75	81	86	91
LOADS (4)				30	60	79.5	99	118.5	138	153	183	213	243	273	292.5	312	331.5	351	351
MOMENTS				0	150	690	1088	1682	2274	3378	4602	5517	6582	7797	10254	11717	13589	15246	17001
DISTANCES					0	9	14	20	25	33	41	46	51	56	65	70	76	81	86
LOADS (5)					30	49.5	69	88.5	108	123	153	183	213	243	262.5	282	301.5	321	321
MOMENTS					0	270	518	932	1374	2238	3222	3987	4902	5967	8154	9467	11159	12666	14271
DISTANCES						0	5	11	16	24	32	37	42	47	56	61	67	72	77
LOADS (6)						19.5	39	58.5	78	93	123	153	183	213	232.5	252	271.5	291	291
MOMENTS						0	97.5	332	624	1248	1992	2607	3372	4287	6204	7367	8879	10236	11691
DISTANCES							0	6	11	19	27	32	37	42	51	56	62	67	72
LOADS (7)							19.5	39	58.5	73.5	103.5	133.5	163.5	193.5	213	232.5	252	271.5	271.5
MOMENTS							0	117	312	780	1368	1886	2553	3371	5112	6177	7572	8832	10190
DISTANCES								0	5	13	21	26	31	36	45	50	56	61	66
LOADS (8)								19.5	39	54	84	114	144	174	193.5	213	232.5	252	252
MOMENTS								0	97.5	410	842	1262	1832	2552	4118	5085	6363	7526	8786
DISTANCES									0	8	16	21	26	31	40	45	51	56	61
LOADS (9)									19.5	34.5	64.5	94.5	124.5	154.5	174	193.5	213	232.5	232.5
MOMENTS									0	156	432	755	1227	1850	3240	4110	5271	6336	7499
DISTANCES										0	8	13	18	23	32	37	43	48	53
LOADS (10)										15	45	75	105	135	154.5	174	193.5	213	213
MOMENTS										0	120	345	720	1245	2460	3233	4277	5244	6309

TWO 213 TON ENGINES + 6000 LB. PER FT.

MOMENTS IN THOUSAND FOOT POUNDS FOR ONE RAIL

LOADS IN THOUSANDS OF POUNDS FOR ONE RAIL

TABLE III

MOMENT TABLE FOR M-50 LOADING.

LOADS	12.5	25	25	25	25	25	31.25	31.25	31.25	31.25	31.25	UNIFORM LOAD 2500 LB. PER FT.									DISTANCES O-11 LOADS O-11 MOMENTS O-11 ABOUT O
SPACING	10	5	5	5	5	15	5	5	5	5	5										
WHEEL NO.	1	2	3	4	5	6	7	8	9	10	11										
DISTANCES	0	10	15	20	25	30	45	50	55	60	65	70	75	80	85	90	95	100	105	110	65
LOADS (1)	12.5	37.5	62.5	87.5	112.5	137.5	168.75	200	231.25	262.5	293.75	294	306	319	331	344	356	369	381	394	293.75
MOMENTS	0	125	313	625	1063	1625	3688	4531	5531	6688	8000	9469	10969	12531	14156	15844	17594	19406	21281	23219	11094
DISTANCES		0	5	10	15	20	35	40	45	50	55	60	65	70	75	80	85	90	95	100	55
LOADS (2)		25	50	75	100	125	156.25	187.5	218.75	250	281.25	281	294	306	319	331	344	356	369	381	281.25
MOMENTS		0	125	375	750	1250	3125	3906	4844	5938	7188	8594	10031	11531	13094	14719	16406	18156	19969	21844	8281
DISTANCES			0	5	10	15	30	35	40	45	50	55	60	65	70	75	80	85	90	95	50
LOADS (3)			25	50	75	100	131.25	162.5	193.75	225	256.25	256	269	281	294	306	319	331	344	356	256.25
MOMENTS			0	125	375	750	2250	2906	3719	4688	5813	7094	8406	9781	11219	12719	14281	15906	17594	19344	7000
DISTANCES				0	5	10	25	30	35	40	45	50	55	60	65	70	75	80	85	90	45
LOADS (4)				25	50	75	106.25	137.5	168.75	200	231.25	231	244	256	269	281	294	306	319	331	231.25
MOMENTS				0	125	375	1500	2031	2719	3563	4563	5719	6906	8156	9469	10844	12281	13781	15344	16969	5844
DISTANCES					0	5	20	25	30	35	40	45	50	55	60	65	70	75	80	85	40
LOADS (5)					25	50	81.25	112.5	143.75	175	206.25	206	219	231	244	256	269	281	294	306	206.25
MOMENTS					0	125	875	1281	1844	2563	3438	4469	5531	6656	7844	9094	10406	11781	13219	14719	4813
DISTANCES						0	15	20	25	30	35	40	45	50	55	60	65	70	75	80	35
LOADS (6)						25	56.25	87.5	118.75	150	181.25	181	194	206	219	231	244	256	269	281	181.25
MOMENTS						0	375	656	1094	1688	2438	3344	4281	5281	6344	7469	8656	9906	11219	12594	3906
DISTANCES							0	5	10	15	20	25	30	35	40	45	50	55	60	65	20
LOADS (7)							31.25	62.5	93.75	125	156.25	156	169	181	194	206	219	231	244	256	156.25
MOMENTS							0	156	469	938	1563	2344	3156	4031	4969	5969	7031	8156	9344	10594	1563
DISTANCES								0	5	10	15	20	25	30	35	40	45	50	55	60	15
LOADS (8)								31.25	62.5	93.75	125	125	138	150	163	175	188	200	213	225	125
MOMENTS								0	156	469	938	1563	2219	2338	3719	4563	5469	6438	7469	8563	938
DISTANCES									0	5	10	15	20	25	30	35	40	45	50	55	10
LOADS (9)									31.25	62.5	93.75	94	106	119	131	144	156	169	181	194	93.75
MOMENTS									0	156	469	938	1438	2000	2625	3313	4063	4875	5750	6688	469
DISTANCES										0	5	10	15	20	25	30	35	40	45	50	5
LOADS (10)										31.25	62.5	62.5	75	88	100	113	125	138	150	163	62.5
MOMENTS										0	156	469	813	1219	1688	2219	2813	3469	4188	4969	156

ONE $293\frac{3}{4}$ TON ENGINE + 5000 LB. PER FT.

MOMENTS IN THOUSAND FOOT POUNDS FOR ONE RAIL

LOADS IN THOUSANDS OF POUNDS FOR ONE RAIL

TABLE IV.—COOPER'S E 60 LOADING

Length of uniform load	Summations of Loads and Moments about the Right End of the Uniform Load, the Last Load to the Left Being Wheel Load																			
	1		2		3		4		5		6		7		8		9		10	
	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment	Load	Moment
5	441	26,714	426	25,004	396	21,824	366	18,794	336	15,914	306	13,184	286.5	11,585	267	10,083	247.5	8,699	228	7,412
10	456	28,956	441	27,171	411	23,841	381	20,661	351	17,631	321	14,751	301.5	13,055	282	11,456	262.5	9,974	243	8,589
15	471	31,274	456	29,414	426	25,934	396	22,604	366	19,424	336	16,394	316.5	14,600	297	12,903	277.5	11,324	258	9,842
20	486	33,666	471	31,731	441	28,101	411	24,621	381	21,291	351	18,111	331.5	16,220	312	14,426	292.5	12,749	273	11,169
25	501	36,134	486	34,124	456	30,344	426	26,714	396	23,234	366	19,904	346.5	17,915	327	16,023	307.5	14,249	288	12,572
30	516	38,676	501	36,591	471	32,661	441	28,881	411	25,251	381	21,881	361.5	19,685	342	17,696	322.5	15,824	303	14,049
35	531	41,294	516	39,134	486	35,054	456	31,124	426	27,344	396	23,714	376.5	21,530	357	19,443	337.5	17,474	318	15,602
40	546	43,986	531	41,751	501	37,521	471	33,441	441	29,511	411	25,731	391.5	23,450	372	21,266	352.5	19,199	333	17,229
45	561	46,754	546	44,444	516	40,064	486	35,834	456	31,754	426	27,824	406.5	25,445	387	23,163	367.5	20,999	348	18,932
50	576	49,596	561	47,211	531	42,681	501	38,301	471	34,071	441	29,991	421.5	27,515	402	25,136	382.5	22,874	363	20,709
55	591	52,514	576	50,054	546	45,374	516	40,844	486	36,464	456	32,234	436.5	29,660	417	27,183	397.5	24,824	378	22,562
60	606	55,506	591	52,971	561	48,141	531	43,461	501	38,931	471	34,551	451.5	31,880	432	29,306	412.5	26,849	393	24,489
65	621	58,574	606	55,964	576	50,984	546	46,154	516	41,474	486	36,944	466.5	34,175	447	31,503	427.5	28,949	408	26,492
70	636	61,716	621	59,031	591	53,901	561	48,921	531	44,091	501	39,411	481.5	36,545	462	33,776	442.5	31,124	423	28,569
75	651	64,934	636	62,174	606	56,894	576	51,764	546	46,784	516	41,954	496.5	38,990	477	36,123	457.5	33,374	438	30,722
80	666	68,226	651	65,391	621	59,961	591	54,681	561	49,551	531	44,571	511.5	41,510	492	38,545	472.5	35,699	453	32,949
85	681	71,594	666	68,684	636	63,104	606	57,674	576	52,394	546	47,264	526.5	44,105	507	41,043	487.5	38,099	468	35,252
90	696	75,036	681	72,051	651	66,321	621	60,741	591	55,311	561	50,031	541.5	46,775	522	43,616	502.5	40,574	483	37,629
95	711	78,554	696	75,494	666	69,614	636	63,884	606	58,304	576	52,874	556.5	49,520	537	46,263	517.5	43,124	498	40,082
100	726	82,146	711	79,011	681	72,981	651	67,101	621	61,371	591	55,791	571.5	52,340	552	48,986	532.5	45,749	513	42,609
105	741	85,814	726	82,604	696	76,424	666	70,394	636	63,884	606	58,784	586.5	55,235	567	51,783	547.5	48,449	528	45,212
110	756	89,556	741	86,271	711	79,941	681	73,761	651	67,731	621	61,851	601.5	58,205	582	54,656	562.5	51,224	543	47,889
115	771	93,374	756	90,014	726	83,534	696	77,204	666	71,024	636	64,994	616.5	61,250	597	57,603	577.5	54,074	558	50,642
120	786	97,266	771	93,831	741	87,201	711	80,721	681	74,391	651	68,211	631.5	64,370	612	60,626	592.5	56,999	573	53,469
125	801	101,234	786	97,724	756	90,944	726	84,314	696	77,834	666	71,504	646.5	67,565	627	63,723	607.5	59,999	588	56,372
130	816	105,276	801	101,691	771	94,7[illegible]	741	87,981	711	81,351	681	74,871	661.5	70,835	642	66,896	622.5	63,074	603	59,349
135	831	109,394	816	105,734	786	98,654	756	91,7[illegible]4	726	84,944	696	78,314	676.5	74,180	657	70,143	637.5	66,224	618	62,402
140	846	113,586	831	109,851	801	102,621	771	95,541	741	88,611	711	81,831	691.5	77,600	672	73,466	652.5	69,449	633	65,529
145	861	117,854	846	114,044	816	106,664	786	99,434	756	92,354	726	85,424	706.5	81,095	687	76,863	667.5	72,749	648	68,732
150	876	122,196	861	118,[illegible]11	831	110,781	801	103,401	771	96,171	741	89,091	721.5	84,665	702	80,336	682.5	76,214	663	72,009